Appendix

Extension of $\Omega$ to a normalized metric space

Let $\mathcal{A} := \{A_1, \ldots, A_q\}$ be a set of not necessarily ordered domains and define $\Omega := \bigotimes_{i=1}^q A_i := \{(a_1, \ldots, a_q)^T \mid a_i \in A_i, i = 1, \ldots, q\}$. Further let $V \subset \Omega$ be any finite subset of $\Omega$.

We suppose that any attribute $A_i$ is finite or at least bounded. Otherwise we replace it by $A_i(V) := \{x \in A_i \mid (\exists v = (v_{*,1}, \ldots, v_{*,q})^T \in V) v_{*,i} = x\}$. We define an unique projection $\pi$ from $\Omega$ into a normalized metric space, as follows:

1. Let $A_i$ any attribute of $\Omega$ with $A_i = \{x_{i,1}, \ldots, x_{i,m_i}\} \not\subseteq \mathbb{R}$, $m_i \in \mathbb{N}$. For $1 \leq j \leq m_i$ set $A_{i,j} := \{0, 1\}$ and define $\pi_i : A_i \longrightarrow \bigotimes_{j=1}^{m_i} A_{i,j} \subset \mathbb{R}^{m_i}$ via

$$\pi_i(x_{i,j}) := (\delta_{i,1}, \ldots, \delta_{i,m_i})^T \text{ for } j = 1, \ldots, m_i$$

with

$$\delta_{i,j} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{else.} \end{cases}$$

2. Let $A_i$ any attribute of $\Omega$ with $A_i \subset [l_i, r_i] \subset \mathbb{R}$ and $l_i, r_i \in \mathbb{R}$. Set $A_{i,1} := [0, 1]$, $m_i := 1$ and define $\pi_i : A_i \longrightarrow A_{i,1} \subset \mathbb{R}$ via

$$\pi_i(x) := \frac{x - l_i}{r_i - l_i} \text{ for } x \in A_i.$$

Then $\pi := (\pi_1, \ldots, \pi_q)^T$ is a projection from $\Omega$ into a $\hat{q} := \sum_{i=1}^q m_i$ dimensional normalized subspace $\Omega_{\mathbb{R}} := \bigotimes_{i=1}^q \bigotimes_{j=1}^{m_i} A_{i,j} \subset \mathbb{R}^{\hat{q}}$.

Obviously we have: $\pi(v) = \pi(w) \iff v = w$ for all $v, w \in \Omega$. 