Chapter 2

Introduction to polarization of light

This Chapter treats the polarization of electromagnetic waves. In Section 2.1 the concept of light polarization is discussed and its Jones formalism is presented. Next, Section 2.2 shows the coupling between the Jones vector and the parameters of polarization ellipses, its orientation, and the ratio of the major and minor axes. The last Section 2.3 analyzes the interaction of a plane linear polarized wave with the liquid crystal used in spatial light modulators.

2.1 Description of polarization by a Jones vector

The polarization of light is due to the orientation of the electrical field vector. Since an electromagnetical wave is a three dimensional object, it can be described by superposition of its two orthogonal components orientated in a plane perpendicular to the propagation direction [56].

$$E_x = E_{0x}e^{i(kz-\omega t+\varphi_x)}$$
$$E_y = E_{0y}e^{i(kz-\omega t+\varphi_y)} \tag{2.1}$$

These components are oscillating in time with the same frequency, however the amplitudes and phases may differ. Eq. 2.1 can be rewritten in a vector form with the fast oscillating term separated from the both components

$$\begin{aligned} \mathbf{E} &= [\mathbf{x}E_{0x}e^{i\varphi_x} + \mathbf{y}E_{0y}e^{i\varphi_y}]e^{i(kz-\omega t+\varphi_y)} \\ &= \widetilde{\mathbf{E}}_0 e^{i(kz-\omega t+\varphi_y)} \end{aligned} \tag{2.2}$$

where

$$\widetilde{\mathbf{E}}_0 = [\mathbf{x}E_{0x}e^{i\varphi_x} + \mathbf{y}E_{0y}e^{i\varphi_y}]. \tag{2.3}$$

The expression in the bracket is no longer dependent from the fast oscillating component, but from phase shifts and amplitudes. It can be expressed in form of a vector.

$$\widetilde{\mathbf{E}}_0 = \begin{bmatrix} E_{0x}e^{i\varphi_x} \\ E_{0y}e^{i\varphi_y} \end{bmatrix} \tag{2.4}$$

The vector 2.4 is called Jones vector and reflects the polarization state. Since the relative amplitudes and phases fully determine the state of polarization, the Jones vector is a complete description of it. To comprehend the physical meaning of complex amplitudes one can consider Eq. 2.1 with $z = 0$, $\varphi_x = 0$, $\varphi_y = \epsilon$ and take the real part of it.

$$E_x = E_{0x}\cos(\omega t)$$

$$E_y = E_{0y}\cos(\omega t + \epsilon) \tag{2.5}$$

This has the form of restricted Lissajous figures with ϵ as the relative phase between the components. The restriction is due to the same factor ω placed in front of the argument t, so in consequence, three possible types of figures are allowed. A linear polarization is obtained if the relative phase is set to $0° \pm 180°$ where $n \in \mathbb{Z}$. An elliptical polarization will result for any other set of amplitudes and relative phases, including a circular case, when the amplitudes are equal and the relative phase is $90° \pm 180°$. Figure 2.1 illustrates the influence of the phase difference on the resulting Lissajous figures for fixed, nonequal amplitudes.

It is interesting to notice that for given amplitudes one can get a linear polarization in two directions depending on the phase difference set to $0°$ or $180°$, as it is shown in Figure 2.1. The difference of $180°$ in phase means that instead of a maximum of an electric field, a minimum is encountered,
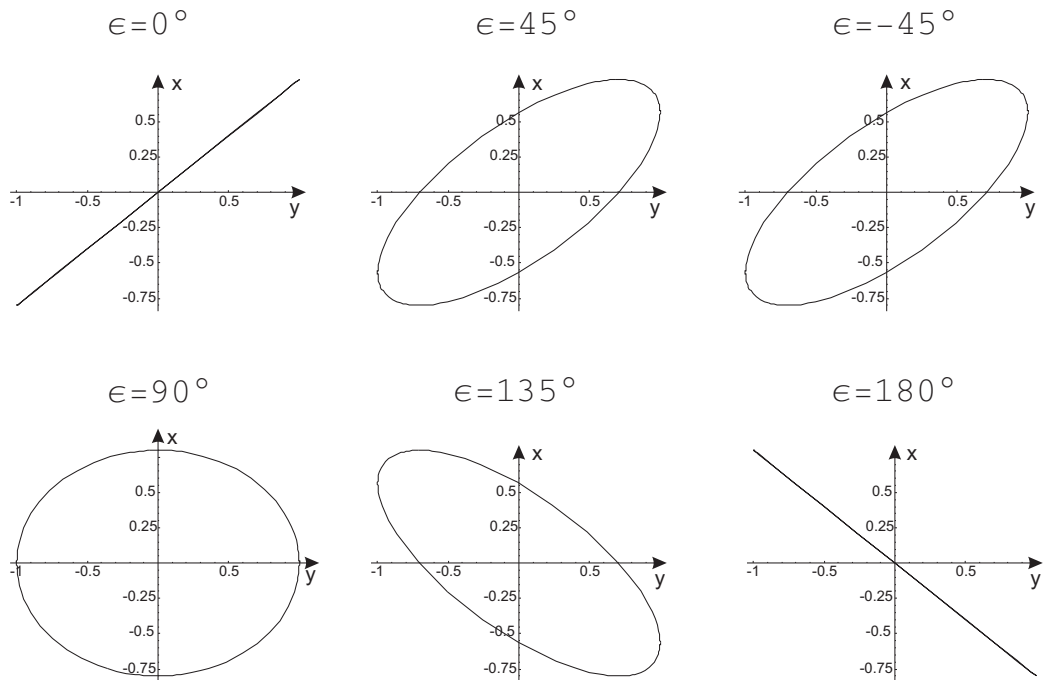
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Figure 2.1: Example Lissajous figures plotted for various relative phase shifts ϵ and an amplitude ratio of 0.75. As indicated on top of the graphs, in the upper row, from left $\epsilon = 0°$ is plotted, which corresponds to linear polarization. Next, two plots are shown for $\epsilon = 45°$ and $\epsilon = -45°$ with the same tilted ellipse in both cases. The first plot in the lower row presents an ellipse with the major axis aligned along the X axis of the coordinate system as a result of $\epsilon = 90°$. The following plot shows the ellipse similar to the one for $\epsilon = -45°$ but with different orientation. The last plot shows the second possible linear polarization for the fixed amplitudes.
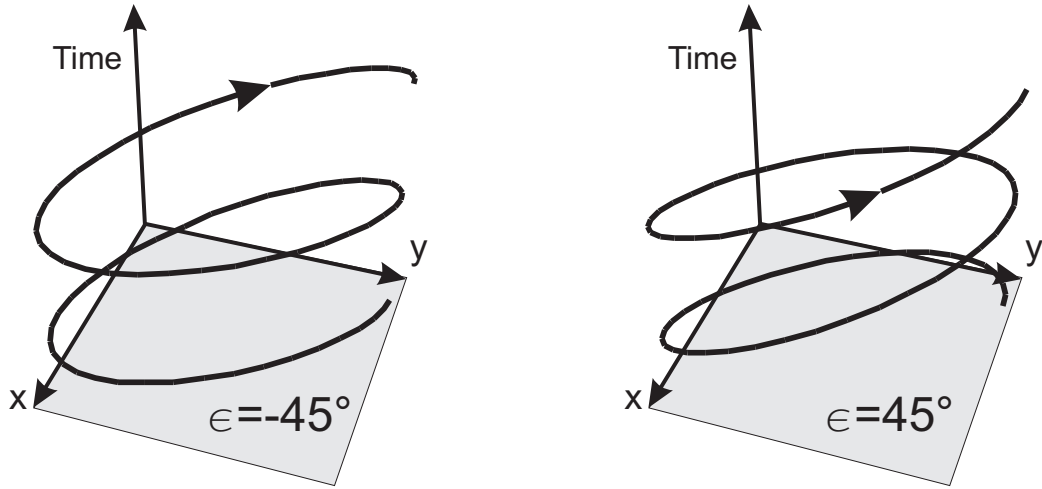
Figure 2.2: 3D Lissajous figures plotted for negative (upper graph, $\epsilon = -45°$) and positive (bottom graph, $\epsilon = 45°$). As depicted, the negative ϵ results in a clockwise rotation of the electrical field vector (right hand helicity) and the positive ϵ in counterclockwise rotation (left hand helicity) in time.

which results in a change of the direction of the electrical field vector. For the general case of an ellipse, a 180° change of relative phase will mirror the figure with the symmetry axis along one of the polarization component axis, as it is for the cases $\epsilon = -45°$ and $\epsilon = 135°$.

Another interesting property is that there is no alternation of the Lissajous figure if the sign of the phase shift is changed. This is not exactly true in terms of polarization of an electromagnetical wave. Adding a time axis to the Lissajous figures helps to clarify the relevance of the phase sign.

$$x = A_x \cos(\omega t)$$
$$y = A_y \cos(\omega t + \epsilon)$$
$$t = t \tag{2.6}$$

The parametrical equations 2.6 are plotted in Figure 2.2. For simplicity, the amplitudes A_x and A_y are chosen to be equal. The influence of the relative phase sign is apparent in Figure 2.2. The upper plot represents the Lissajous figure where ϵ is equal to $-45°$. The negative shift results in clockwise rotation of the vector of the electrical field and the positive shift results in a counterclockwise rotation.

There is a more general way to prove the direction of the rotation. One can transform Eq. 2.5 to the cylindrical coordinates, where

$$
\begin{aligned}
r &= \sqrt{E_x{}^2 + E_y{}^2} \\
\tan(\psi) &= \frac{E_x}{E_y} \\
z &= t
\end{aligned}
$$

and then one can directly see the behavior of the electrical field vector.

$$
\psi(t, \epsilon) = \arctan\left(\frac{E_{0x}}{E_{0y}} \frac{\cos(\omega t)}{\cos(\omega t + \epsilon)}\right) \tag{2.7}
$$

Plotting the orientation of the electrical vector ψ as a function of phase shift ϵ and time shows the regions where ψ is rising with time, (change from black to white) and where it is falling. It is illustrated as well, by the right insets in Figure 2.3 on which one can see the ψ function plot for a fixed ϵ equal $45°$ and $-45°$. This example proves that indeed a positive phase shift between the polarization components corresponds to counterclockwise rotation and negative relative phase to clockwise rotation of the electrical field vector. The plot is consistent with the obvious fact described by Eq. 2.7, that setting the phase shift to $0°\pm180°$ will give no rotation and therefore yield the linear polarization.

The transition from the Lissajous figure picture to the Jones vector formalism is quite apparent. The short summary of Jones vectors describing a few selected polarizations is presented in Table 2.1.

2.2   Parametrization of the polarization ellipse

In this Section it is explained how to calculate the resulting Jones vector from the polarization ellipse with desired major and minor axes and orientation. The calculation is based on comparing two ellipse equations, one obtained from the Lissajous figures described by Eq. 2.5 and a rotated Cartesian ellipse equation.

Obtaining the first equation is done by eliminating the fast oscillating factor ωt from the Eq. 2.5.
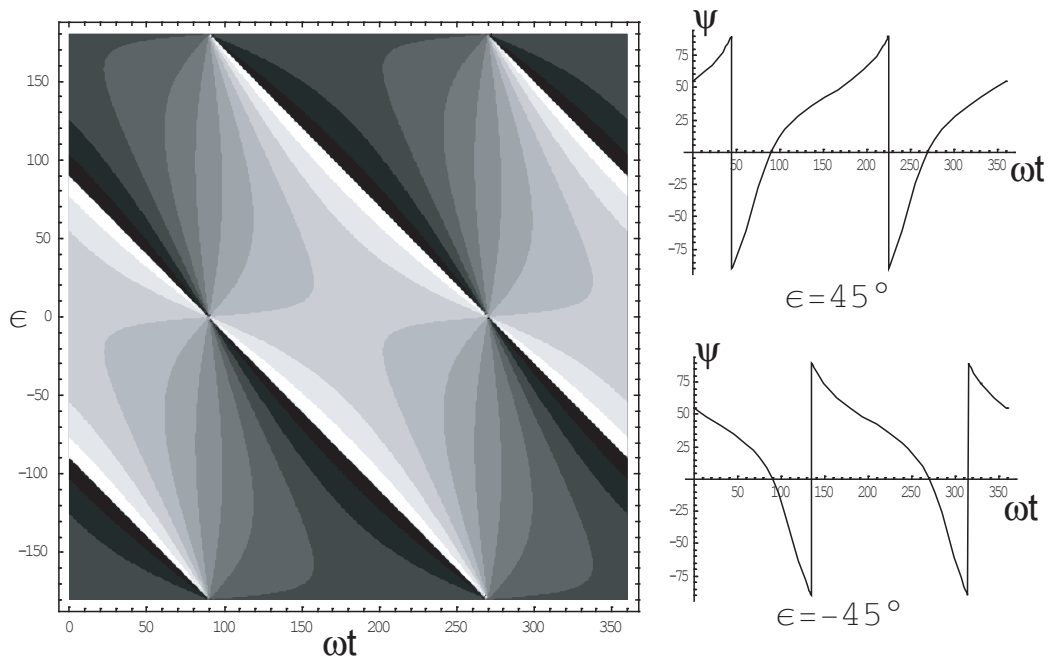
Figure 2.3: The left side of the Figure shows the contour plot of the ψ function. The relative phase ϵ is on the vertical axis and the horizontal axis is proportional to time multiplied by the factor ω. The shades of gray indicate $\psi(t, \epsilon)$ values in a way that the change from dark to light colors depict positive change and opposite, negative change. Two graphs on the right side present the plot of $\psi(t)$ for ϵ equal 45° and −45° and they can be regarded as horizontal cuts through the contour plot.

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Table 2.1: Jones vectors

Polarization	Jones vector
linear, general	$\begin{bmatrix} \sin(\alpha) \\ \cos(\alpha) \end{bmatrix}$
linear, vertical (S)	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
linear, horizontal (P)	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
circular, ($+i$ counterclockwise, $-i$ clockwise)	$\begin{bmatrix} 1 \\ \pm i \end{bmatrix}$
elliptical, principal axes parallel to x,y axes	$\begin{bmatrix} A \\ \pm iB \end{bmatrix}$
elliptical, general	$\begin{bmatrix} A \\ B \pm iC \end{bmatrix}$

$$\frac{E_x}{E_{0x}} = \cos(\omega t) \tag{2.8}$$

$$\frac{E_y}{E_{0y}} = \cos(\omega t + \epsilon)$$

$$= \cos(\omega t)\cos(\epsilon) - \sin(\omega t)\sin(\epsilon)$$

Thus

$$\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}}\cos(\epsilon) = -\sin(\omega t)\sin(\epsilon). \tag{2.9}$$

Since from Eq. 2.8 one can obtain

$$\sin(\omega t) = \sqrt{1 - \frac{E_x}{E_{0x}}^2}, \tag{2.10}$$

the Eq. 2.9 is reformulated into

$$\frac{1}{\sin^2(\epsilon)}\left[\left[\frac{E_x}{E_{0x}}\right]^2 + \left[\frac{E_y}{E_{0y}}\right]^2 - 2\frac{E_x}{E_{0x}}\frac{E_y}{E_{0y}}\cos(\epsilon)\right] = 1 \tag{2.11}$$

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which represents the ellipse in the same way as the Lissajous curves presented in the previous Section [56]. The free parameters of the equation are the amplitudes E_{0x} and E_{0y} and the relative phase shift ϵ, which are exactly the same parameters, that are used in the Jones vector formalism.

The Cartesian ellipse equation is given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \, . \tag{2.12}$$

It describes an ellipse with principal axes a and b, with the orientation of the principal axes fixed. In order to vary the orientation one can transform this ellipse to rotated Cartesian coordinates. This yields a rotated ellipse equation with the rotation angle γ.

$$\frac{(x\cos(\gamma) + y\sin(\gamma))^2}{a^2} + \frac{(x\cos(\gamma) - y\sin(\gamma))^2}{b^2} = 1 \tag{2.13}$$

Both equations, 2.11 and 2.13, describe the same ellipse, one in basis of the electric field components E_x and E_y, and the second in the Cartesian coordinates x and y. Therefore the corresponding coefficients ($E_x \Leftrightarrow x, E_y \Leftrightarrow y$) are equal.

$$\frac{1}{E_{0x}{}^2 \sin^2(\epsilon)} = \frac{\cos^2(\gamma)}{a^2} + \frac{\sin^2(\gamma)}{b^2}$$

$$\frac{1}{E_{0y}{}^2 \sin^2(\epsilon)} = \frac{\sin^2(\gamma)}{a^2} + \frac{\cos^2(\gamma)}{b^2}$$

$$\frac{\cos(\epsilon)}{\sin^2(\epsilon)} \frac{1}{E_{0x}E_{0y}} = \frac{\sin(\gamma)\cos(\gamma)}{b^2} - \frac{\sin(\gamma)\cos(\gamma)}{a^2} \tag{2.14}$$

This provides the coupling of the principal axes a and b of an ellipse, the orientation γ with the amplitudes E_{0x} and E_{0y}, and the relative phase ϵ. Depending on the requirements, Eq. 2.14 can be used to solve two inverse problems. The first asks which polarization ellipse (a, b, γ) is generated with the given electric field (E_{0x}, E_{0y}, ϵ). The inverse problem is what electric field is needed to obtain the desired polarization state, which seems to be much more interesting, since one would like to control the polarization state by choosing proper electric field amplitudes E_{0x} and E_{0y} and the relative phase ϵ. The solution is

$$E_{0x}(a, b, \gamma) = \sqrt{\frac{1}{2}(a^2 + b^2 + (a^2 - b^2)\cos(2\gamma))}$$

$$E_{0y}(a, b, \gamma) = \sqrt{\frac{1}{2}(a^2 + b^2 + (b^2 - a^2)\cos(2\gamma))}$$

$$\epsilon(a, b, \gamma) = \frac{\sin(2\gamma)}{|\sin(2\gamma)|} \arccos \sqrt{\frac{(a^2 + b^2)^2}{(a^4 + b^4 + a^2 b^2(\cot^2(\gamma) + \tan^2(\gamma)))}} \quad (2.15)$$

which describe E_{0x}, E_{0y}, and ϵ as a function of the principal axes and the orientation of the polarization ellipse.

This description of an ellipse is not very useful in terms of light polarization since a change in the major axis results in a change of the polarization state of an electromagnetic wave together with the intensity. Other ways to parameterize an ellipse are better and more intuitive in this case. Instead of this parametrization one can manipulate the ratio of the principal axes r and the intensity independently. Substituting

$$r = \frac{b}{a}$$

$$I = a^2 + b^2 \quad (2.16)$$

the relations 2.15 take the form:

$$E_{0x}(I, r, \gamma) = \sqrt{\frac{1}{2}I\left[1 - \frac{(r^2 - 1)}{(r^2 + 1)}\cos(2\gamma)\right]}$$

$$E_{0y}(I, r, \gamma) = \sqrt{\frac{1}{2}I\left[1 + \frac{(r^2 - 1)}{(r^2 + 1)}\cos(2\gamma)\right]}$$

$$\epsilon(I, r, \gamma) = \frac{\sin(2\gamma)}{|\sin(2\gamma)|} \arccos \sqrt{\frac{(r^2 - 1)^2}{(1 + r^4 + r^2(\cot^2(\gamma) + \tan^2(\gamma)))}} \quad (2.17)$$

One can see that the parameter I is the intensity since

$$E_{0x}{}^2 + E_{0y}{}^2 = I$$

is fulfilled.

Applying relations 2.17 allows to change the polarization state parameters like the ratio of principal axes and orientation in an independent way by

choosing the suitable electrical fields. This relation could as well be used to calculate the Jones vectors for given polarization states.

2.3   Jones matrices for the phase retarder and the nematic crystal modulator

A number of several optical elements influence the orientation of the polarization. The nature of their interaction with an optical wave allows to categorize those devices into three classes: polarizers, phase retarders, and rotators.

The transmission (or reflectivity) of a polarizer depends very strongly on the relative orientation of the polarizer and the incoming polarization of the light. In other words, a horizontally orientated polarizer will transmit all of the horizontally polarized light and none of the vertically polarized.

Phase retarders do not cut out any of the polarization components, instead of that they introduce a phase retardation between them. As it was explained in the Section before, by changing the relative phase between the components an restrained control over the polarization can be achieved.

The last type, a rotator, turns the incident light polarization by some predefined angle. The rotators usually take the advantage of a property of optically active materials like quartz, or they rotate the whole beam spatially, so unlike retarders, the angle of rotation is not dependent from the incident polarization. Apart from these categories one has to remember that the reflectivity of every high reflective mirror design for a nonzero incidence angle is polarization sensitive.

An effect of such elements on a light wave can be described in a similar way as the polarization state. The matrices that are multiplied by the Jones vector result in a vector describing the polarization state after the element are called Jones matrices. There are found by solving Eq. 2.18.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} = \begin{bmatrix} x_i' \\ y_i' \end{bmatrix} \tag{2.18}$$

Therefore, if the influence on the incident polarizations $\begin{bmatrix} x_i \\ y_i \end{bmatrix}$ is known, the

Table 2.2: Jones matrix

Component	Jones matrix
Horizontal (P) polarizer [PP]	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
Vertical (S) polarizer [PS]	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
Phase retarder	$\begin{bmatrix} e^{i\epsilon_x} & 0 \\ 0 & e^{i\epsilon_y} \end{bmatrix}$
Rotator	$\begin{bmatrix} \cos(\beta) & -\sin(\beta) \\ \sin(\beta) & \cos(\beta) \end{bmatrix}$

corresponding Jones matrix can be found. A set of Jones matrices for a few elements is presented in Table 2.2.

Let us consider a phase retarder, which is a plate cut from a birefringent crystal. The light traveling along the phase retarder will undergo the double refraction. More precisely, there is a slow and a fast optical axis. The electrical field oscillating along the fast axis will experience less retardation than in the case when it oscillates along the perpendicular, slow axis. The retardation of the fast and the slow axis are ϵ_x and ϵ_y, respectively, and when the difference between them is 180°, the retarder is called a half waveplate and in case of 90° a quarter waveplate. A half wave plate in a beam of linearly polarized light rotates the polarization by the angle given by twice the difference between the fast axis and the orientation of the polarization. To clarify the mechanism, one can imagine linear polarization of light as a superposition of two waves perpendicularly polarized along the fast and the slow axis of a waveplate. Introducing phase shift of a half wave (180°) will result in a change of polarization. The angle of rotation depends on the amplitudes of these components along the waveplate axes. Applying the Jones formalism, Eq. 2.19 shows the effect of a half waveplate with the fast

axis orientated parallel to the table (0°) on a parallel (P) polarized light.

$$\begin{bmatrix} e^{-i*\pi/2} & 0 \\ 0 & e^{i*\pi/2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix} \tag{2.19}$$

The outcome is a vector representing exactly the same P polarization multiplied by a factor $-i$ which is due to the phase shift of 180° experienced by passing the plate. This example shows that if the fast axis is parallel to the polarization no rotation occurs. It is not the case for the rotated half waveplate. Corresponding Jones matrices for rotated half waveplate $HW[\beta]$ and quarter waveplate $QW[\beta]$ can be found by applying the rotation operator on a waveplate.

$$HW[\beta] = \begin{bmatrix} \cos[\beta] & -\sin[\beta] \\ \sin[\beta] & \cos[\beta] \end{bmatrix} \begin{bmatrix} e^{-i*\pi/2} & 0 \\ 0 & e^{i*\pi/2} \end{bmatrix} \begin{bmatrix} \cos[-\beta] & -\sin[-\beta] \\ \sin[-\beta] & \cos[-\beta] \end{bmatrix}$$

$$QW[\beta] = \begin{bmatrix} \cos[\beta] & -\sin[\beta] \\ \sin[\beta] & \cos[\beta] \end{bmatrix} \begin{bmatrix} e^{-i*\pi/4} & 0 \\ 0 & e^{i*\pi/4} \end{bmatrix} \begin{bmatrix} \cos[-\beta] & -\sin[-\beta] \\ \sin[-\beta] & \cos[-\beta] \end{bmatrix} \tag{2.20}$$

where β is the angle of rotation.

$$HW[45°]P = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -i \end{bmatrix} \tag{2.21}$$

Multiplying this matrix by P polarization has the effect of rotating it 90° of the plane of the electrical field oscillation, as expected.

Waveplates and nematic liquid crystals used in a spatial light modulator are very similar in the way of interacting with the electromagnetical waves since the crystals are birefringent as well. The crucial difference between liquid crystals and waveplates is, that an extraordinary index of refraction of the nematic crystals can be changed in a controlled way by applying a voltage [37, 57, 58]. This is why one can regard these crystals as waveplates with a variable phase retardance. In order to describe and analyze this phenomenon let us first define a few matrices for simplifying the calculations.

The rotation matrix as a function of the rotation angle is given by Eq. 2.22 as

$$R[\beta] = \begin{bmatrix} cos[\beta] & -sin[\beta] \\ sin[\beta] & cos[\beta] \end{bmatrix}. \tag{2.22}$$

For a nematic crystal with the extraordinary axis orientated parallel to the table one finds

$$LC[\phi] = \begin{bmatrix} e^{\frac{1}{2}i\phi} & 0 \\ 0 & e^{-\frac{1}{2}i\phi} \end{bmatrix} e^{\frac{1}{2}i\phi} \tag{2.23}$$

where ϕ denotes the retardation or the phase shift difference between the fast and the slow axis in the crystal (difference retardance), which is controlled by an external voltage. Having these Jones matrices we are able to show the interaction of light with crystals for a single pixel in the modulator and in this way analyze shaping in frequency domain.

It is important to point out that it is very common to use P polarized light in pulse shapers, while the efficiency of the incorporated gratings is much higher, so from this point on we will assume the incoming light polarization as P. As the example of a waveplate orientated parallel to the polarization shows, for crystals with parallel axes only the phase alteration will occur.

$$LC[\phi] \cdot E_0 e^{i\omega t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = E_0 e^{i\omega t} e^{i\phi} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{2.24}$$

Therefore this structure of the crystals is used in single array spatial light modulators to influence the phase of the pulse. As exhibited, for the case of an incoming P polarized light, the amplitude and the polarization state remains unchanged but the phase is altered. It is relatively easy to demonstrate that for polarizations perpendicular to the crystal optical axis the phase as well as the polarization will remain unaltered. The optical axes of the crystals have to be rotated in order to influence the phase and the polarization state. The appropriate matrix of this confinement is calculated in 2.25.

$$R[45°] \cdot LC[\phi] \cdot R[-45°] = e^{\frac{1}{2}i\phi} \begin{bmatrix} \cos[\frac{\phi}{2}] & i\sin[\frac{\phi}{2}] \\ i\sin[\frac{\phi}{2}] & \cos[\frac{\phi}{2}] \end{bmatrix} \tag{2.25}$$

The electric field after such a layer is found by multiplying matrix 2.25 by a P polarized electric field.

$$e^{\frac{1}{2}i\phi} \begin{bmatrix} \cos[\frac{\phi}{2}] & i\sin[\frac{\phi}{2}] \\ i\sin[\frac{\phi}{2}] & \cos[\frac{\phi}{2}] \end{bmatrix} \cdot E_0 e^{i\omega t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = E_0 e^{i\omega t} e^{\frac{1}{2}i\phi} \begin{bmatrix} \cos[\frac{\phi}{2}] \\ i\sin[\frac{\phi}{2}] \end{bmatrix} \tag{2.26}$$
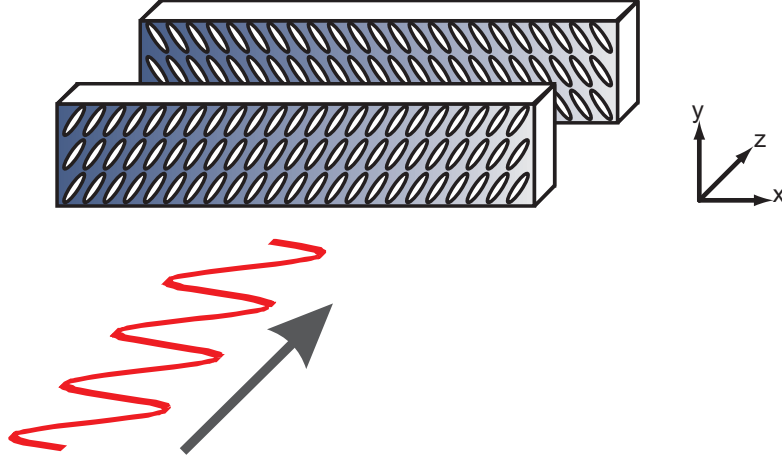
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Figure 2.4: Arrangement of two arrays of crystal. Optical axes are orientated at 45° in the first array and −45° in the second.

For a single array of nematic crystals orientated 45° with regard to the incoming polarization, the change of the phase is inseparably coupled with the polarization state. When two layers of crystals rotated 45° in a first row (b) and −45° in the second (a), are combined as shown in Figure 2.4, then it is possible to influence the phase and polarization separately.

Analyzing the Jones matrices in the same matter, as for a single array, gives the Jones matrix of such a modulator.

$$R[-45°] \cdot LC[\phi_b] \cdot R[45°] \cdot R[45°] \cdot LC[\phi_a] \cdot R[-45°]$$

$$= e^{\frac{1}{2}i(\phi_a+\phi_b)} \begin{bmatrix} \cos[\frac{(\phi_a-\phi_b)}{2}] & i\sin[\frac{(\phi_a-\phi_b)}{2}] \\ i\sin[\frac{(\phi_a-\phi_b)}{2}] & \cos[\frac{(\phi_a-\phi_b)}{2}] \end{bmatrix} \quad (2.27)$$

Next, the electrical field is calculated as in the case above.

$$e^{\frac{1}{2}i(\phi_a+\phi_b)} \begin{bmatrix} \cos[\frac{(\phi_a-\phi_b)}{2}] & i\sin[\frac{(\phi_a-\phi_b)}{2}] \\ i\sin[\frac{(\phi_a-\phi_b)}{2}] & \cos[\frac{(\phi_a-\phi_b)}{2}] \end{bmatrix} \cdot E_0 e^{i\omega t} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= E_0 e^{i\omega t} e^{\frac{1}{2}i(\phi_a+\phi_b)} \begin{bmatrix} \cos[\frac{\phi_a-\phi_b}{2}] \\ i\sin[\frac{\phi_a-\phi_b}{2}] \end{bmatrix} \quad (2.28)$$

This solution includes the Jones vector identifying the polarization and phase factor. Since one can independently handle the difference of the phase shifts $\phi_a - \phi_b$, and the sum of them $\phi_a + \phi_b$, autonomous manipulation of

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polarization and phase is possible. It is important at this point to stress that the Jones vector in the above described cases takes a form of $\begin{bmatrix} A \\ \pm iB \end{bmatrix}$ and as shown in the Table 2.1, it corresponds to a restricted elliptical polarization with the major axis parallel or perpendicular to the incoming polarization. In terms of polarization manipulation, this solution cannot provide the full range of possibilities since it is not capable of simply turning the linear polarization. By tuning $\phi_a - \phi_b$ one can only change the ratio of the principal axes.

Let us consider cutting off the S polarization component of the laser field described by the equation 2.28. It is experimentally accomplished by adding a P adjusted polarizer after the modulator crystals.

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \cdot E_0 e^{i\omega t} e^{\frac{1}{2} i(\phi_a + \phi_b)} \begin{bmatrix} \cos\left[\frac{(\phi_a - \phi_b)}{2}\right] \\ i\sin\left[\frac{(\phi_a - \phi_b)}{2}\right] \end{bmatrix} = E_0 e^{i\omega t} e^{\frac{1}{2} i(\phi_a + \phi_b)} \begin{bmatrix} \cos\left[\frac{(\phi_a - \phi_b)}{2}\right] \\ 0 \end{bmatrix}$$

$$(2.29)$$

Since there is only one polarization component present, modulating the difference of phase shifts results in a change of the amplitude of the transmitted light. The corresponding electrical field will take the form

$$E_{ph+amp} = E_0 e^{i\omega t} e^{\frac{1}{2} i(\phi_a + \phi_b)} \cos\left[\frac{(\phi_a - \phi_b)}{2}\right]. \tag{2.30}$$

The transmission and phase shift introduced by such a setup is given by Eq. 2.31

$$T = \cos^2\left[\frac{(\phi_a - \phi_b)}{2}\right]$$

$$\varphi_{ph+amp} = \frac{1}{2}(\phi_a + \phi_b) \tag{2.31}$$

Therefore combining a double layer modulator with a polarizer is a most common way to design an independent phase and amplitude modulator.

2.4 Summary

In the first Section of this Chapter the light polarization was discussed. The analogy between the Lissajous curves and the polarization resulting from the two perpendicular polarization components of the light wave was shown. Next, this approach was used to present the influence of the components

amplitudes of the orthogonal polarization components together with their relative phase shift on the the resulting light polarization.

In the next Section the polarization ellipse was described by the ratio of the major axes, the orientation, and the rotation direction. Then, the amplitudes of the orthogonal polarization components together with relative phase shift are found as a function of the ellipse parameters. It is done by comparing ellipses described in basis of electric field components with the one in Cartesian coordinates. This relation allows for the generation of the desired polarization states by choosing the proper amplitudes and the relative phase shift.

The last Section uses at the beginning the Jones formalism to show the interaction of the polarization optics with light. Then, by applying the analogy between waveplates and liquid crystals used in shapers, an analysis of the light interaction with the single and the double array shaper was provided. The mechanism of phase only, phase, and amplitude, and limited phase and polarization shaping was presented.