# Chapter 5

# Conclusions

In this thesis, I have demonstrated with several examples from natural resource management that even very general model ensembles can reveal robust and valuable features. The new restriction and abstraction techniques for model ensembles as well as techniques for viable control design substantially improved the capacity for yielding such results. The framework of model ensembles as the organising principle of the thesis proved to be a common ground for various methods like qualitative differential equations (QDEs), differential inclusions and causal loop diagrams. The achievements of this thesis can be summarised on three levels: theoretical, with respect to the concrete resource use problems investigated in Chapter 4, and for future applications.

- The concept of a model ensemble provides common ground for linking QDEs, differential inclusions and viability theory, and meets methodological demands of sustainability science. Together with the graph theoretical formulation, abstraction and restriction can be investigated in a systematic way.
- The experience with the new abstraction and restriction methods proves an increased applicability of qualitative reasoning methods. They allow the identification of robust properties of infinite ensembles of ODEs as they occur under uncertainty and generality.
- The applications demonstrate that relevant conclusions for natural resource management can be made on the base of model ensembles. Viable control strategies can be designed with these methods.

## **Theoretical Achievements**

The thesis contributes to the theory of qualitative reasoning from different perspectives. In section 2.1, I proposed model ensembles as a framework to embed both qualitative differential equations (QDEs) and differential inclusions. It is designed to meet essential needs of sustainability science and can be expected to be useful for other domains where uncertainty or generality of models have to be faced. It also suggests a base to combine qualitative and quantitative dynamics, and by providing a systematic description of the relation between QDEs and ODEs, we can take a new view on qualitative modelling. In former approaches, a

QDE is portrayed as an abstraction of a single ODE which yields – more or less "involuntarily" – a large set of systems including ODEs which are not intended as valid representations. In contrast, within this thesis the modelling already starts from a set of systems which may all be valid under uncertainty or generality.

This way of formalising the relation between ODEs and QDEs further profited from the use of graph theoretical concepts, which have not been used explicitly in the qualitative reasoning literature previously (section 2.2). These concepts not only allowed for a more concise formalisation, but also for a generic definition of abstraction (section 2.2.4) and the development of new and advanced abstraction techniques.

The no-return abstraction I developed in section 3.1 was motivated by detecting irreversible system development and emerged from a discrete analogon of invariant sets as defined in viability theory. It was shown that sets of qualitative states which cannot be reentered once they are left are closely related to the well-known concept of strongly connected components of directed graphs. It can also be proved how this method can be combined with established abstraction techniques as chatter-box abstraction and projection.

The use of abstraction techniques is limited if the state-transition graph is not very well structured. Further model assumptions are needed to restrict the model ensemble. Some steps in this direction were made in this thesis. I presented automated elimination of marginal edges as one new method in section 3.2. Its power stems from a clear definition of "marginality" which fosters the use of two complementary algorithms, and from the fact that the number of marginal edges tends to be large due for combinatorial reasons.

Another achievement is the restriction of monotonic (landmark) ensembles by making ordinal assumptions as defined in section 3.3. These restrict the model ensemble to a subset where certain expressions involving the coefficients of the Jacobian of the systems are larger or smaller than other expressions. This restriction and the proposed ORDAS algorithm yields stronger structural properties by eliminating paths of length 2.

A hybrid method combining QDEs and differential inclusions was developed in section 3.4. It can be applied if the modeller has information about quantitative intervals for each component of the Jacobian. The viability algorithm computes whether given edges in the state-transition graph can be eliminated under this restriction of the model ensemble and determines velocities which necessarily lead to a given qualitative successor state. This method is limited by the capacities to compute viability kernels. If progress is made in the computation of viability kernels, the results of the hybrid method are still valid such that there is a potential for its future improvement.

In some of the applications in Chapter 4, control problems play a prominent role (section 4.3, section 4.4). QDEs can easily be used to design qualitative control rules. Since the qualitative state space is a finite set, all possible qualitative constraints for the control variable can be explored to identify conditions under which the structure of the state-transition graph significantly improves, e.g. by producing invariant sets. Such constraints are important candidates for management strategies which are robust under uncertainties and transferable to different natural resource use cases.

### **Management of Natural Resources**

The applications yield several robust properties of natural resource use although model ensembles are very general in nature. In section 4.1 about agriculture on marginal land it is shown that the so called poverty-degradation spiral does not necessarily occur for all ODEs which are consistent with the qualitative description of the underlying mechanisms. On the other hand, a slight restriction of the model model ensemble identifies states which lead to a certain outcome for all ODEs: If poverty and agricultural activity are low and decreasing at the same time, the soil cannot degrade at a later stage.

For unregulated fisheries, I demonstrated in section 4.2 that every ODE describing capital dynamics according to some general qualitative assumptions exhibits a phase of overcapitalisation, i.e. every fishery described by the model will invest in fishing gear and technology for some time although harvests are already decreasing. In section 4.3 the effects of participatory fisheries management and of ichthyocentric decision-making were assessed in combination. The results from a general ODE model complemented with viability criteria and from a QDE model both show that participatory management is not sustainable per se and that management which focuses solely on the ecological view necessarily produces trajectories violating environmental or economic viability criteria. A more complex qualitative control strategy was designed which at least outperforms ichthyocentric control.

The example of lake management in section 4.4 demonstrates how ordinal assumptions and numerical bounds can yield new insights into a model which is prone to uncertainties although a theoretical structure and quantitative data are available. The original quantitative model contains one functional relationship for phosphorus recycling, of which little is known empirically. A model ensemble allows generalisation of this relationship. To design a management strategy on this base, ordinal assumptions proved to be essential to bring about a more advantageous structure. The linear-interval version of the model shows, counter intuitively, that in a certain qualitative state a management which is less sensitive to changes in the phosphorus content of the lake is more likely to prevent eutrophication.

All these applications analyse problematic interactions of society and ecology which mainly occur on a regional level. However, although every lake, every fishery and every agricultural system is different, the problems investigated are not singular: They are observed in a more or less similar way at many places on Earth. Using QDEs, their communalities can be precisely formulated as a syndrome of global change without ignoring particularities. If such a description does not yield robust management strategies, more particularities have to be considered and the models have to be refined. In contrast, management strategies which are already robust in the general setting can be transferred between different instances of a problem.

#### Model Ensembles for Sustainability Science

I conclude with a reflection on how adequate the framework of model ensembles is for sustainability science and about the potential of the new techniques for future use. The set of applications in this thesis demonstrates several advantages and limits. In all case studies, removing marginal edges (section 3.2) was one of the basic techniques to restrict the statetransition graph, thereby revealing more structure and improving the graphical representation. Also the no-return abstraction was used for every model. It reveals important structural features even of large state-transition graphs and can substantially improve the graphical representation. This was particularly important for the control design exercises, where the number of non-trivial no-return sets is a highly valuable indicator for the predictability of a given management strategy. On the other hand, this method also made some limits of monotonic landmark ensembles more visible. Many qualitative models are so weakly constrained that the state-transition graph consists only of one strongly connected component and several final states. This stresses the urgency to restrict the model ensemble – if possible – by making further model assumptions beyond pure QDEs.

Ordinal assumptions are one such possibility which is appealing since it does not require pure quantitative information about the system. This restriction method proved to be valuable for some applications, but the ORDAS algorithm did not produce stronger results for every case. Not all ordinal assumptions a modeller can make yield a graph with more no-return sets. A systematic assessment of all combinatorially possible ordinal assumptions and sign matrices of dimension three reveals that in some cases the ORDAS algorithm does not change the state-transition graph. The stronger restriction of partial derivatives to intervals can be combined with ordinal assumptions. More importantly, this method yields information about the tendency of transitions to a given successor state – a valuable result, since qualitative states have multiple successors in most of the relevant applications.

Considering not a single model but a set of models defined by common structural features takes account of uncertainty and generality at the same time. Using monotonic (landmark) ensembles has a further advantage if some system variables are difficult or impossible to measure quantitatively. Currently, the methods discussed and developed in the thesis reach their limits if the issue of complexity has to be modelled, since they are restricted to a small or intermediate number of variables. It must be noted that simple qualitative management models based only on knowledge expressed by a causal loop diagram are, in many cases, too general to make crisp predictions about systems behaviour. Hence, if there is really not more information available, management of such systems seems hardly possible except for good-natured cases. Using landmarks and restriction by ordinal assumptions or quantitative intervals can substantially improve results. However, for very large models ambiguities due to uncertainty or generality make it difficult to detect robust properties of model ensembles. Therefore, the strength of monotonic (landmark) ensembles lies more in the context of conceptual modelling where dynamical systems are reduced to the most relevant components. The impact of such models should not be underestimated, since they can be used in social learning processes where decision-makers or lay people draw basic conclusions from a model which they can transfer to the much more complex environment they live in. Finally, we have seen that viability theory can be flexibly combined with model ensembles, which allows for complementing conceptual models with normative knowledge - another basic requirement to contribute to decision-making for sustainable development.

For the future, more implications of ordinal assumptions could be investigated. These assumptions are rather strong in comparison to a monotonic (landmark) ensemble. Since in some models only a small number of paths can be excluded by the ORDAS algorithm, this suggests that there are further ways to reduce the state-transition graphs by exploiting ordinal assumptions. One interesting possibility for future consideration could be to expand the representation of qualitative states by including not only signs of derivatives with respect to time, but a (partial) ordering of the velocities. Together with ordinal assumptions on the coefficients of the Jacobian this provides rules about whether the order of velocities can change in time. If this or other new approaches yield a state-transition graph with less ambiguity and more strongly connected components, the usability of the no-return abstraction will increase. It may then be valuable to exploit further concepts from graph theory, which currently does not seem fruitful. A third direction of future research is to explore model ensembles such that QDEs can better cross-fertilise with well established simulation techniques. Within the context of sustainability science, model ensembles offer a promising conceptual framework to discuss uncertainties at the same time with issues of classification of "syndromatic" and "paradigmatic" archetypes. By reasoning about restrictions and subsets of model ensembles, and by investigating how policy options and institutional arrangements change the solution operator, this is central to the design of sustainable futures. I hope this work encourages the appropriate use of qualitative reasoning methods for this task and shows important paths for further development.