Appendix

Extension of Ω to a normalized metric space

Let $\mathcal{A} := \{A_1, \ldots, A_q\}$ be a set of not necessarily ordered domains and define $\Omega := \bigotimes_{i=1}^{q} A_i := \{(a_1, \ldots, a_q)^T \,|\, a_i \in A_i\,, i = 1, \ldots, q\}$. Further let $V \subset \Omega$ be any finite subset of Ω.

We suppose that any attribute A_i is finite or at least bounded. Otherwise we replace it by $A_i(V) := \{x \in A_i \,|\, (\exists\, v = (v_{*,1}, \ldots, v_{*,q})^T \in V)\, v_{*,i} = x\}$. We define an unique projection π from Ω into a normalized metric space, as follows:

1. Let A_i any attribute of Ω with $A_i = \{x_{i,1}, \ldots, x_{i,m_i}\} \not\subseteq \mathbf{R}$, $m_i \in \mathbf{N}$. For $1 \leq j \leq m_i$ set $A_{i,j} := \{0, 1\}$ and define $\pi_i : A_i \longrightarrow \bigotimes_{j=1}^{m_i} A_{i,j} \subset \mathbf{R}^{m_i}$ via

   $$\pi_i(x_{i,j}) := (\delta_{i,1}, \ldots, \delta_{i,m_i})^T \;\text{ for } j = 1, \ldots, m_i$$

   with

   $$\delta_{i,j} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{else.} \end{cases}$$

2. Let A_i any attribute of Ω with $A_i \subset [l_i, r_i] \subset \mathbf{R}$ and $l_i, r_i \in \mathbf{R}$. Set $A_{i,1} := [0, 1]$, $m_i := 1$ and define $\pi_i : A_i \longrightarrow A_{i,1} \subset \mathbf{R}$ via

   $$\pi_i(x) := \frac{x - l_i}{r_i - l_i} \;\text{ for } x \in A_i.$$

Then $\pi := (\pi_1, \ldots, \pi_q)^T$ is a projection from Ω into a $\dot{q} := \sum_{i=1}^{q} m_i$ dimensional normalized subspace $\Omega_{\mathbf{R}} := \bigotimes_{i=1}^{q} \bigotimes_{j=1}^{m_i} A_{i,j} \subset \mathbf{R}^{\dot{q}}$.

Obviously we have: $\pi(v) = \pi(w) \iff v = w$ for all $v, w \in \Omega$.