Chapter 4

Management of Natural Resources

In this chapter I apply the framework of model ensembles proposed Chapter 2, in particular qualitative differential equations (QDEs), differential inclusions and viability theory to develop novel conceptual models for three examples of natural resource problems, namely subsistence farming, marine capture fisheries and fresh water use. These models demonstrate the scope and limits of the abstraction and restriction techniques developed in Chapter 3 (which are intensively used) and show how the investigation of model ensembles yields robust results for resource management.

I first give a short outline of the resource problems. In many cases, natural resources are *common pool resources*, so it is often difficult to exclude users from the resource or limit extraction and pollution. While users benefit individually from resource utilisation, the costs of a degraded resource are likely to be shared by the community (Hardin 1968). Thus, to avoid degradation, management strategies and institutional arrangements are needed which guarantee sustainable use. Actually, some kind of management is implemented for most exploited natural resources, but not always in a sustainable way (Ostrom 1990).

Agricultural production increased substantially during the 20th century, primarily due to industrialised agriculture, intensification of cultivated systems, and expansion of cultivated areas (Millennium Ecosystem Assessement 2005). One key factor for agriculture is the quality of soil. Much land is still used for subsistence farming, which is important for food safety in developing countries. Subsistence farming can evolve along the so called impoverishment-degradation spiral: existential rural poverty forces farmers to intensify their land use. This leads to soil degradation, which reduces yield and thereby further exacerbates rural poverty (Leonhard 1989; Reenberg and Paarup-Laursen 1997). There is ongoing research on adequate practices to stop or prevent this problematic interaction of processes (e.g. Lüdeke et al. 1999; Reij et al. 2005).

Marine fish stocks are degrading worldwide. Due to globally decreasing catches, in many cases the fishing industry can only be sustained at an economic level by paying high subsidies, while at the same time increased capitalisation puts additional pressure on the stocks (Banks 1999; Munro 1999; Pauly et al. 2002). As a consequence of this intricate situation there is an ongoing debate on adequate control and management instruments. Recent years have seen a number of bio-economic models examining the effects of commercial fishery on marine resources.

Fresh water is threatened by eutrophication, triggered by high nutrient loads of urban runoff and excessive agricultural use of fertilisers. In this sense, inland waters are overused as a sink. Eutrophication is typically associated with algal blooms, declining fish populations, and loss of recreation opportunities (Lathrop et al. 1998).

Within the list proposed by the German Advisory Council on Global Change (WBGU 1996) the above land use problem can be classified as Sahel Syndrome (overcultivation of marginal land), the fishery problem as Overexploitation Syndrome (overexploitation of natural ecosystems), and the eutrophication problem as Waste Dumping Syndrome (environmental degradation due to controlled and uncontrolled waste disposal). As discussed in Chapter 1, several properties of these problems make it promising to use model ensembles and further methods I presented and developed in Chapters 2 and 3:

- They appear at different places in a similar way, making a generalised identification of patterns of global environmental change valuable.
- They are characterised by various uncertainties. For example, in the domain of fisheries we must live with the fact that the amount of fish and its growth properties as well as the functions describing changes in behaviour of fishing firms are not exactly known (Clark 1999; Whitmarsh et al. 2000; Charles 2001). The latter also holds for the behaviour of subsistence farmers. In the case of eutrophication, storages and flows of nutrients are not all easily measured (e.g. in the mud of a lake), and some highly variable processes are not completely understood in a quantitative sense (Lathrop et al. 1998).
- They involve normative considerations, since a *problematic* pattern has to be understood.
- In addition to addressing generality and uncertainty, we demonstrate in this chapter how QDEs can be used to advance from identifying dynamical patterns to designing management options.

4.1 Land-Use Changes in Developing Countries

A well-known QDE model from sustainability science is presented in this section to demonstrate the most basic ensemble methods. Its state-transition graph is computed, which has a simple no-return abstraction, and examples are given how the methods contribute to the understanding of the motivating real-world problem. Some management interventions can be analysed in a straightforward way within this framework.

The Sahel Syndrome Model

The model studies regional land-use changes due to subsistence farming in developing countries (for details see Petschel-Held et al. 1999; Petschel-Held and Lüdeke 2001; Eisenack and Petschel-Held 2002). State variables are the quality of the resource R, agricultural activities A, and the poverty level P. Obviously, high agricultural activities reduce R due to overuse, while low A has a positive effect on the quality of the resource. Poverty increases agricultural activity, being the constituting behavioural assumption in a context of subsistence farming. Poverty inversely depends on agricultural yield, which increases with activity and the quality of the resource. The first question is whether these mechanisms necessarily bring about the poverty-degradation spiral. We are interested in measures shifting the system in a favourable direction. The model is described by the following equations:

$$\begin{aligned}\dot{A} &= b(P),\\ \dot{R} &= r(A),\\ P &= y(A,R),\end{aligned}$$

with $A, R, P \in \mathbb{R}_+$. The behavioural function $b \in C^1(\mathbb{R}_+, \mathbb{R}), D_P b > 0$ assigns to a given poverty level the change of agricultural activity. The soil regeneration function $r \in C^1(\mathbb{R}_+, \mathbb{R})$ is strictly decreasing with respect to A. Poverty is reduced by yield via the function $y \in C^1(\mathbb{R}_+ \times \mathbb{R}_+, \mathbb{R})$ with $D_A y, D_R y < 0$: economic production increases with effort A and resource quality R, thus reducing poverty.

By substituting y for P, poverty can be eliminated from the model, but as poverty is an important component of the Sahel Syndrome, we want to keep it as a state variable. By differentiation,

$$\begin{aligned}\dot{P} &= D_A y \cdot \dot{A} + D_R y \cdot \dot{R}\\ &= D_A y(A,R) \cdot b(P) + D_R y(A,R) \cdot r(A),\end{aligned}$$

yielding some sign ambiguities in the Jacobian since

$$\begin{aligned}\left[D_A \dot{P}\right] &= \left[D_{AA} y \cdot b + D_{RA} y \cdot r + D_R y \cdot D_A r\right] = [?],\\ \left[D_R \dot{P}\right] &= \left[D_{AR} y \cdot b + D_{RR} y \cdot r\right] = [?],\\ \left[D_P \dot{P}\right] &= \left[D_A y \cdot D_P b\right] = [-],\end{aligned}$$

unless we make appropriate assumptions on the second derivatives of the yield function. Thus, taking $(A\,R\,P)^t$ as state vector, the basic monotonicity properties of the model are captured by the sign matrix

$$\Sigma = \begin{pmatrix} 0 & 0 & [+] \\ [-] & 0 & 0 \\ [?] & [?] & [-] \end{pmatrix}.$$

The model is refined by introducing landmarks and setting up a monotonic landmark ensemble. The maximum sustainable agriculture is denoted by the landmark $\mathtt{ms}$, and it is assumed that $r(\mathtt{ms}) = 0$, i.e. below $\mathtt{ms}$ the soil regenerates, while it degrades above. Similarly, we introduce $\mathtt{ex}$ for poverty (existential poverty level) with $b(\mathtt{ex}) = 0$, meaning that for a poverty level above $\mathtt{ex}$ agricultural activities increase (to offset yield losses). Furthermore, landmarks for the upper bounds of the variables are set. As introduced in section 2.2.3 (p. 32), the state space is augmented with the velocity variables dA, dR and dP. We obtain the quantity spaces

$$\begin{aligned} Q_A &:= (0, \{0, \mathtt{ms}\}, \mathtt{ms}, \{\mathtt{ms}, \mathtt{Amax}\}, \mathtt{Amax}), \\ Q_R &:= (0, \{0, \mathtt{Rmax}\}, \mathtt{Rmax}), \\ Q_P &:= (0, \{0, \mathtt{ex}\}, \mathtt{ex}, \{\mathtt{ex}, \mathtt{Pmax}\}, \mathtt{Pmax}), \\ Q_{dA} &:= (\{-\infty, 0\}, 0, \{0, \infty\}), \\ Q_{dR} &:= (\{-\infty, 0\}, 0, \{0, \infty\}), \\ Q_{dP} &:= (\{-\infty, 0\}, 0, \{0, \infty\}), \end{aligned}$$

the resulting quantity space Q and the qualitative state space S. By default, three constraints link state and velocity variables:

$$\begin{aligned} C_1 &:= \{v \in S \mid \mathrm{qdir}_A(v) = [\mathrm{qmag}_{dA}(v)]\}, \\ C_2 &:= \{v \in S \mid \mathrm{qdir}_R(v) = [\mathrm{qmag}_{dR}(v)]\}, \\ C_3 &:= \{v \in S \mid \mathrm{qdir}_P(v) = [\mathrm{qmag}_{dP}(v)]\}. \end{aligned}$$

The basic properties of the yield function are expressed by the constraint

$$\begin{aligned} C_4 := \{v \in S \mid \\ & [\mathrm{qmag}_A(v)][\mathrm{qmag}_R(v)] = -[\mathrm{qmag}_P(v)] \wedge \\ & [\mathrm{qmag}_A(v)]\mathrm{qdir}_R(v) + [\mathrm{qmag}_R(v)]\mathrm{qdir}_A(v) = -\mathrm{qdir}_P(v)\}, \end{aligned}$$

which accounts for the cases where one or more of the qualitative directions or magnitudes vanish. The zeros of r and m are expressed by

$$\begin{aligned} C_5 &:= \{v \in S \mid [\mathrm{qmag}_P(v)]_{\mathtt{ex}} = \mathrm{qdir}_A(v)\}, \\ C_6 &:= \{v \in S \mid [\mathrm{qmag}_A(v)]_{\mathtt{ms}} = -\mathrm{qdir}_R(v)\}. \end{aligned}$$

Defining $C := \{C_1, \dots, C_6\}$ and the constant mapping $\mu : Q \to \mathcal{A}^{3\times3}, q \mapsto \Sigma$, we obtain a monotonic landmark ensemble $\mathcal{M}(\mu, C)$. All solutions of ODEs with right-hand side $f \in \mathcal{M}(\mu, C)$ on the state space $X = \mathbb{R}^3_+$, i.e. $S_{\mathcal{M}(\mu,C)}(\mathbb{R}^3_+)$, are possible evolutions of agricultural systems as described by the Sahel Syndrome model. These can be computed using the QSIM algorithm with a model code as follows:

```
(quantity-spaces
 (A  (0 ms Amax)    "Activity")
 (dA (minf 0 inf)   "dA")
 (R  (0 Rmax)       "Resource")
 (dR (minf 0 inf)   "dR")
 (P  (0 ex Pmax)    "Poverty" ))

(constraints
 ((d//dt A dA))
 ((d//dt R dR))
 ((M+ P dA) (ex 0))
 ((M- A dR) (ms 0))
 (((M - -) A R P)))
```

The landmarks for the upper and lower bounds of the quantity spaces are not designed to appear in any constraint, but have another purpose. If a qualitative state v is considered by the QSIM algorithm where one $\mathrm{qmag}_i(v), i \in \{1,2,3\}$ attains one of these landmarks and $\mathrm{qdir}_i(v)$ is such that the landmark will be transgressed, the state v is regarded as a final state, i.e. no further successors of this state are generated. Thus, states are automatically detected where the soil totally degrades, where efforts come their limits, etc.

Results

The quantity space of the model consists of 2025 qualitative states. By applying the QSIM algorithm, we obtain a state-transition graph with 158 edges and 49 vertices. Some basic abstraction and restriction techniques further simplify the result (see section 2.2.4, p. 36). We end up with 20 edges and 20 vertices, of which 16 are final states where at least one variable attains its bound (see Fig. 4.1, Tab. 4.1). The no-return abstraction (see section 3.1, p. 52) of the graph is simple in this case because the graph contains no strongly connected components, i.e. every edge is irreversible.

The irreversibility of all edges expresses an important feature of the model – it brings the agricultural system to a situation which cannot be changed without an intervention. Since the qualitative model subsumes a set of ODEs defined by right-hand sides f in a monotonic landmark ensemble, interventions which change the quantitative state of the system without crossing a landmark or which replace f by another right-hand side $f' \in \mathcal{M}(\mu, C)$ have no substantial effect. Fig. 4.1 shows that not every final state is problematic: although there are cases where the resource quality is reduced to a minimum level or poverty comes to a maximum, there are also final outcomes with a recovered resource or a level of well-being above the existential level.

Value judgements enter the analysis at this stage. In Fig. 4.1 an example is provided for such a valuation of final states, based on the qualitative magnitudes and directions of P and R. A degrading resource and existential poverty are considered as problematic, while

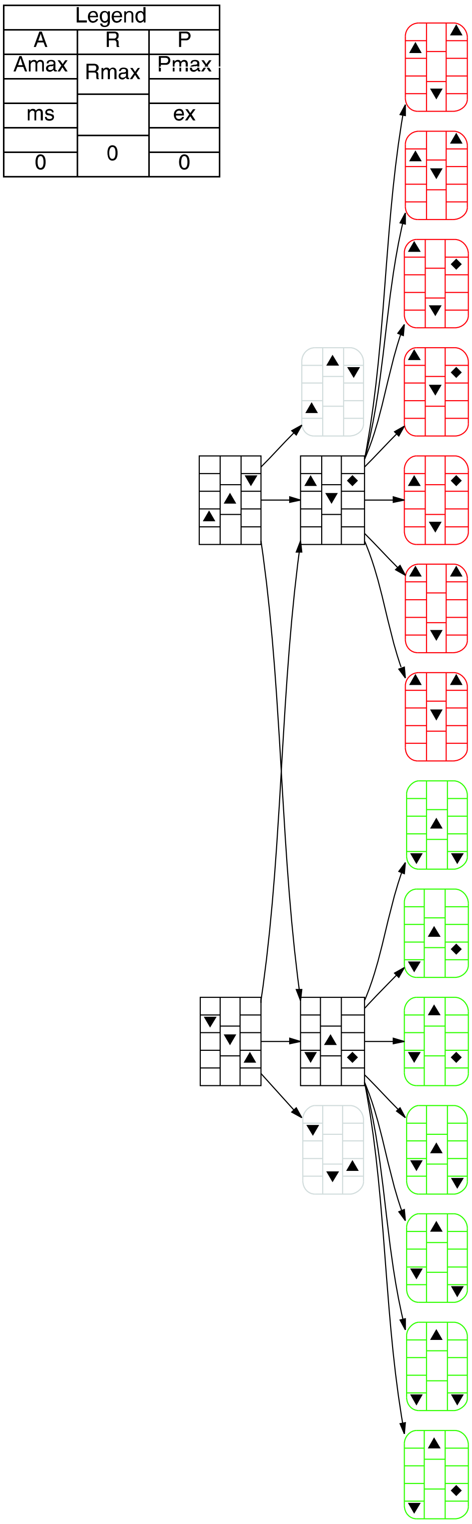

Figure 4.1: Abstracted and restricted state-transition graph of the core of the Sahel Syndrome (computer-generated output). The columns in the vertices represent qualitative values as given in the legend, where landmarks and intervals between landmarks alternate. Diamonds abstract multiple qualitative directions detected by chatter-box abstraction. Colours indicate value judgements as explained in the text.

Graph	Vertices	Edges
State-transition graph	49	158
After chatter-box abstraction	33	42
Removing marginal edges	33	36
Removing non-analytical states	20	20

Table 4.1: Number of vertices and edges resulting from different restriction techniques applied subsequently to the Sahel Syndrome model.

a recovering resource or low poverty is preferable. If a variable is in a preferable state and the other not a problematic one, it is coloured green. If one is problematic and the other not preferable, it is red. The ambiguous cases where one variable is in a problematic and the other in a preferable state are grey.

Both problematic and preferable outcomes are possible for an initial state with increasing agricultural activities below the maximum sustainable level and decreasing, but existential poverty. The same applies for decreasing activities which degrade soils, combined with increasing poverty below the existential level. But, once activity *and* poverty are above the critical landmarks, it is *inevitable* for every solution of the monotonic landmark ensemble $\mathcal{M}(\mu, C)$ that the resource totally degrades or poverty remains critical. Conversely, a positive development necessarily occurs if poverty and agricultural activity are low at the same time.

Management

Three types of potential interventions into systems without control variables can be distinguished (Eisenack and Petschel-Held 2002):

> **External interventions**: A manager is temporarily introduced who alters state variables to shift the system directly to another qualitative state. During the intervention, the mechanism of the QDE is postponed but becomes active afterwards again.
>
> **Structural management**: The social-ecological conditions are changed such that another model ensemble has to be chosen, e.g. by another sign matrix or the introduction of new landmarks and variables. This results in a different state-transition graph where, e.g. problematic invariant sets may be resolved.
>
> **Micro-management**: Management changes parameters such that the ODE describing the system is defined by a new right-hand side which is a member of the same model ensemble as before. The effect can be a change in the tendency of the system to shift to one or another successor state. As this does not change the state-transition graph, the evaluation of micro-management is beyond the scope of qualitative reasoning.

Petschel-Held et al. (1999) discuss three external interventions into the Sahel Syndrome dynamics. (i) A policy to combate poverty is initialised if P is existential, and results in poverty reduction to below the existential level. (ii) The agricultural impact on soils is mitigated when $\mathrm{qmag}_A(v) > \mathtt{ms}$, with the effect that activity is below the critical level afterwards. (iii) Application of both policies at the same time. Interestingly, only the latter guarantees

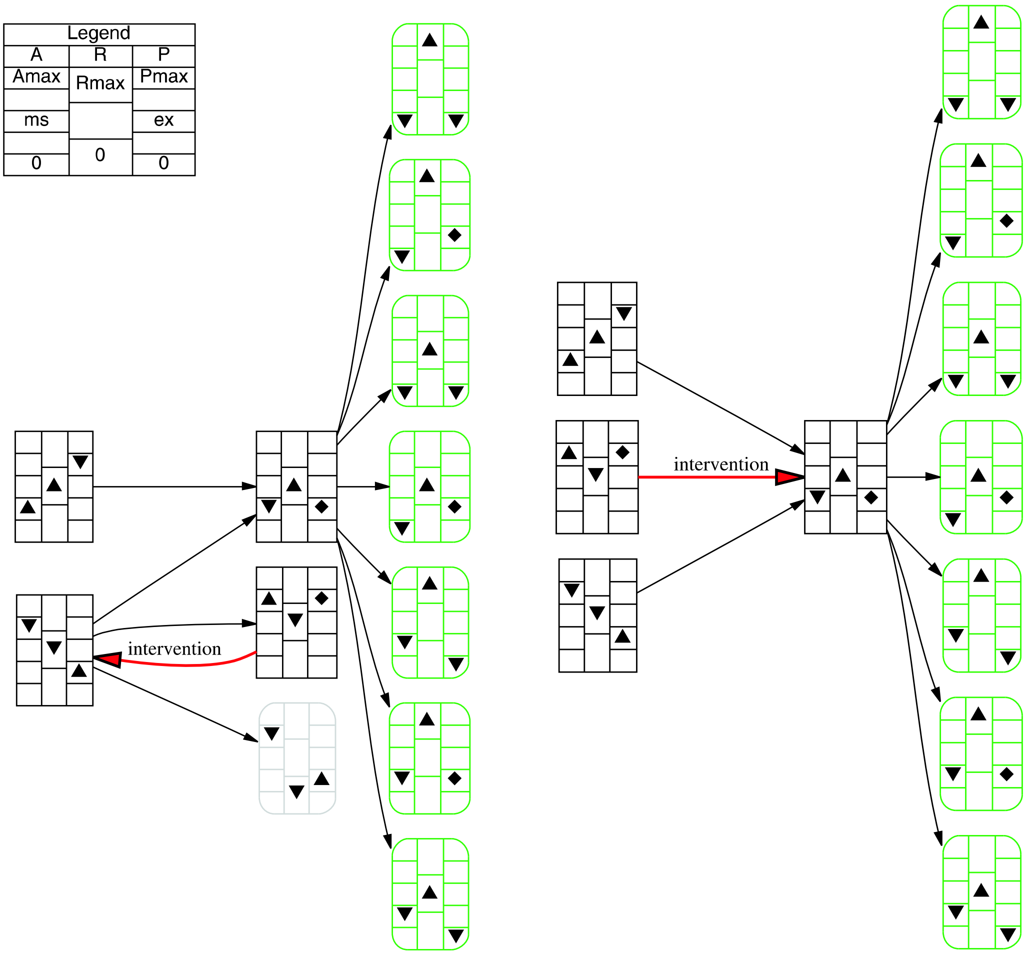

Figure 4.2: State-transition graph modified by external interventions (left: only combating poverty, right: combating poverty and mitigating agricultural impact). The columns represent the same qualitative values as in Fig. 4.1.

an improvement in every case (cf. Fig. 4.2). Combating poverty shifts the system back to a state where the resource may recover, but does not necessarily have to. It may also come back to the situation where the intervention has to be applied again. If only the agricultural impact is changed, the effect is symmetrical. This is avoided if both types of intervention are combined.

Now we analyse an example of structural management. In Eq. (4.1) we observed two sign ambiguities. Suppose some policy influences the agriculture such that these ambiguities are resolved so that $[D_A\dot{P}] = [+]$ and $[D_R\dot{P}] = [-]$. This means that a high resource quality always reduces poverty (if nothing else changes), while high activities have an adverse effect. This can be a consequence of introducing agricultural techniques more sensitive to the resource and measures so as to dampen the influence of income on poverty. Would such a policy be beneficial or not? By including the constraint

```
(((M + - -) A R P dP))
```

in the model description, i.e. defining the sign matrix

$$\Sigma' = \begin{pmatrix} 0 & 0 & [+] \\ [-] & 0 & 0 \\ [+] & [-] & [-] \end{pmatrix},$$

and the sign map $\mu' : Q \to \mathcal{A}_*^{3\times 3}, q \mapsto \Sigma'$, we obtain a new QDE defined by the monotonic landmark system $\mathcal{M}(\mu', C)$ on the same state space as $\mathcal{M}(\mu, C)$. The resulting state-transition graph can be computed and compared to that of $\mathcal{M}(\mu, C)$ (see Fig. 4.3). As expected, certainties increase slightly: In the state with low agricultural activity, recovering resource and low as well as decreasing poverty (a), it is already sure that the outcome will be positive. In the original model it is also possible that poverty begins to increase again (there, the edge (b) is bidirectional, making both states a chatter-box). The situation is symmetric for edge (c), making high and increasing poverty and high agricultural activity a safe predictor for a bad outcome: it would be dangerous to recommend the structural management proposed here as a panacea, since its success or failure depends on the actual situation of the system. This is emphasised by the fact that some parts of the state-transition graph cannot be reached from every initial state. This has the consequence that combining this structural management with intervention (i) from above (combating poverty) is sufficient for a good outcome. However, there is a caveat to investigating structural management in a way like here. Expanding the two assumptions about the effect of management, we obtain

$$\begin{aligned} D_{AA}y \cdot b + D_{RA}y \cdot r + D_R y \cdot D_A r &> 0, \\ D_{AR}y \cdot b + D_{RR}y \cdot r &< 0. \end{aligned}$$

Thus, the monotonic landmark ensemble $\mathcal{M}(\mu', C)$ contains all functions $f \in \mathcal{M}(\mu, C)$ for which both relations hold for every state in X. A restriction of this kind may make $\mathcal{M}(\mu', C)$ an empty set, although this is not always obvious. In this case the required relations can still hold on a restricted region of the state space $X' \subsetneq X$. If it can be justified that an investigated system stays in X', the state-transition graph remains meaningful. Otherwise, the solution can be determined independently for monotonic landmark ensembles on different regions of the state space and the solutions have to be combined appropriately.

Summing up, we have learned from the qualitative Sahel Syndrome model that the underlying mechanism does not always bring about the impoverishment-degradation spiral. On the other hand, there are qualitative states where the outcome is more predictable and management can avoid a critical development. Two management options were analysed using QDEs and the state-transition graph. It can be seen that simple interventions are not sufficient: combining different external interventions, or external interventions with structural management are more efficient.

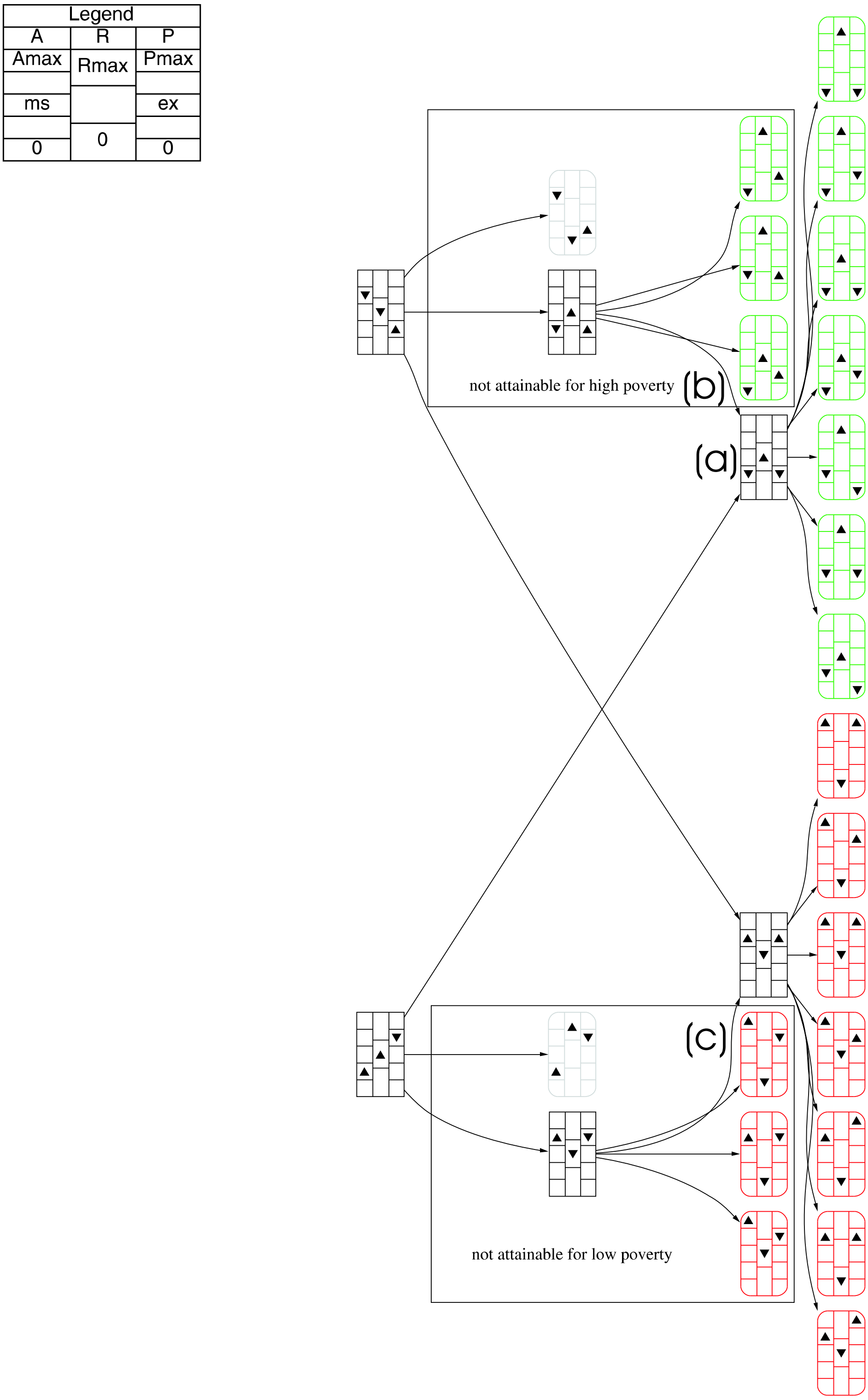

Figure 4.3: State-transition graph of the Sahel Syndrome with additional assumptions ($D_A\dot{P} > 0, D_R\dot{P} < 0$). The boxes indicate sets of states which are not attainable from a state with high or from a state with low poverty. The columns of the state representation are again the same as in Fig. 4.1. The vertex (a) and the edges (b), (c) are discussed in the text.

4.2 Capital Accumulation in Unregulated Fisheries

In this section I present a qualitative model about the problematic interaction of capital and biological stocks in marine fisheries (for details, I refer to Eisenack and Kropp 2001; Kropp and Eisenack 2001; Kropp, Zickfeld, and Eisenack 2002; Eisenack et al. 2006). I concentrate on the demonstration of the abstraction and restriction techniques developed and presented in this thesis: projection (section 2.2.4, p. 36), elimination of marginal edges (section 3.2, p. 62) and no-return abstraction (section 3.1, p. 52).

Capital accumulation has been a major issue in fishery economics over the last two decades; commercial fishery is portrayed as a system in which a biological stock and a capital stock interact dynamically (Clark et al. 1979; McKelvey 1985; Boyce 1995; Jørgensen and Kort 1997; Munro 1999; Pauly et al. 2002). The biological stock is the amount (number of fish or biomass) of the target species, whereas the capital stock consists of fishing gear (boats, nets, technical equipment etc.). As the capital stock is highly specialised and cannot readily be converted to other uses, investment decisions are irreversible. In many contributions this is understood as a major cause of over-fishing. If a fish stock is overexploited, making the fishery less profitable, there is no opportunity to sell the fishing gear. Consequently, more capital than efficient is allocated to the fishery, or equipment is transfered to other fisheries, putting other target stocks at risk – a pattern known as serial overfishing (Goñi 1998). With the model below I reveal one major cause of overfishing. It is shown that every solution of the monotonic landmark ensemble necessarily produces a period where excess capacities are built up, making the fishery less efficient and contributing to the risk of serial overfishing. Since this is driven by profit-oriented resource use, the model is qualified as a representative system for the Overexploitation Syndrome (cf. Cassel-Gintz and Petschel-Held 2000; Kropp, Eisenack, and Scheffran 2006).

First I set up an analytical model based on standard bio-economics. Then we use a monotonic landmark ensemble for its analysis for the following reasons:

- Due to tractability, previous efforts in this field have relied on a variety of simplifying assumptions, and many of them are restricted to equilibrium analysis. As QDEs extend the possibilities to handle the global dynamic properties of a system, the model can be substantially extended and some simplifications can be avoided.

- We are uncertain about exact functional relationships and parameters in marine fisheries (see p. 86).

The Capital Fisher Model

The capital fisher model investigates the dynamics of capital accumulation in an unregulated marine fishing industry with nonlinear investment costs and stock-dependent harvesting productivity. It describes a situation where N identical and profit maximising firms compete for an unregulated resource, i.e. a marine fish stock of size x. Assuming that any harvesting requires capital k (e.g. ships, fishing gear), and that the productivity of these inputs depends on the biological stock x, we can set up a variable cost function $v(h, x, k) : \mathbb{R}^3_+ \to \mathbb{R}_+$ which describes the harvesting costs at a given time for a given harvest h, fish stock x and capital stock k. We assume that this function has the following monotonicity properties:

$$D_h v > 0, D_x v, D_k v < 0,$$
$$D_{hh} v, D_{kk} v, D_{xx} v > 0,$$
$$D_{kx} v > 0, D_{hx} v, D_{hk} v < 0.$$

These inequalities describe consequences of economic standard properties of positive but decreasing marginal productivity, and that certain attributes of capital enhance the accessibility of the fish stock (improved fishing gear and technology, increased horsepower of boats, etc.). Additionally, we assume that the Hessian of v is positive definite, which is no contradiction to the above inequalities (see Eisenack et al. 2006 for details).

The regeneration of the resource is given by a recruitment function $R \in C^1(\mathbb{R}_+, \mathbb{R}_+), x \mapsto R(x)$ of logistic type. It attains a unique maximum sustainable yield (MSY) for $x = x_{MSY}$ (both parameters do not have to be known quantitatively). Furthermore, $R(0) = R(Q) = 0$, where $Q > x_{MSY}$ the carrying capacity of the biological system (which also does not have to be known qualitatively). For $x < x_{MSY}$, $D_x R > 0$, but $D_x R < 0$ if $x > x_{MSY}$. The fish stock changes according to

$$\dot{x} = R(x) - (h + h'), \tag{4.1}$$

where h denotes the harvest of a firm under consideration and h' that of all the others. The change of each firm's capital stock is described by

$$\dot{k} = I - \delta k, \tag{4.2}$$

where $I \geq 0$ represents the investment rate and δ a depreciation rate which is assumed to be constant. Investment costs are expressed by a strictly convex increasing function $c \in C^1(\mathbb{R}_+, \mathbb{R}_+), I \mapsto c(I)$. The convexity reflects inelastic supply of highly specialised equipment and rising adjustment costs for higher investment. The demand for fish is described by the downward sloping inverse demand function $p \in C^1(\mathbb{R}_+, \mathbb{R}_+), h + h' \mapsto p(h + h')$, assigning a market price to a given amount of harvested fish.

The decision of each firm about h and I now has to be determined. We assume that the harvest decision is myopic in contrast to the investment decision, i.e. fishing firms only take the current state of the system into account when deciding about h, while they take long-term effects into consideration when choosing the level of investment. The latter is justified by the long time scale of capital dynamics (ships are typically used for 10 to 50 years). The former is partly because of a lack of knowledge about the recruitment function, and partly because firms consider their own influence on the fish stock to be negligible. Moreover, they tend to assume that other firms behave in the same way. Thus, we suppose that the impact of harvesting on the biological stock are neglected by the individual firms in their short-term decision making. If each fishing company acts in an economically rational way, it chooses harvest to maximise profits $p(h+h')h - v(h, x, k)$ at each time. By using the implicit function theorem, some standard properties of the inverse demand function, the monotonicity and convexity properties of v, it can be guaranteed that the solution to this static problem is a harvest supply function $h : \mathbb{R}_+^2 \to \mathbb{R}_+, (x, k) \mapsto h(x, k)$ with $D_x h, D_k h > 0$. The investment plan is chosen such that it maximises the discounted profit given by

$$\Pi := \int_J e^{-rt} \Big(p(h + h')h - v(h, x, k) - c(I) \Big) dt,$$

subject to Eq. (4.1), Eq. (4.2) and $h = h(x,k)$. Here, r denotes a constant discount rate and $J = [0,T]$ a planning interval. Assuming that all firms are characterised by the same technology and behave in the same way (i.e. $h + h' = Nh$), the decision problem can be solved with a theorem on dynamical optimisation by Mangasarian (1966). The resulting analytical model can be written as:

$$\begin{aligned}
\dot{x} &= R(x) - Nh,\\
\dot{k} &= I - \delta k,\\
h &= h(x,k),\\
\dot{I} &= \frac{1}{D_{II}c(I)}\big((r+\delta)D_I c(I) + D_k v(h,x,k)\big).
\end{aligned}$$

Unfortunately, many signs of the Jacobian display ambiguities, and only some of these can be resolved by introducing landmarks. To distinguish the monotonically increasing part of R from the monotonically decreasing part, x is supplied with the landmark `xmsy`:

$$\begin{aligned}
D_x\dot{x} &= D_x R - N D_x h = \begin{cases} < 0 & \text{if } x > \texttt{xmsy},\\ \gtrless 0 & \text{otherwise}, \end{cases}\\
D_k\dot{x} &= -N D_k h < 0,\\
D_I\dot{x} &= D_x\dot{k} = 0,\\
D_k\dot{k} &= -\delta < 0,\\
D_I\dot{k} &= 1 > 0,\\
D_x\dot{I} &= \frac{1}{D_{II}c(I)}\big(D_{kx}v + D_{kh}v D_x h\big) \gtrless 0,\\
D_k\dot{I} &= \frac{1}{D_{II}c(I)}\big(D_{kk}v + D_{kh}v D_k h\big) \gtrless 0,\\
D_I\dot{I} &= (r+\delta)\big(1 - \frac{D_I c D_{III} c}{(D_{II}c(I))^2}\big).
\end{aligned}$$

Assuming $\frac{D_I c D_{III} c}{(D_{II}c(I))^2}$ to be small, for a qualitative state q with $\mathrm{qmag}_x(q) < \texttt{xmsy}$, we have the sign matrix

$$\mu(q) = \begin{pmatrix} [?] & [-] & 0\\ 0 & [-] & [+]\\ [?] & [?] & [+] \end{pmatrix},$$

while for $\mathrm{qmag}_x(q) < \texttt{xmsy}$

$$\mu(q) = \begin{pmatrix} [-] & [-] & 0\\ 0 & [-] & [+]\\ [?] & [?] & [+] \end{pmatrix}.$$

To formulate a set of constraints C we introduce the landmark $\texttt{MSY} := R(\texttt{xmsy})$ for R and h. Also, harvest increases monotonically with x and k. As in the previous section 4.1 (p. 87),

Graph	Vertices	Edges
State-transition graph		
after chatter-box abstraction	134	599
Removing marginal edges in runtime	103	330
Removing non-analytical states	59	103
Removing further marginal edges	59	93
Simple projection	30	47

Table 4.2: Number of vertices and edges for abstraction and restriction techniques subsequently applied to the Capital Fisher model.

upper and lower bounds are introduced into the quantity spaces to detect cases with extreme outcome, e.g. a diminishing fish stock or harvest rate. Several `cornot` constraints (cf. section 3.2, p. 62) are defined to eliminate marginal edges during computation (see Appendix for the model code).

Results

Without the `cornot` constraints (but using simple chatter-box abstraction, cf. section 2.2.4, p. 36), the state-transition graph has 134 vertices and 599 edges. Several restriction techniques are applied (chatter-box abstraction, projection and restriction to analytical functions, cf. section 2.2.4, p. 36, and elimination marginal edges, cf. section 3.2, p. 62; see Tab. 4.2). The result is presented in Fig. 4.4 in a manually improved form, where equilibria are omitted and all remaining final states are classified into two categories: (A) represents a catastrophic state where the fish stock is fully exploited ($x = 0$), while in (B) the stock recovers ($\dot{x} > 0$) but no harvest takes place ($h = 0$). The former is an environmental and economical disaster, while the latter is only an economic disaster. (A) can only directly be reached if $x <$ `xmsy` and $\dot{x} < 0$, while for type (B) $\dot{x} > 0$ and $\dot{h} < 0$ is a precondition.

The no-return abstraction (cf. section 3.1, p. 52) yields that the subgraph containing all except the final states is a strongly connected component, i.e. as long as no final state is reached, every vertex can *possibly* be re-entered. This is in contrast to established bioeconomic ODE models, where the system evolves monotonically towards equilibrium, or where equilibrium is reached after one turning point (e.g. Clark et al. 1979; McKelvey 1985; McKelvey 1986; Boyce 1995). However, the occurrence of boom-and-bust cycles is an empirical fact in many industrial fisheries (Hilborn and Walters 1992; Charles 2001): the state-transition graph can be used to reconstruct case studies which cannot be reconstructed by the older models (e.g. the collapse of the North Atlantic cod fishery or the historical development of the blue whale industry, cf. Eisenack et al. 2006). This shortcoming is mainly due to various linearity assumptions which are used to derive tractable solutions. QDEs allow for greater flexibility in this respect.

The non-linearities also bring about another strong feature of the state-transition graph: every fishery described by the model *necessarily* undergoes a phase of over-capitalisation, i.e. capital increases although catches are declining. In Fig. 4.4 this is the case in vertices #3, #6, #8 and #18. It is easy to see that every path in the abstracted state-transition graph which starts from vertex #1 and has at least length 3 reaches one of these vertices or results

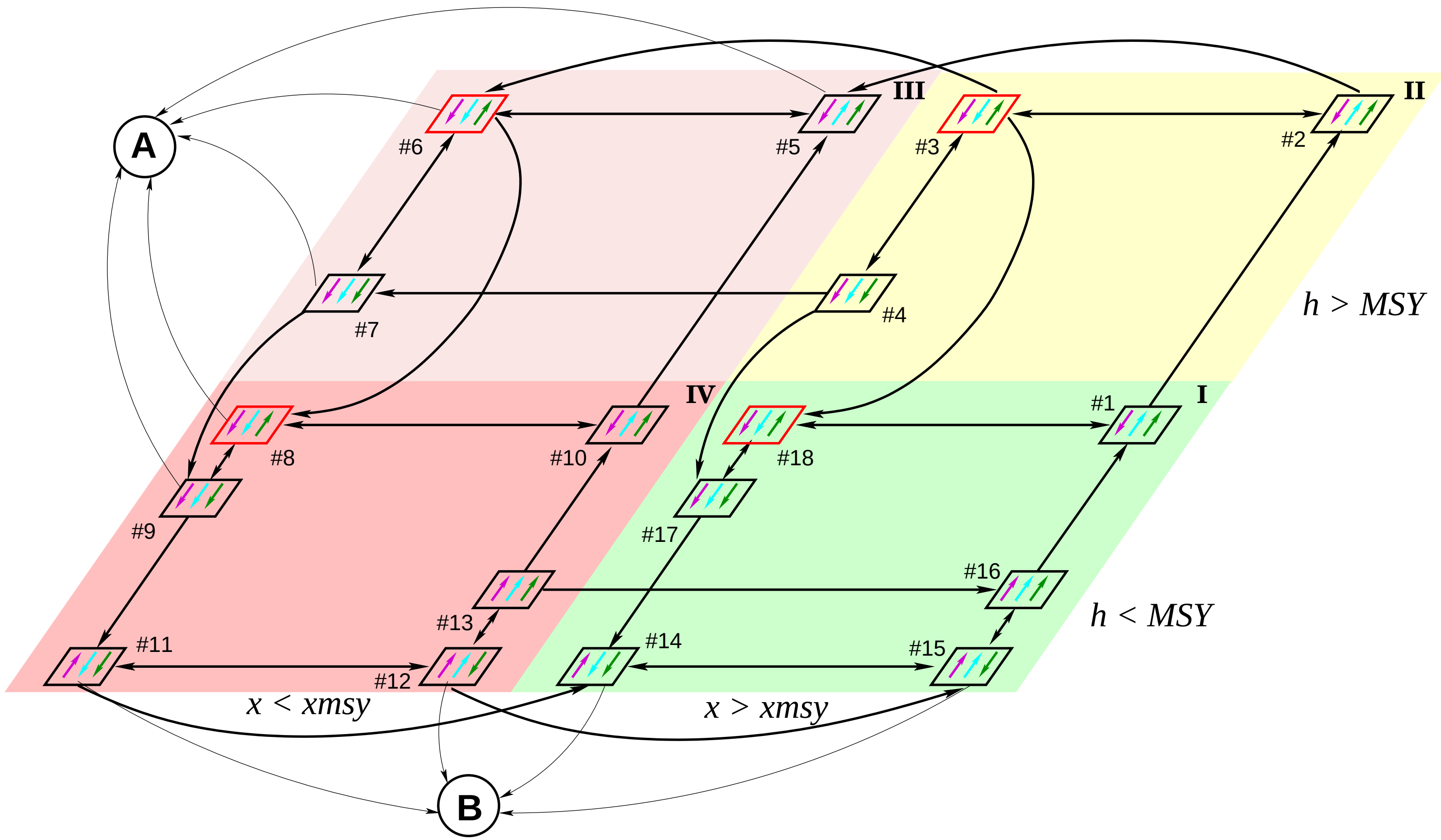

Figure 4.4: Projection of the state-transition graph of the capital fisher model with respect to x, h and k (after several restriction techniques are applied, cf. text). The arrows in the vertices (boxes) label the direction of the first derivative (from left to right $\dot{x}$, $\dot{h}$, $\dot{k}$), the red boxes indicate situations where overcapitalisation occurs; (A) and (B) are final states (adapted from Kropp, Zickfeld, and Eisenack 2002).

in a collapse. State #1 is a typical situation in a fishery where exploitation begins: Fish stocks are still high but declining, while investment increases the capital stock and harvests. But overcapitalisation also occurs for other initial conditions unless the system remains in a chatter-box forever or reaches a final state. This property is rooted in the fact that $v_{hx} < 0$ and $v_{hk} < 0$, i.e. that the harvest supply function h increases in x and k. As long as increased harvest is observed although the fish stock is reduced, net investment must be positive to compensate losses from increasing marginal costs. In other words, an increase in marginal costs due to a decreasing fish stock may trigger additional investment in an effort to keep marginal costs from rising excessively. Therefore, k cannot start to decrease before h.

The above model is less constrained than the Sahel Syndrome model, which becomes obvious from the no-return abstraction: there are no invariant sets except final states – it concludes that substantially more knowledge than considered by a monotonic landmark ensemble is needed to make crisper predictions. This is supported by experiments in which all ordinal assumptions consistent with the basic monotonicity properties were tested (cf. section 3.3, p. 68). Although some paths can be excluded such that several no-return sets consisting of single states emerge, they do not improve the overall situation. However, in spite of this ill-posed nature, diverse abstraction and restriction techniques substantially simplify the state-transition graph, and robust properties are revealed which are common to all systems given by the monotonic landmark ensemble.

4.3 Participatory Fishery Management

In this section I present and assess various management schemes of participatory resource management using viability theory and qualitative differential equations. The analysis is based on two models which are examined as dynamic control systems (cf. previous work in Eisenack 2003; Kropp, Eisenack, and Scheffran 2004; Eisenack et al. 2006). I show that serious problems may result if participatory management is purely resource-based. The analysis investigates whether a less risky management strategy can be implemented even with only limited data.

Viability criteria are imposed on both models of a management framework. We assess whether different management strategies comply with these requirements and how they change the structure of the resulting state transition graph. The first model is investigated solely by analytical techniques from viability theory (cf. section 2.4, p. 45). The second model describes a closely related but more complex setting. It uses QDEs to account for uncertainties more thoroughly and to design a qualitative closed-loop control in a systematic way. But would a management strategy which is promising in the setting of the first model remains robust in the second? The new methods developed in Chapter 3, elimination of marginal edges (section 3.2, p. 62) and no-return abstraction (section 3.1, p. 52), are important tools when addressing this question. The model also demonstrates another way in which viability theory can be fruitful for qualitative reasoning by restricting the qualitative state space to a region close to the boundary of a constrained set.

Recently, participatory strategies to fisheries management have been seen as a promising way. The basic idea is to include stakeholders – e.g. fishing firms, processing companies, scientific institutions and NGOs – in the decision-making process on catch restrictions. (Jentoft et al. 1998; Noble 2000; Charles 2001; Potter 2002). This is in contrast to the common type of top-down management where a government agency imposes restrictions on the fishery. When a fishery reaches a state of crisis, scientific institutions are criticised for putting too much emphasis on conservation objectives and neglecting the economic sustainability. If fishermen are involved in the decision-making process, it is assumed that economic objectives will complement conservation goals of government organisations and that compliance with regulations will be better (Pinkerton 1989; Mahon et al. 2003). In many cases, this type of management is exercised via a fishery council where the representatives of stakeholder groups negotiate, e.g. about the total allowable catch. This plan is executed by a management organisation which works in close collaboration with local fishermen.

One precondition for the success of both top-down and participatory frameworks is a proper information base, usually delivered by scientific institutions (e.g. ICES 2002). In this context it is sometimes argued that fishery management is focused too much on an ecological viewpoint – compared with efforts to examine the behaviour of the resource users, their economic settings, and aims – sometimes referred to as “ichthyocentrism” (Lane and Stephenson 2000; Davis and Gartside 2001). Thus, the following models are designed to discuss the potential benefits and risks of participatory and ichthyocentric management frameworks. We want to know if there is still a risk of overexploitation.

4.3.1 Viability Analysis of Management Frameworks

The quota negotiation model of a participatory management framework uses game theory and includes scientific catch recommendations as control variable (for details, I refer to Kropp, Eisenack, and Scheffran 2004; Eisenack, Scheffran, and Kropp 2006). The model is supplied with viability constraints to identify conditions for viable control. Different recommendation strategies are assessed against these conditions. One of them will also be assessed within the extended model in the next subsection.

The Quota Negotiation Model

As in section 4.2 (p. 95), the basic state variables is the biomass of a fish stock x, which is influenced by the total harvest h and the recruitment function R, yielding for the stock dynamics the ordinary differential equation

$$\dot{x} = R(x) - h.$$

Recall that $R(x_{MSY}) = MSY$, $R(0) = R(Q) = 0$ and for $x > x_{MSY}$ we have $D_x R(x) < 0$, while $D_x R(x) > 0$ for $x = x_{MSY}$. Due to the complexity of ecosystems we have only limited knowledge about the behaviour of a fish stock. Thus, no additional assumptions about R are made.

In a participatory framework the total harvest h is determined in a negotiation process about the allocation of catch quotas, written as vector $q \in \mathbb{R}_+^N$, to N groups of fishing firms. The resulting total harvest is $h = \sum_{i=1,\ldots,N} q_i$. The negotiation process is modelled with the following assumptions: A scientific institution and representatives from the fishing industry bargain for the total harvest h and the individual quotas q_i. When these pressure groups agree on an allocation, the result is transformed into practice by the management authority. The negotiations are opened by the scientific institution, which makes a recommendation $r \geq 0$ for the total catch. Each group of the fishing industry tries (i) to get an optimum share of the total harvest h and (ii) to increase h above the catch recommendation r if it is profitable.

The optimum share and the optimum increase $h - r$ may differ between the groups, e.g. due to technical and economic parameters. There is a trade-off between higher profits resulting from higher quotas and deviation costs d_i imposed by exceeding the scientific recommendation. These costs are linked to the legitimation of bargaining positions which challenge scientific advice and increasing transaction costs of fierce negotiations. They may differ between the pressure groups. It is further assumed that the fishing groups act to optimise short-term profits. This is realistic if single fishing firms perceive their impact on the resource as negligible (Banks 1999; Kropp, Eisenack, and Scheffran 2004). This entails that they only account for short-term deviation costs.

Each group i is supplied with a profit function $\pi_i : \mathbb{R}_+ \times \mathbb{R}_+^N \to \mathbb{R}_+$, depending on the fish stock and the overall allocation plan,

$$\pi_i(x, q) = pq_i - c_i(q_i, x) - d_i\left(\sum_{j=1,\ldots,N} q_j - r\right), \tag{4.3}$$

where the first term represents revenues on markets, p corresponding to the market price (which is assumed to be exogenous), while $c_i \in C^1(\mathbb{R}_+ \times \mathbb{R}_+, \mathbb{R}_+)$, $i = 1, \ldots, N$ are cost functions, assigning variable costs of a single group to its catch q_i and the amount of fish x. It is economically reasonable to assume that all cost functions c_i are convex and increasing in q_i, while costs are decreasing in x due to higher densities of fish. The deviation costs, described by the functions $d_i \in C^1(\mathbb{R}_+, \mathbb{R}_+), i = 1, \dots, N$ depend on the individual quota q_i, on the quotas allocated to the other pressure groups, and on the scientific catch recommendation r. If $\sum_{j=1,\dots,N} q_j - r$ becomes negative, we assume that d_i vanishes, since deviation costs do not apply if the sum of all quotas is below the recommendation. It is reasonable to assume that each d_i is a monotonically increasing, convex function.

Each group i negotiates for a quota which maximises their profit π_i for given p, x and r. This problem can be described as a non-cooperative game with the so called Nash equilibrium as solution, where each participant takes the decision of the other participants as given (Nash 1951). Since all profit functions $\pi_i, i = 1, \dots, N$ are concave and continuously differentiable with respect to q_i, the Nash equilibrium is given by the equation system

$$\forall i = 1, \dots, N : D_{q_i}\pi_i = 0. \tag{4.4}$$

In the following analysis we restrict this general approach to the case of two specific fishery groups (e.g. artisanal and industrial fishers) and provide a possible functional specification for variable and deviation costs by

$$c_i(q_i, x) := \frac{\alpha_i q_i + \beta_i q_i^2}{x}, \tag{4.5}$$

$$d_i(q_1 + q_2 - r) := \begin{cases} 0 \quad \text{if} \quad q_1 + q_2 < r, \\ \kappa_i(q_1 + q_2 - r)^2 \quad \text{otherwise}. \end{cases} \tag{4.6}$$

The parameters $\alpha_i, \beta_i, \kappa_i$ $(i = 1, 2)$ are not completely known, but positive.

PROPOSITION 44: *In the Nash bargaining solution for the profit functions given by Eq. (4.3) with Eq. (4.5), Eq. (4.6) and $N = 2$ results in the total catch*

$$h(x,r) = \begin{cases} h_b(x,r) & \textit{if} \quad r \le \hat{r}(x) \quad \textit{and} \quad h_b(x,r) \ge 0, \\ \hat{r}(x) & \textit{if} \quad r \ge \hat{r}(x) \ge 0, \\ 0 & \textit{if} \quad \hat{r}(x) \le 0, \\ 0 & \textit{if} \quad 0 \le r \le \hat{r}(x) \quad \textit{and} \quad h_b(x,r) \le 0, \end{cases} \tag{4.7}$$

$$= \max\big(0, \min(h_b(x,r), \hat{r}(x))\big), \tag{4.8}$$

where

$$\hat{r}(x) = \frac{upx - v}{\beta_1\beta_2},$$
$$h_b(x,r) = \frac{upx + wxr - v}{\beta_1\beta_2 + wx},$$

and

$$u := \frac{1}{2}(\beta_2 + \beta_1) > 0,$$
$$w := \beta_1\kappa_2 + \beta_2\kappa_1 > 0,$$
$$v := \frac{1}{2}(\alpha_1\beta_2 + \alpha_2\beta_1) > 0.$$

The continuous total harvest function h increases monotonically in x and r. The functions $\hat{r}, h_b$ are strictly increasing. Additionally, it holds that

$$h_b(x, \hat{r}(x)) = \hat{r}(x), \tag{4.9}$$
$$\textit{if } \hat{r}(x) \geq 0 : h(x, r) \leq \hat{r}(x). \tag{4.10}$$

The function h (Eq. 4.7), depending on the fish stock x and the harvest recommendation r, is called the total harvest function. The different cases result from that fact that for recommendations $r \geq \hat{r}(x)$ it is not profitable for fishing firms to exceed them. In this case the total harvest equals $\hat{r}(x)$, which is the optimum catch if $\kappa_1 = \kappa_2 = 0$. We refer to this case as non-binding harvest recommendations. However, if $r < \hat{r}(x)$, recommendations are binding, resulting in the catch $h_b(x, r)$.

PROOF: At first we derive Eq. (4.7). Suppose the bargaining is constrained to the case $q_1 + q_2 < r$ where deviation costs vanish. In this case the conditions Eq. (4.4) can be solved independently from each other for q_1 and q_2 by elementary calculations, yielding $h(x, r) = q_1 + q_2 = \hat{r}(x)$. Thus, if $0 \leq \hat{r}(x) \leq r$, the harvest is $\hat{r}(x)$. Now suppose that $0 \leq r \leq \hat{r}(x)$, making the conditions Eq. (4.4) a linear equation system. The solution for q_1 and q_2 yields the harvest $h(x, r) = h_b(x, r)$. Of course, if $\hat{r}(x) \leq 0$ or $h_b(x, r) \leq 0$, then $h(x, r) = 0$ because harvest cannot be negative.

The monotonicity properties of $h, x, \hat{r}, h_b$ and Eq. (4.9) can easily be shown with elementary calculations.

For $r \geq \hat{r}(x)$, Eq. (4.10) is true because $h(x, r) = \hat{r}(x)$. For $r \leq \hat{r}(x)$ and $h_b(x, r) \geq 0$, $h(x, r) = h_b(x, r) \leq h_b(x, \hat{r}(x)) = \hat{r}(x)$ due to monotonicity of h_b and Eq. (4.9). If $0 \leq r \leq \hat{r}(x)$ and $h_b(x, r) \leq 0$, then $h(x, r) = 0 \leq \hat{r}(x)$. This covers all possible cases for Eq. (4.10).

Eq. (4.8) is valid because by Eq. (4.7), harvest does not vanish iff $0 < \hat{r}(x) < r$ or $0 < h_b(x, r) \wedge r \leq \hat{r}(x)$. Using Eq. (4.9) and the monotonicity properties, in the first case

$$h = \hat{r}(x) = h_b(x, \hat{r}(x)) \leq h_b(x, r),$$

i.e. $h = \min(h_b(x, r), \hat{r}(x))$. Similarly, in the second case

$$h = h_b(x, r) \leq h_b(x, \hat{r}(x)) = \hat{r}(x).$$

□

We end up with the ODE

$$\dot{x} = R(x) - h(x, r),$$

where $r \in \mathbb{R}_+$ is a control variable. In the following we will express different management regimes as different strategies for choosing r. To assess them, we need quality criteria – these are provided using the framework of viability theory.

Viability Constraints

Two reasonable viability constraints are defined and investigated for our examination of marine fisheries, and conditions are deduced under which a control rule for r exists, which respects both constraints:

1. Ensure that the biomass of a stock always remains above a minimum level $\underline{x} > 0$:
$$\forall t : x(t) \geq \underline{x}.$$
2. Require that a minimum total harvest $\underline{h} > 0$ can always be realized or exceeded:
$$\forall t : h(t) \geq \underline{h}.$$

We refer to the first criterion as environmental and to the second one as economic viability. We define the set-valued map $F : \mathbb{R}_+ \rightsquigarrow \mathbb{R}$ by

$$F(x) := \{R(x) - h(x,r) | r \in \mathbb{R}_+ \text{ and } h(x,r) \geq \underline{h}\}. \tag{4.11}$$

It assigns to a system state x all possible velocities resulting from a harvest recommendation r which ensures economic viability. We are now looking for a set of states in the interval $[\underline{x}, \infty]$ which is viable with respect to F. In such a set it is possible to choose an open-loop control function $r(\cdot)$ such that both viability constraints are met forever (cf. section 2.4, p. 45). This will serve as the basis to assess whether concrete control rules for r keep this set viable.

To apply the viability theorem (PROP. 10, p. 48), we have to evaluate the regularity of F. This includes determining the fish stocks for which $F(x) = \varnothing$ – which yields the cases where economic viability cannot be met.

PROPOSITION 45: *The set-valued map F defined by Eq. (4.11) equals*

$$F(x) = \begin{cases} [R(x) - \hat{r}(x), R(x) - \max\left(\underline{h}, h_b(x,0)\right)] & \textit{if } \hat{r}(x) \geq \underline{h}, \\ \varnothing & \textit{otherwise}, \end{cases} \tag{4.12}$$

and is Marchaud on every compact set $K \subset \mathbb{R}_+$ where $\exists x \in K : \hat{r}(x) \geq \underline{h}$.

PROOF: At first we show that $F(x) \neq \varnothing$ if and only if $\hat{r}(x) \geq \underline{h}$. The set $F(x)$ does not vanish if and only if there is one $r \in \mathbb{R}_+ : h(x,r) \geq \underline{h}$. Then, from Eq. (4.7), Eq. (4.9) and Eq. (4.10),

$$0 \leq h(x, \hat{r}(x)) = \hat{r}(x) \geq h(x,r) \geq \underline{h}.$$

Now suppose that $\hat{r}(x) \geq \underline{h} > 0$. With choosing $r \geq \hat{r}(x)$ the total harvest function Eq. (4.7) yields $h(x,r) = \hat{r}(x) \geq \underline{h}$, i.e $F(x) \neq \varnothing$.

Next, we determine the concrete form of F. We only have to consider the case $\hat{r}(x) \geq \underline{h}$, and denote the lower and upper bounds of F as $\underline{F}(x)$ and $\bar{F}(x)$, respectively. By definition, $R(x) - h(x,r)$ cannot be below $\underline{F}(x)$, and choosing $r = \hat{r}(x)$ yields exactly $\underline{F}(x)$. Now choose $r = 0 < \hat{r}(x)$. If $h_b(x, 0) > \underline{h} > 0$, then $\forall r \geq 0 : h(x, r) \geq h(x, 0) = h_b(x, 0)$ due to Eq. (4.7) and the monotonicity of the total harvest function. Since r cannot be negative, $R(x) - h(x, r)$ cannot be above $R(x) - h_b(x, 0)$. If, on the other hand, $h_b(x, 0) \leq \underline{h} \leq \hat{r}(x)$, there is one $\underline{r} \in [0, \hat{r}(x)]$ such that $h(x, \underline{r}) = h_b(x, \underline{r}) = \underline{h}$ due to Eq. (4.7), Eq. (4.9) and continuity of h. In summary, continuity of h guarantees that $F(x)$ contains exactly the values between $\underline{F}(x)$ and $\bar{F}(x)$.

That the set-valued map F is Marchaud when it is restricted to K can be verified using the characterisation from p. 43. Obviously, F has convex values and $\mathrm{Dom}(F) = K \cap \{x \in \mathbb{R}_+ \mid \hat{r}(x) \geq \underline{h}\}$ is an intersection of closed sets and nonempty since $\exists x : \hat{r}(x) \geq \underline{h}$. The $\mathrm{Graph}(F)$ is closed because because $\underline{F}$ and $\bar{F}$ depend continuously on x and $\mathrm{Dom}(F)$ is compact. It has linear growth since it is bounded on a compact set. □

Choosing a compact subset of $\mathbb{R}_+$ is only a technicality to account for the case that the recruitment function $R(x)$ may decrease faster than linear for $x > Q$ (when it has negative values). Alternatively, this can be excluded by an additional model assumption. However, since a harvested fish stock will not be above the equilibrium of a non-utilised stock Q, this is irrelevant in our case: we can simply choose a compact set $K \supseteq [0, Q]$, as long as there is one $x \in K$ with $\hat{r}(x) \geq \underline{h}$.

We are now ready to apply the viability theorem, stating that for a Marchaud map a closed set K is viable iff K is a viability domain (cf.PROP. 8, p. 47); in our case

$$\forall x \in K : F(x) \neq \varnothing, \tag{4.13}$$

$$F(\inf(K)) \cap [0, \infty] \neq \varnothing, \tag{4.14}$$

$$F(\sup(K)) \cap [-\infty, 0] \neq \varnothing. \tag{4.15}$$

PROPOSITION 46: *An interval $J = [\underline{x}, b]$ is a viability domain of F iff*

(i) $\exists x' \leq \underline{x} : \hat{r}(x') = \underline{h}$,

(ii) *and* $R(\underline{x}) \geq \max\left(\underline{h}, h_b(\underline{x}, 0)\right)$,

(iii) *and* $R(b) \leq \hat{r}(b)$.

PROOF: Condition (i) is equivalent to Eq. (4.13): It follows from monotonicity of $\hat{r}$ that $\forall x \geq x' : \hat{r}(x) \geq \underline{h}$, such that $F(x) \neq \varnothing$ by PROP. 45. Conversely, if $\forall x \in J : \hat{r}(x) \geq \underline{h}$ there exists an appropriate x' since $\hat{r}$ is continuous increasing and has a positive zero.

Condition (ii) holds iff $\bar{F}(\underline{x}) \geq 0$ which is equivalent to Eq. (4.14) by PROP. 45.

Condition (iii) holds iff $\underline{F}(b) \geq 0$ which is equivalent to Eq. (4.15). □

This proposition can be interpreted as follows. If a fish stock is in a viability domain, it is possible to choose an appropriate harvest recommendation r which keeps the fishery in the domain. Condition (i) guarantees that for sufficient large recommendations r the economic viability criterion can be met. This must be possible for all states in the viability domain. The second condition safeguards that for a fish stock at the lower boundary of the interval, r can be decreased sufficiently to prevent the fish stock from declining further. If the catch has to be decreased below $\underline{h}$ to obtain $\dot{x} \geq 0$, the economic viability constraint would be violated. Condition (iii), being more technical in nature, implies that fishing firms would voluntarily catch more than $R(b)$ if recommendations are non-binding, such that $\dot{x} \leq 0$ in this situation.

Note also that the viability kernel $\mathrm{Viab}_F(K)$ – which is the largest closed viability domain contained in K (cf.PROP. 10, p. 48) – is the largest interval $J \subseteq K$ satisfying the conditions. It has to be an interval, since condition (i) holds for all $x \geq x'$ and never holds for $x < x'$, and the other conditions only apply on the boundary of the viability kernel.

Management

We now use PROP. 46 to assess the viability of two different recommendation strategies for r. Although it is *possible* to keep the system viable if an appropriate strategy is selected, the proposition does not ensure that *every* control strategy is successful. Formally, such a strategy assigns a value for r to a given system state, i.e. a closed-loop control according to the following schemes:

- *Ichthyocentric control*: The harvest recommendation is purely based on an exact estimate of the stock recruitment, i.e.

$$r = R(x).$$

- *Conservative control*: The harvest recommendation is based on economic viability in the sense that recommendations are adjusted to yield $h = \underline{h}$, i.e.

$$r = \underline{r}(x),$$

  where $\underline{r}(x)$ is defined as the smallest $r \geq 0$ such that $h(x, r) \geq \underline{h}$ or as $+\infty$ if no such r exists.

These strategies can be regarded as extreme cases of management where only ecological or where only economic criteria matter. The latter is called "conservative" because this strategy only guarantees a minimum aspiration level for harvest, but avoids increased catches.

Ichthyocentric control: Assuming that the harvest recommendation r equals recruitment presupposes that the scientific institution is able to estimate $R(x)$ correctly. This is a challenging task, since an exact estimation of the stock biomass is bound to fail due to unavoidable measurement deficits (see the discussion of uncertainties above, p. 86). However, let us assume that the estimator is correct. We show that even in this ideal case, the strategy cannot guarantee viability. The main argument stems from the crucial fact that

$$h(x, r) \geq r \Leftrightarrow r \leq \hat{r}(x) \tag{4.16}$$

i.e. that the negotiated harvest is always above the scientific catch recommendation, except in the case of non-binding recommendations. This relation holds by Eq. (4.7) and Eq. (4.9), since $D_r h_b < 1$, and it holds for $0 \leq r \leq \hat{r}(x)$ that $h(x,r) = h_b(x,r) \geq r$, while for $r > \hat{r}(x)$ it holds that $h(x,r) = \hat{r}(x) < r$. Thus, if recommendations are binding, then

$$h(x,r) = h(x, R(x)) > R(x),$$

harvest is above recruitment, resulting in a decreasing fish stock. Comparing with PROP. 46, even if there exists a viability domain, the ichthyocentric strategy necessarily violates the environmental viability criterion if $\underline{h} \leq R(\underline{x}) \leq \hat{r}(\underline{x})$. It is only viable if, contrarily, $\underline{h} \leq \hat{r}(\underline{x}) \leq R(\underline{x})$ holds. The latter means that the realized catch $\hat{r}$ must be significantly lower than the scientific recommendation, a situation which normally does not occur in industrial capture fisheries (Eisenack et al. 2006). This makes it impossible that the fish stock recovers once it is below $\underline{x}$. We can summarise that even in the case of a perfect stock assessment, the ichthyocentric strategy exposes the fishery to a risky development.

Conservative control: If a viability domain $J = [\underline{x}, b]$ exists for the fishery, then $\underline{r}(x) < \infty$ for all $x \in J$, since economic viability is guaranteed. It also holds that $\max\left(\underline{h}, h_b(\underline{x}, 0)\right) \leq R(\underline{x})$ (cf. PROP. 46). By definition of $\underline{r}$,

$$h(\underline{x}, \underline{r}(\underline{x})) = \underline{h} \leq R(\underline{x})$$

if $\underline{r}(x) > 0$; and if $\underline{r}(x) = 0$,

$$h(\underline{x}, \underline{r}(\underline{x})) = h_b(\underline{x}, 0) \leq R(\underline{x}).$$

Together, this has the consequence that conservative control guarantees also environmental viability. If the fish stock is below $[\underline{x}, b]$, i.e. outside the viability domain, environmental or economic viability is no longer sustained. However, it is possible that only the economic criterion is violated, resulting in $h(x, \underline{r}(x)) \leq R(x)$, i.e. x may increase until a viability domain is reached again. In contrast, if only the environmental criterion is violated, there is no chance of a recovery although harvest remains above $\underline{h}$ for some time: the strategy is always viable if the management begins in the viability domain of a fishery, and is thus less risky. Additionally, conservative control allows for a recovery in some fisheries.

Conservative control was claimed to be based purely on economic observations. For binding catch recommendations, the control equals

$$\underline{r}(x) = \frac{\underline{h}(wx + \beta_1\beta_2) + v}{wx} - \frac{up}{w}.$$

This might raise the objection that the parameters u, v, w, which depend on technical and political conditions, may be uncertain to the scientific institutions. In this case, one idea is to approximate conservative control by an adaptive strategy

$$\begin{aligned} \dot{r} &= f(h), \\ f &\in C^1(\mathbb{R}_+, \mathbb{R}_+), \\ f(\underline{h}) &= 0, D_h f < 0. \end{aligned} \tag{4.17}$$

The result is that for harvest above $\underline{h}$, recommendations are decreased and vice versa. This is close to a qualitative control rule which can also work in data-poor settings, and motivates an expanded qualitative model of participatory resource management in the next subsection. It shows how the conservative strategy performs if capital dynamics come into play.

4.3.2 Qualitative Viability Analysis and Control Design

The negotiation model with capital extends the previous model by introducing capital as a further state variable (see Eisenack 2003 for details). As shown in section 4.2 (p. 95), this can fundamentally change the dynamics. If the conservative control strategy is also useful in this setting, this indicates its robustness. The model is transferred to a QDE formulation, again to take account of uncertainty and generality (see p. 86) and to allow for a systematic state space scan in a higher dimensional setting. It is also an example of how viability constraints can be incorporated in the definition of a monotonic landmark ensemble. Finally, it admits a methodological innovation for the design of a closed-loop control in following way: first, a control variable u is included in the model as an open-loop control, and the sign matrix Σ and the constraints C are formulated such that the monotonic landmark ensemble admits all solutions which result from any continuously differentiable control $u(\cdot)$ on $\mathbb{R}_+$. Thanks to the guaranteed coverage theorem (see PROP. 3, p. 34), the resulting state-transition graph contains the abstraction of all trajectories brought about by all possible open-loop controls $u(\cdot)$. This graph can be used to identify the controls which are promising. In a second step the model is refined by introducing qualitative constraints for u – defining a class of closed-loop controls and producing a subgraph of the state-transition graph from the first step. The state-transition graphs of alternative controls can be compared against each other to choose the best option. Using QDEs has two advantages in this context:

- Solving the model for an unconstrained control is possible since the qualitative state space is finite.
- The search space of possible qualitative constraints for u is also finite (as a consequence of the finite qualitative state space) – in contrast to the design of a quantitative closed-loop control.

The Negotiation Model with Capital

As in the previous sections, the basic state variable is the resource stock x, supplied with a logistic recruitment function R and reduced by harvest h. For simplicity, the investigation is focused on stocks $x \leq x_{MSY}$ since non-viable behaviour is more likely to happen in this situation. In addition, the amount of capital k accumulated in the fishery is introduced as a second state variable. Capital is important in this context because (i) it represents the technological efficiency and has an effect on optimum harvest, (ii) inertia is introduced in the model (cf. section 4.2, p. 95), and (iii) it is assumed that capital is an indicator of the political pressure the fishing industry can exercise.

The harvest h is determined in a negotiation process in the fishery council. Again assume full compliance, with catches exceeding the initial catch recommendation r. In contrast to the previous subsection, catch recommendations are always binding. The model is extended in that the outcome of the negotiations, depending on the political power of the fishing industry which is assumed to increase with k such that the so-called negotiation equilibrium is expressed by a total harvest function $h \in C^1(\mathbb{R}^3_+, \mathbb{R}_+), (x, k, r) \mapsto h(x, k, r)$ with $D_x h, D_k h, D_r h > 0$. The stock size x has a positive effect since higher catches are profitable for higher abundance of fish. Catches also increases with r since they are larger when recommendations are less restrictive. Both is in accordance with Eq. (4.7) for binding recommendations.

The dynamics of capital k are described by the investment rate I' and depreciation, given by a (quantitatively unknown) depreciation rate $\delta > 0$ (similar to section 4.2, p. 95):

$$\dot{k} = I(h, k) := I'(h, k) - \delta k.$$

The investment function $I' \in C^1(\mathbb{R}^2_+, \mathbb{R}_+)$ is related to the profit expectations of fishing firms. It is assumed to decrease in k for economic reasons, including the law of diminishing returns (Eisenack 2003). All together, I decreases with k, but increases with h due to better profit expectations.

To design a control strategy for the catch recommendations, we make no assumptions about r at this stage as outlined above. The associated sign matrix with state vector $(x\, k\, h\, r)^t$ (note that $\dot{h}$ is implicitly determined) is:

$$\Sigma = \begin{pmatrix} [+] & 0 & [-] & 0 \\ 0 & [-] & [+] & 0 \\ [?] & [?] & [?] & [?] \\ [?] & [?] & [?] & [?] \end{pmatrix}.$$

The last ambiguous row results from the yet undetermined control rule for r which propagates to the monotonicity properties of $\dot{h}$. The latter are also uncertain due to the unspecified Hessian of h. The same viability criteria as in the previous subsection are introduced. To include them in the QDE we define `xmin` $= \underline{x}$ as landmark for the state variable x, and `hmin` $= \underline{h}$ for h. Additional corresponding values are defined by setting the landmarks `hv` $= R(\underline{x})$ and `xv` with $R($`xv`$) = \underline{h}$. To define a quantity space for the variables, a choice has to be made about the order of the landmarks. We investigate the more interesting case where `xmin` $<$ `xv` and `hv` $<$ `hmin`, which corresponds to the case where $R(\underline{x}) < \underline{h}$ (see Appendix for the model code). If the results from the previous subsection are robust to extending the model by capital k, we expect from PROP. 46 (p. 106) that stocks below `xv` are not part of a viability domain.

Results

The state-transition graph of the QDE is very large due to the unconstrained control variable r. To keep it tractable, several abstraction and restriction techniques are applied. At first, we restrict the quantity space to `xmin` $< x <$ `xmsy` and $h >$ `hv`, since states where both environmental and economic viability is violated are not our main interest, as well as fish stocks $x > x_{MSY}$ (see above). The QSIM algorithm automatically detects states where this quantity space is left. We further introduce multiple `cornot` constraints to exclude most marginal edges by preprocessing, and apply simple chatter-box abstraction (cf. section 2.2.4, p. 36).

Graph	Vertices	Edges
After chatter-box abstraction		
and removing marginal edges in runtime	137	407
Removing further marginal edges	137	321
Removing non-analytical states	78	170
Projection on x and h	18	38

Table 4.3: Number of vertices and edges resulting from different restriction techniques applied to the negotiation model with capital.

More simplifications are possible by removing marginal edges (cf. section 3.2, p. 62) and non-analytical states (see Tab. 4.3 for the effect of the methods). The no-return abstraction (cf. section 3.1, p. 52) reveals 2 non-trivial no-return sets. All other no-return sets are final states where the boundaries of the restricted quantity space are hit. To make this structure visible, a simple projection (cf. section 2.2.4, p. 36) is performed with respect to the variables x and h (see Fig. 4.5). Once the system enters the "downstream" no-return set, only problematic final states are possible – environmental or economic viability will necessarily be violated. But even in the "upstream" no-return set it is possible that catches fall below $\underline{h}$. The no-return sets can be distinguished by the qualitative value of x: In the upstream subgraph it is above XV, while it is below this landmark in the downstream subgraph. We should bear in mind that the graph covers *all* possible open loop controls $r(\cdot) \in C^1(\mathbb{R}_+ \to \mathbb{R}_+), t \mapsto r(t)$. This means that however $r(\cdot)$ is chosen, and for all ODEs given by $f \in \mathcal{M}(\mu, C)$, viability will be lost once $x <$ XV. This very robust result parallels the conclusions of the previous subsection (cf.PROP. 46, p. 106).

Management

To find successful interventions in the sense of structural management (cf. section 4.1, p. 87), the second step of the design method as outlined above is adopted (p. 109). Since r is still unconstrained, we introduce additional constraints to the model which describe the recommendation strategy of the scientific institution – one subsumes conservative control as introduced in the last subsection, and the other one considers economic *and* ecological indicators. The resulting state-transition graphs are compared. It is clear from the above model results that management interventions only make sense for $x \geq$ XV. The aim of these interventions is to prevent harvest from decreasing below $\underline{h}$ and to keep the system in the upstream no-return set. For simplification, we restrict the model ensemble by considering only qualitative values for x above XV (see Appendix for the modified version of the model including several possible constraints).

Conservative control is implemented in the adaptive version Eq. (4.17) by introducing a quantity space for $\dot{r}$ and the constraint

```
((M- h dr) (hmin 0)).
```

We apply the same restriction and abstraction techniques as before, with the result that the former upstream no-return set splits into two no-return sets (see Fig. 4.6). Observe that eco-

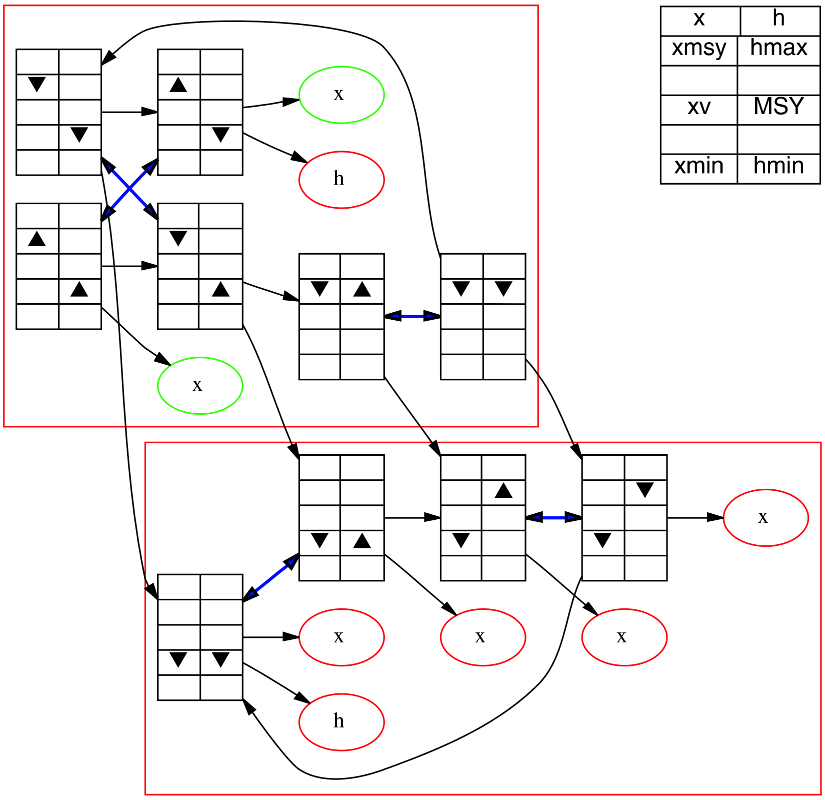

Figure 4.5: Projection of the restricted state-transition graph of the negotiation model with capital with respect to x and h. Final states are indicated by ellipses where a trajectory leaves the pruned state space: the variable transgressing the boundary is printed in an ellipse. Ellipses are red, when the lower boundary is transgressed (x = `xmin` or h = `hmin`), while green ellipses denote a recovering fish stock (x = `xmsy`). Red boxes represent no-return sets. To improve the presentation, final states are printed within the strongly connected no-return sets they are the successors to.

nomic viability cannot be violated in this part of state space. On the other hand, the conservative strategy cannot generally prevent x falling below `xv`. This can be explained if recommendations react too slowly to deviations of h from $\underline{h}$ or if the inertia introduced by k makes the system non-viable. If the knowledge about the fishery only allows for the set-up of a qualitative model as here, it must be admitted that conservative control is also risky. Even if the model is refined by various ordinal assumptions, the ORDAS algorithm presented in section 3.3 (p. 68) does not provide substantial improvements.

Qualitative control subsumes various possibilities for constraints defining control rules. Here, we consider a specific set of constraints which improves the situation:

```
(((M - + -) x dx dh r) (xmsy 0 0 hmin)),
(((M - + -) x k h dh)).
```

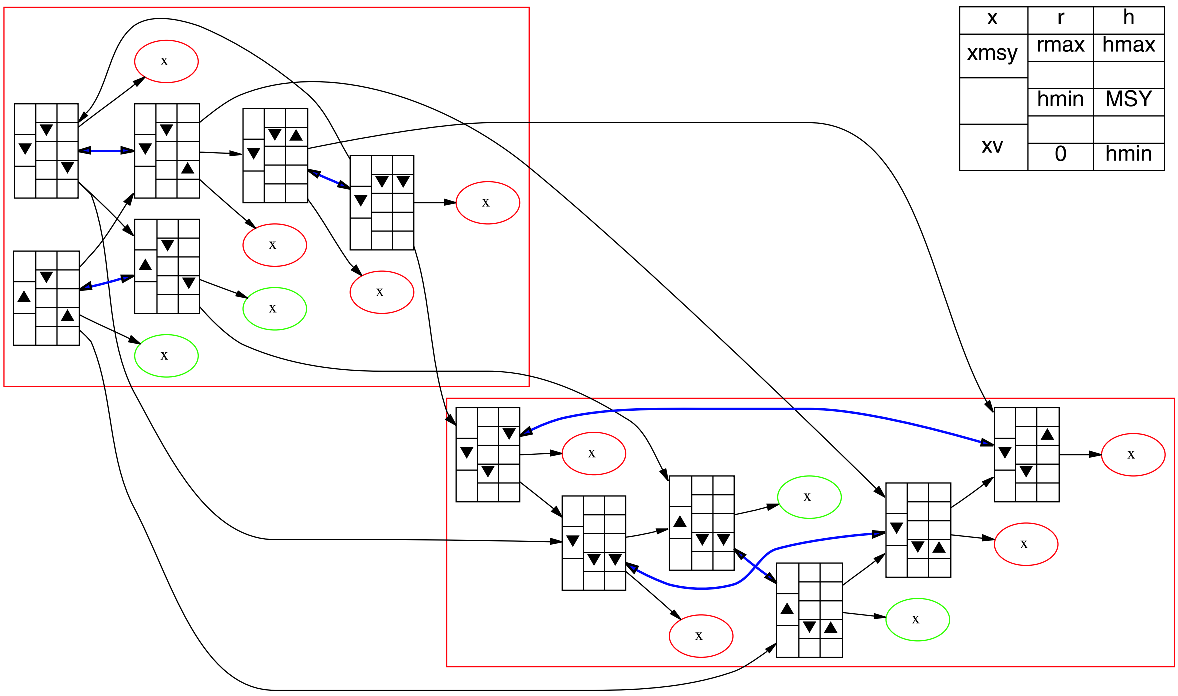

Figure 4.6: Projection of the restricted state-transition graph resulting from conservative control. Red ellipses denote that x drops below xv; other ellipses and clusters are used as in Fig. 4.5.

Consequently,

$$\Sigma = \begin{pmatrix} [+] & 0 & [-] \\ 0 & [-] & [+] \\ [-] & [+] & [-] \end{pmatrix},$$

and we implicitly assume that there exists a control function $u \in C^1(\mathbb{R}_+ \times \mathbb{R}^2, \mathbb{R}_+), r = u(x, \dot{x}, \dot{h})$ with $D_x u, D_{\dot{h}} u < 0$, $D_{\dot{x}} u > 0$ and $u(x_{MSY}, 0, 0) = \underline{h}$. The function associated with the last row of Σ is denoted by f. Such assumptions need a deeper justification, because they could result in $\mathcal{M}(\mu, C) = \varnothing$. We have to show that there are functions u, f with the given monotonicity properties such that

$$\dot{x} = R(x) - h, \tag{4.18}$$

$$\dot{k} = I(k, h), \tag{4.19}$$

$$h = h(x, k, r), \tag{4.20}$$

$$r = u(x, \dot{x}, \dot{h}), \tag{4.21}$$

$$\dot{h} = f(x, k, h). \tag{4.22}$$

This is only possible if u can be chosen such that by inserting Eq. (4.21) in Eq. (4.20) and differentiating with respect to time yields the same monotonicity properties of $\dot{h}$ as in Eq. (4.22). Showing this directly is problematic since $\dot{h}$ also appears as an argument of u. Thus use an indirect approach. Substituting Eq. (4.18) into Eq. (4.21), and then Eq. (4.21) into Eq. (4.20) yields

$$h = h\big(x, k, u(x, R(x) - h, \dot{h})\big),$$

which can be solved for $\dot{h}$ by the implicit function theorem. We obtain

$$D_x\dot{h} = -\frac{D_x h + D_r h(D_x u + D_{\dot{x}} u D_x R)}{D_r h D_{\dot{h}} u}, \tag{4.23}$$

$$D_k\dot{h} = -\frac{D_k h}{D_r h D_{\dot{h}} u} > 0,$$

$$D_h\dot{h} = \frac{1 + D_r h D_{\dot{x}} u}{D_r h D_{\dot{h}} u} < 0,$$

where Eq. (4.23) has an ambiguous sign – based on the sign assumptions made so far. Thus, the signs do not contradict the last row of Σ, i.e. the monotonicity properties of f. It also becomes clear that the qualitative control rule includes choosing u such that Eq. (4.23) is negative. The restricted and abstracted state-transition graph following from these specifications is given in Fig. 4.7. The graph contains 6 strongly connected no-return sets such that the qualitative control rule introduces much more certainty into the system. The rule also introduces a small invariant set of states where fish stock and harvest never violate the viability constraints. There are no problematic outcomes for one intermediate no-return set as long as it is not left. On the other hand, this cannot be safeguarded by the qualitative control rule since the system may also evolve into an invariant set with non-viable final states. In summary, this rule is an improvement but not a perfect solution. However, an extensive explorative test of various other constraints for r and ordinal assumptions has not revealed a strategy which performs substantially better.

Summarising the results from both models in this section, we have seen that a recommendation strategy based purely on the observation of the fish stock necessarily leads to economic or environmental decline. The situation can be improved substantially by a strategy based on purely economic observations. However, it does not generally work in the more complex setting where capital dynamics come into play and only qualitative observations about the system can be made. This also contributes to the insight that sustainable common property harvesting under uncertainty represents really a difficult problem. At least, the more flexible qualitative control requires only little information about the state of the fishery and is less risky than data-rich ichthyocentric management.

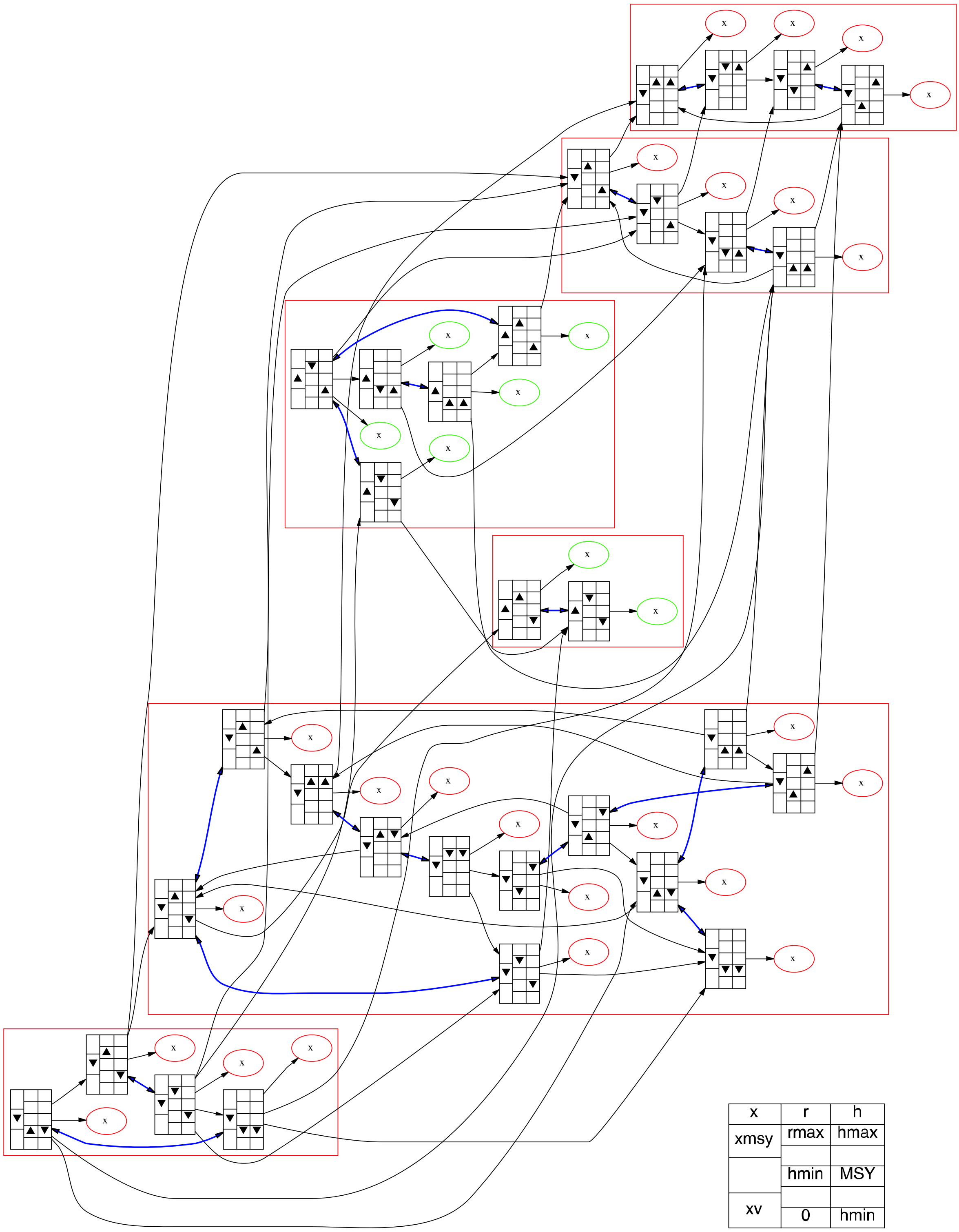

Figure 4.7: Restricted state-transition graph resulting from qualitative control. Ellipses and clusters have the same meaning as in Fig. 4.5.

4.4 Lake Management

In this section I analyse a qualitative model of a lake ecosystem subject to a management system to avoid eutrophication. Based on a model by Carpenter (2003), it is modified to account for various uncertainties. The methods developed in this thesis, in particular ordinal assumptions (cf. section 3.3, p. 68) and quantitative bounds (cf. section 3.4, p. 77) are applied.

Eutrophication of inland waters is a major threat to water quality. It is a process by which a body of water acquires a high concentration of nutrients, especially nitrogen and phosphorus, which promote primary production (Schwörbel 1999). “Many ecosystem services are reduced when inland waters and coastal ecosystems become eutrophic. Water from lakes that experience algal blooms is more expensive to purify for drinking or industrial uses. Eutrophication can reduce or eliminate fish populations.” (Millennium Ecosystem Assessement 2005, p. 69). The major cause of eutrophication is excessive inputs of phosphorus from urban runoff and agricultural areas (Lathrop et al. 1998). Despite decades of water research and management, blooms of blue-green algae are still a major water quality problem in lakes and reservoirs. Deterministic models are frequently used to determine consequences of phosphorus input changes, but they involve large prediction uncertainties (Lathrop et al. 1998).

The modification of the original model allows for further insights in the underlying lake management dynamics. It combines an ecosystem model of phosphorus dynamics in lake water and sediment with a management model. The manager observes the phosphorus content in the water column P and in the sediment M qualitatively and takes measures to change phosphorus input using a specified management strategy, e.g. by constructing and operating sewage plants. As in the previous section 4.3 (p. 101), several qualitative constraints describing the management strategy can be assessed for viability (i.e. to avoid eutrophication in our case). We concentrate on one interesting alternative.

The goal of the original contribution is to explore the possibility of anticipating thresholds before they are crossed in a setting of high uncertainties: The levels of phosphorus input are subject to unpredictable variations (e.g. due to weather). The original model is parameterised such that its state is close to a critical level of eutrophication. The lake manager is assumed not to know the values of all parameters of the ecosystem exactly. This leads Carpenter to formulate the relation between input target and actual phosphorus input as well as parameter estimation of the lake manager stochastically. The model contains a non-linear term describing phosphorus release from the sediment to the water column. This non-linearity helps to explain abrupt transitions from a so called clear-water regime to a turbid regime (high phosphorus in water column or eutrophic lake). For simplicity, Carpenter describes this threshold effect by a sigmoid rational function depending on P, although various other processes contribute to phosphorus release and other sigmoid functions cannot be refuted (Wetzel 2001). These problems are good reasons to develop a qualitative version of the model:

- There is uncertainty about functional relationships.
- Several parameters and some state variables (esp. M) are not known exactly and costly to measure.
- The effects of the decisions of the lake manager are not exactly predictable.

The Model

The basic model equations are

$$\dot{P} = -(s+h)P + r\phi M + L, \tag{4.24}$$
$$\dot{M} = sP - bM - r\phi M, \tag{4.25}$$

with L being the exogenous phosphorus influx into the lake which is the control variable of the lake manager. The parameter s describes the rate of sedimentation from the water column, h the outflow rate from the lake. Phosphorus leaves the sediment by two processes: a part is buried with rate b, and another part may be recycled to the water column, depending on the parameter r, the phosphorus content of the sediment M and a dependent recycling parameter ϕ. It is assumed that ϕ is small for low P, increases quickly near a threshold, and converges to a maximum afterwards. The quantitative simulation performed by Carpenter (2003) requires a functional form for ϕ although little is known about an exact quantification. One possibility is

$$\phi(P) = \frac{P^q}{m^q + P^q},$$

with a threshold parameter m. The parameter values used for the original model are given in Tab. 4.4. I translate this to a model ensemble by replacing the function ϕ with the qualitative constraint

```
((S+ P phi) (l1 0) (l2 1)),
```

claiming that the monotonic landmarks ensemble should contain all ODEs with a functions $\phi \in C^1(\mathbb{R}_+, \mathbb{R}_+), P \mapsto \phi(P)$ for which values $\lambda_1, \lambda_2 \in \mathbb{R}_+$ exist such that

$$\begin{aligned} \forall P \leq \lambda_1 &: \phi(P) = 0, \\ \forall P \geq \lambda_2 &: \phi(P) = 1, \\ \forall P \in (\lambda_1, \lambda_2) &: \phi(P) \in (0,1) \text{ and } D_P\phi(P) > 0. \end{aligned}$$

Also the linear relationships of the original model are generalised to monotonic functions such that a broad variety of non-linear dependencies are also included in the monotonic landmark ensemble. This is justified by our limited knowledge about the exact relationships, especially for this highly simplified model. The generalisation is based on the signs of the partial derivatives of $\dot{P}$ and $\dot{M}$:

$$\begin{pmatrix} D_P\dot{P} & D_M\dot{P} & D_L\dot{P} \\ D_P\dot{M} & D_M\dot{M} & D_L\dot{M} \end{pmatrix} = \begin{cases} \begin{pmatrix} -(s+h) & 0 & 1 \\ s & -b & 0 \end{pmatrix} & \text{for } P \le \lambda_1, \\ \begin{pmatrix} -(s+h)+rMD_P\phi & r\phi & 1 \\ s - rMD_P\phi & -(b+r\phi) & 0 \end{pmatrix} & \text{for } P \in (\lambda_1, \lambda_2), \\ \begin{pmatrix} -(s+h) & r & 1 \\ s & -(b+r) & 0 \end{pmatrix} & \text{for } P \ge \lambda_2. \end{cases} \tag{4.26}$$

The quantity space of P is supplied with the landmarks $\lambda_1 < \lambda_2 < \lambda_{eu}$, the latter denoting a problematic level of eutrophication. In the first version of the qualitative model, the control variable L is left unconstrained as in the previous section (cf. section 4.3, p. 101) – the resulting state-transition graph will contain the consequences of all continuous differentiable phosphorus input functions $L(\cdot)$. We will test possible constraints for L only in the second step. The sign matrices contributing to the definition of the monotonic landmark ensemble $\mathcal{M}(\mu, C)$ are

$$\mu(v) = \begin{cases} \begin{pmatrix} [-] & 0 & [+] \\ [+] & [-] & 0 \\ [?] & [?] & [?] \end{pmatrix} & \text{for } \mathrm{qmag}_P(v) \le \lambda_1, \\ \begin{pmatrix} [?] & [+] & [+] \\ [?] & [-] & 0 \\ [?] & [?] & [?] \end{pmatrix} & \text{for } \mathrm{qmag}_P(v) \in (\lambda_1, \lambda_2), \\ \begin{pmatrix} [-] & [+] & [+] \\ [+] & [-] & 0 \\ [?] & [?] & [?] \end{pmatrix} & \text{for } \mathrm{qmag}_P(v) \ge \lambda_2. \end{cases}$$

The complete model specification is given in the Appendix.

Results

The state-transition graph of this very general model has 72 vertices and 269 edges. Eliminating marginal edges (cf. section 3.2, p. 62) and states which are non-analytical in P, M or L reduces the graph to 34 vertices and 78 edges (see Fig. 4.8). No-return abstraction (cf. section 3.1, p. 52) reveals that all vertices except the final states form a strongly connected component. This is not surprising due to the generality of a model with unconstrained management. However, it is worth comparing this result with the negotiation model with

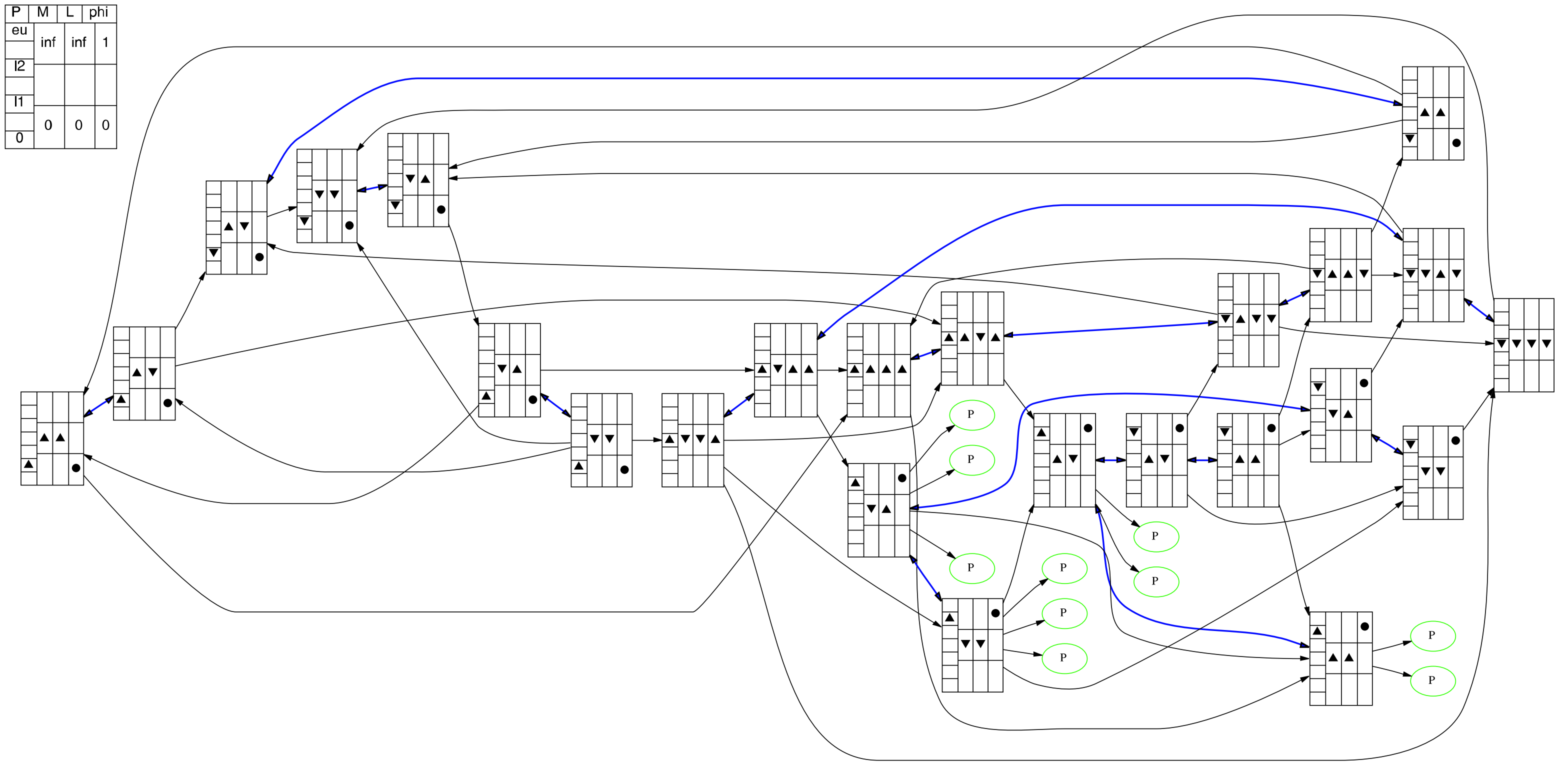

Figure 4.8: Restricted state-transition graph of the lake model with unconstrained management. Final states where phosphorus exceeds the unacceptable threshold λ_{eu} are marked as green ellipses.

capital from section 4.3.2 (p. 109), in particular Fig. 4.5, where even in the unconstrained case no viable control exists for some states.

Management

As in the previous section 4.3 (p. 101) we can in principle test out all possibilities to constrain the control variable L. This corresponds to finding a closed-loop control for the lake manager to influence L, e.g. by the effort put into operating sewage plants, depending on the manager's qualitative observation of the system. I do not describe the whole search process here, but present one of the promising qualitative control rules. It works on a restricted quantity space, obtained by introducing a further landmark $\lambda_* \in (\lambda_1, \lambda_2)$ for P such that

$$\forall P > \lambda_* : D_P\dot{P} < 0 \text{ and } D_P\dot{M} > 0.$$

Only qualitative states v with $\mathrm{qmag}_P(v) \geq \lambda_*$ are considered and the landmark `phil*` $:= \phi(\lambda_*)$ is defined for the quantity space of ϕ. Within the parameterisation of the original model this means that

$$\forall P > \lambda_* : rMD_P\phi(P) < s,$$

which is valid if a sufficiently small upper bound is given for M. Then, since $D_P\phi(\lambda_2) = 0$, such a landmark λ_* always exists if ϕ is differentiable twice. For the original parameter values and a reasonable bound $M < 1500$, the relation is satisfied for $\lambda_* > 4.92 = 2.05m$. Thus, the pruned quantity space restricts the attention to the state space region close to eutrophication, which is in spirit of the original model. In this region the qualitative constraints

```
(((M - + +) P M L dP)),
(((M + - ) P M dM)),
```

are valid. If we take the parameterisation of Eq. (4.24) and Eq. (4.25) as given, and assume that the lake manager can choose a strategy such that L is determined by a monotonic decreasing function $u \in C^2(\mathbb{R}_+, \mathbb{R}_+), P \mapsto u(P)$, differentiating with respect to time yields

$$\dot{L} = D_Pu(P)\big(-(s+h)P + r\phi(P)M + L\big),$$

and

$$D_P\dot{L} = D_{PP}u\dot{P} + D_PuD_L\dot{P},$$
$$D_M\dot{L} = D_PuD_M\dot{P} < 0,$$
$$D_L\dot{L} = D_PuD_L\dot{P} < 0.$$

By assuming $D_{PP}u = 0$, i.e. u to be an affine function, and since $s > rMD_P\phi(P)$ for $P \geq \lambda_*$, the following constraint can be formulated:

```
(((M + - -) P M L dL)),     (4.27)
```

i.e. $\dot{L}$ is described by a function $u \in C^1(\mathbb{R}^3_+, \mathbb{R}_+), u \mapsto u(P, M, L)$ such that $D_Pu > 0$ and $D_Mu, D_Lu < 0$. This means that the lake manager implicitly considers the changes in M, L

if he only observes P and reacts in an affine way – which is much easier to perform. The essence of this situation is entailed by a monotonic landmark ensemble with

$$\mu \equiv \Sigma := \begin{pmatrix} [-] & [+] & [+] \\ [+] & [-] & 0 \\ [+] & [-] & [-] \end{pmatrix} . \tag{4.28}$$

(the model code is given in the Appendix). The state-transition graph contains 61 vertices and 172 edges, which are restricted to 35 vertices and 54 edges after applying the usual restriction techniques. The no-return abstraction of the restricted graph yields one strongly connected component, 3 no-return sets consisting of one state, and 18 final states where the region with $P \in (\lambda_*, \lambda_{eu})$ is left. However, the large no-return set has successors with eutrophication as well as others where the phosphorus level becomes low, making it difficult to evaluate the proposed control rule as preferable or problematic.

The situation changes when appropriate ordinal assumptions are made. According to section 3.3 (p. 68), these are assumption on the signs of

$$d^{i,j}_{k,l} = D_l f_j \cdot D_k f_i - D_k f_j \cdot D_l f_i,$$

(cf. Eq. 3.2, p. 71). In the case of Eq. (4.28), some of these signs are already prescribed, e.g.

$$[d^{1,2}_{2,3}] = -[d^{1,2}_{3,2}] = -[d^{2,1}_{2,3}] = [d^{2,1}_{3,2}] > 0,$$

but others are free to choose. We assume that

$$\begin{aligned} &d^{1,2}_{1,2}, d^{2,3}_{1,2} > 0, \\ &d^{1,3}_{1,2} = d^{1,3}_{2,3} = d^{1,3}_{1,3} = 0, \end{aligned}$$

and the further ordinal assumptions resulting from symmetry (see Eq. 3.11, p. 73). These assumptions do not make the model ensemble empty since

$$\begin{aligned} d^{1,3}_{1,2} &= D_P\dot{P} D_P u D_M \dot{P} - D_P u D_P \dot{P} D_M \dot{P} = 0, \\ d^{1,3}_{2,3} &= D_M \dot{P} D_P u D_L \dot{P} - D_P u D_M \dot{P} D_L \dot{P} = 0, \\ d^{1,3}_{1,3} &= D_P \dot{P} D_P u D_L \dot{P} - D_L \dot{P} D_P u D_P \dot{P} = 0, \\ d^{1,2}_{1,2} &= h(b + r\phi) + b(s - rMD_P\phi) > 0, \\ d^{2,3}_{1,2} &= -D_P u d^{1,2}_{1,2} > 0, \end{aligned}$$

such that all ordinal assumptions are valid. We are now ready to run the ORDAS algorithm developed in section 3.3 (p. 68). The procedure detects 10 paths of length 2 which contradict at least one of the ordinal assumptions. As some of these paths share vertices, only 6 new vertices and 22 new edges have to be introduced. Although this is an increase in number (see Tab. 4.5), the graph now has 3 strongly connected components and 5 non-final no-return sets consisting of a single state, i.e. the structure fosters stronger predictions of system behaviour (see Fig. 4.9). Even more preferable, there a two invariant sets of vertices which can be clearly evaluated: One admits only final states with eutrophication, the other admits only

Parameter	Value	Units
b	$0.1 \cdot 10^{-2}$	a^{-1}
h	0.15	a^{-1}
r	$1.9 \cdot 10^{-2}$	a^{-1}
s	0.7	a^{-1}
m	2.4	$kg\ m^{-2}$
q	8	dimensionless

Parameter	Interval
b	$[0.05, 0.15] \cdot 10^{-2}$
h	$[0.10, 0.20]$
r	$[1.50, 2.50] \cdot 10^{-2}$
s	$[0.60, 0.80]$
β	$[0.80, 1.20]$
ϕ	$[0.00, 1.00]$

Table 4.4: Left: parameter values of the lake model of Carpenter (2003). Right: interval estimates for model parameters, containing the values of the original model.

Graph	Vertices	Edges
State-transition graph		
after removing marginal edges in runtime	61	172
Removing marginal edges	55	105
Removing non-analytical states	33	54
After applying ORDAS algorithm	41	72

Table 4.5: Number of vertices and edges resulting from different restriction techniques applied subsequently to the lake management model.

final states where P decreases below λ_*. Thus, we cannot conclude that the proposed rule in its general form is always successful, but it is successful once the positive invariant set is reached. In the large no-return set this cannot be predicted from our general assumptions. For crisper results we pick out some interesting states in the graph and investigate the tendency that the system shifts into one or the other successor state by using quantitative bounds (cf. section 3.4, p. 77).

A Linear-Interval Version of the Management Model

We consider the qualitative state v_1 where $\mathrm{qmag}_P(v_1) = \{\lambda_*, \lambda_2\}$, $\mathrm{qdir}_P(v_1) = \mathrm{qdir}_\phi(v_1) = [+]$, and $\mathrm{qdir}_M(v_1) = \mathrm{qdir}_L(v_1) = [-]$; denoted as state (A) in Fig. 4.9. In this state the next successor can determine the fate of the system: If the input level L begins to increase, the large neutral no-return set is entered. In this case there are several succeeding paths of length 2 leading to the negative invariant set or to an eutrophic state. If the phosphorus in the sediment begins to increase, entering the neutral no-return set is just postponed, with the same of risk of reaching the negative invariant set. In contrast, if P decreases, the phosphorus in the water column will necessarily fall below λ_* – no matter which affine control function the lake manager chooses. The question is whether there is a reasonable tendency for the latter outcome.

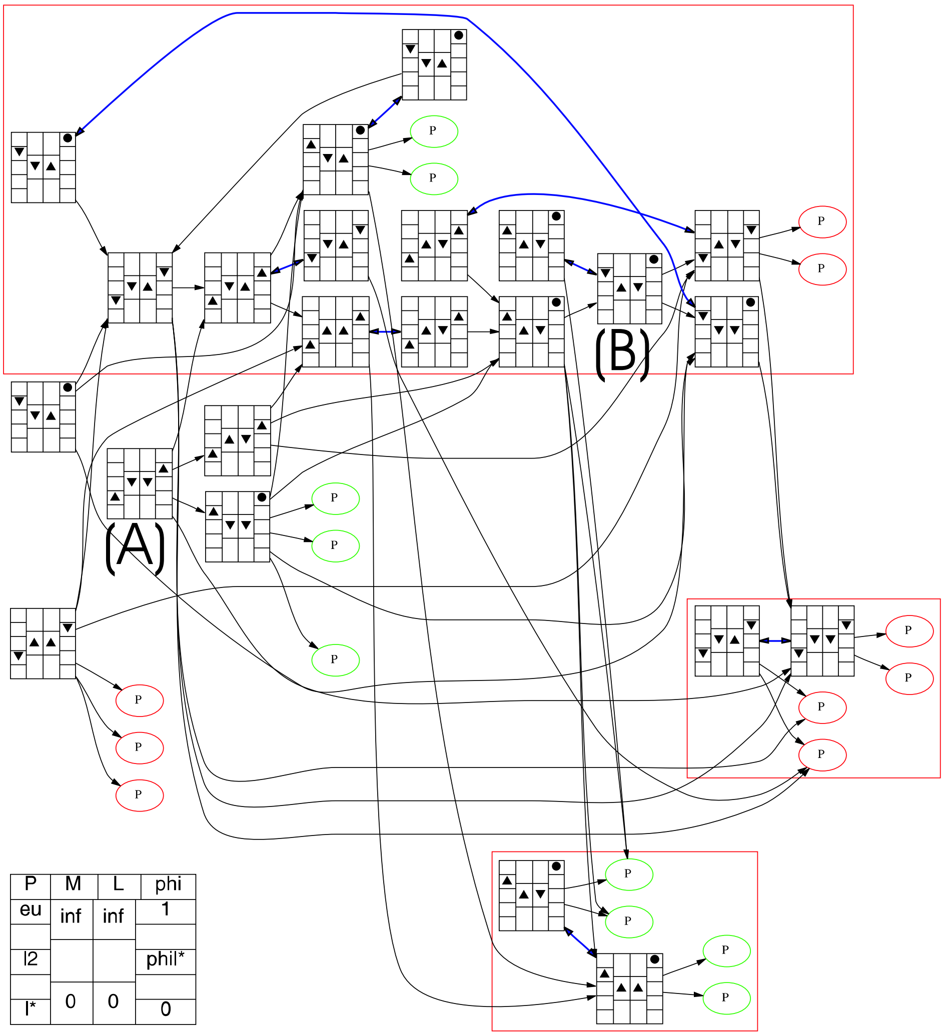

Figure 4.9: Restricted state-transition graph of the lake model with qualitative control rule and ordinal assumptions as discussed in the text. Final states where phosphorus exceeds the unacceptable threshold λ_{eu} are marked as green ellipses, while final states where P falls below λ_* are red.

We now formulate a linear-interval differential inclusion as introduced in section 3.4 (p. 77) which is valid in the qualitative state described above. We compute the absorption basin of each successor state, i.e. all initial velocities which necessarily lead to a given successor state. For this task quantitative intervals for the components of the Jacobian of the system are needed. The values of the original model are replaced by intervals to account for uncertainties. To define the affine closed-loop control function $u(P) = \alpha - \beta P$, also choose an interval for β since the actual phosphorus input L can substantially differ from an input target $u(P)$ (Carpenter 2003). The intervals are given in Tab. 4.4. Little is known about actual values of the non-linear part ϕ. Although by assumption $\mathrm{qmag}_\phi(v) = \{\texttt{phil*}, 1\}$, a tight estimate is difficult since also `phil*` is not known quantitatively. For sake of generality we only assume that $\phi \in [0, 1]$. This has two important consequences: First, in this formulation ϕ is no longer a function of P, but an independent parameter. Secondly, this parameter can

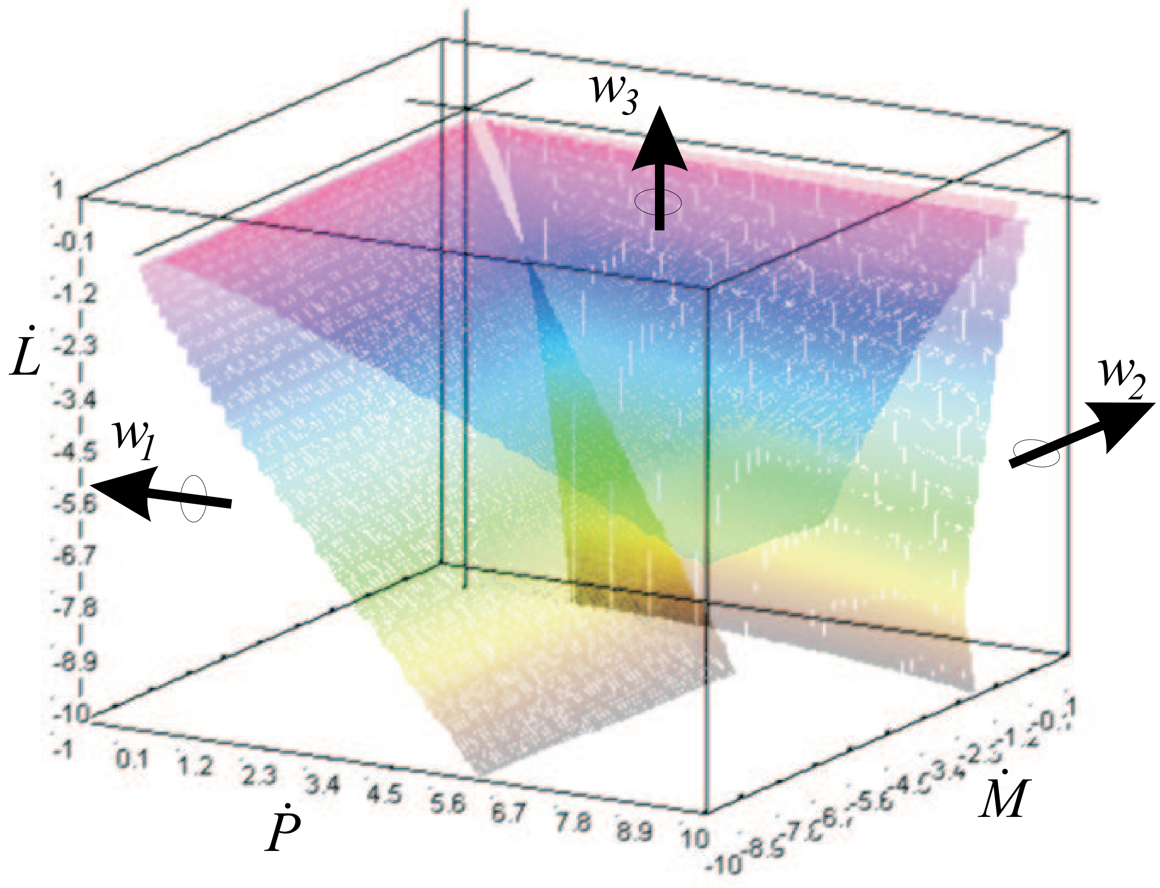

Figure 4.10: The absorption basins $\mathrm{Abs}_F(\bar{K}(v_1), \bar{K}(v_1) \cap \bar{K}(w_1)) \cap Q$, $\mathrm{Abs}_F(\bar{K}(v_1), \bar{K}(v_1) \cap \bar{K}(w_2)) \cap Q$ and $\mathrm{Abs}_F(\bar{K}(v_1), \bar{K}(v_1) \cap \bar{K}(w_3)) \cap Q$.

change arbitrarily in time (as long as $\phi(\cdot)$ remains measurable). The interpretation of the dynamics brought about by a linear-interval differential inclusion is thus different from the QDE dynamics as already discussed in section 3.4 (p. 77). Having said this, the appropriate set-valued map (cf. Eq. 4.26, p. 118) is defined by the interval matrix

$$U = \begin{pmatrix} [-1.00, -0.70] & [0, 0.0250] & 1 \\ [0.60, 0.80] & [-0.0265, -0.0005] & 0 \\ [0.56, 1.20] & [-0.0300, 0] & [-1.2, -0.8] \end{pmatrix}.$$

We use the viability kernel algorithm to compute the absorption basins

$$\begin{gathered} \mathrm{Abs}_F(\bar{K}(v_1), \bar{K}(v_1) \cap \bar{K}(w_1)), \\ \mathrm{Abs}_F(\bar{K}(v_1), \bar{K}(v_1) \cap \bar{K}(w_2)), \\ \mathrm{Abs}_F(\bar{K}(v_1), \bar{K}(v_1) \cap \bar{K}(w_3)), \end{gathered}$$

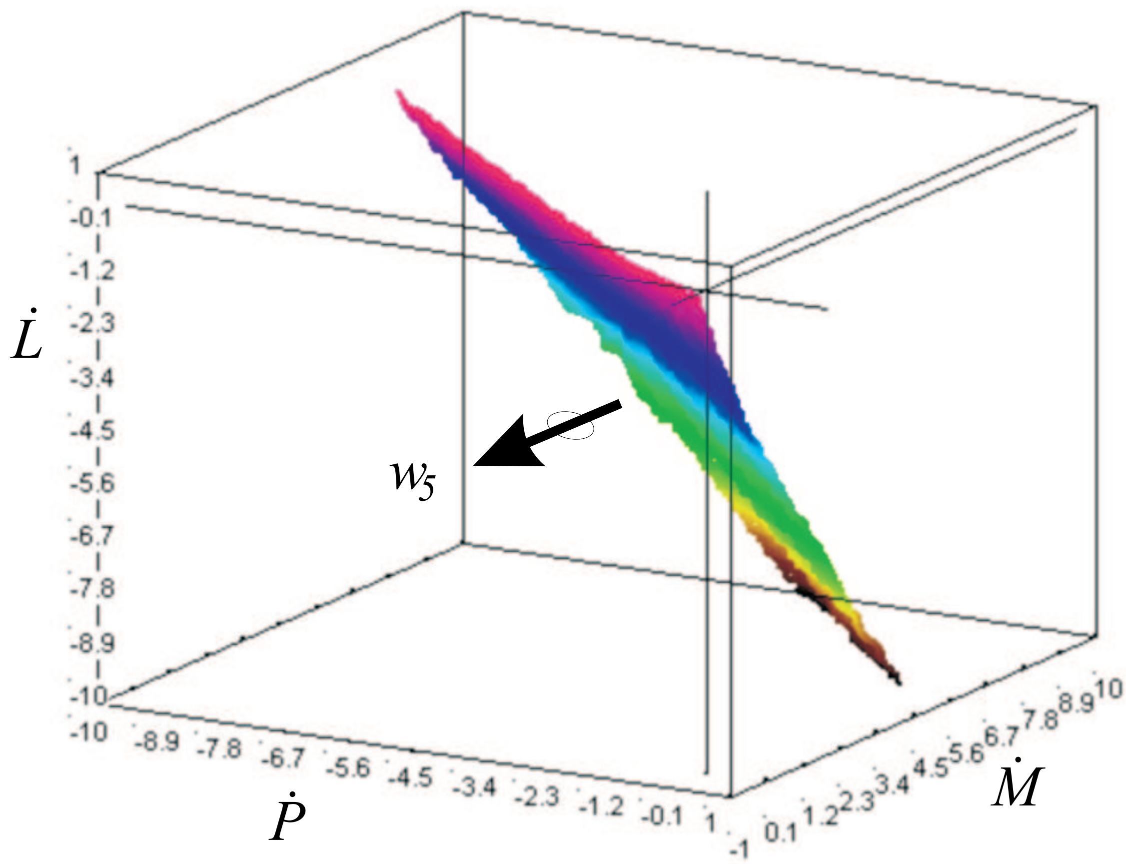

Figure 4.11: The absorption basins $\mathrm{Abs}_F(\bar{K}(v_2), \bar{K}(v_2) \cap \bar{K}(w_4)), \mathrm{Abs}_F(\bar{K}(v_2), \bar{K}(v) \cap \bar{K}(w_5))$, restricted to a cube. The former is too small to be seen in this presentation.

with $F : \mathbb{R}^3 \rightsquigarrow \mathbb{R}^3, (\dot{P}, \dot{M}, \dot{L}) \rightsquigarrow U(\dot{P}\,\dot{M}\,\dot{L})^t$ and

$$\begin{aligned} v_1 &= ([+]\,[-]\,[-])^t, \\ w_1 &= ([-]\,[-]\,[-])^t, \\ w_2 &= ([+]\,[+]\,[-])^t, \\ w_3 &= ([+]\,[-]\,[+])^t. \end{aligned}$$

Recall that for $v \in \mathcal{A}^n, K(v) = \{\dot{x} \in \mathbb{R}^n \mid \mathrm{sgn}(\dot{x}) = v\}$, such that the absorption basins contain all initial values for which the system necessarily shifts from state v_1 to state w_1, w_2 or w_3 (cf. section 3.4, p. 77). The boundary of all three absorption basins, restricted to the cube $Q = [0, 10] \times [-10, 0] \times [-10, 0]$ is depicted in Fig. 4.10. The volume of $\mathrm{Abs}_F(\bar{K}(v_1), \bar{K}(v_1) \cap \bar{K}(w_1)) \cap Q$ is of considerably large compared to the other absorption basins (restricted to Q) – this can be interpreted that within the given parameter ranges there is a considerable chance that the system enters the invariant set where the

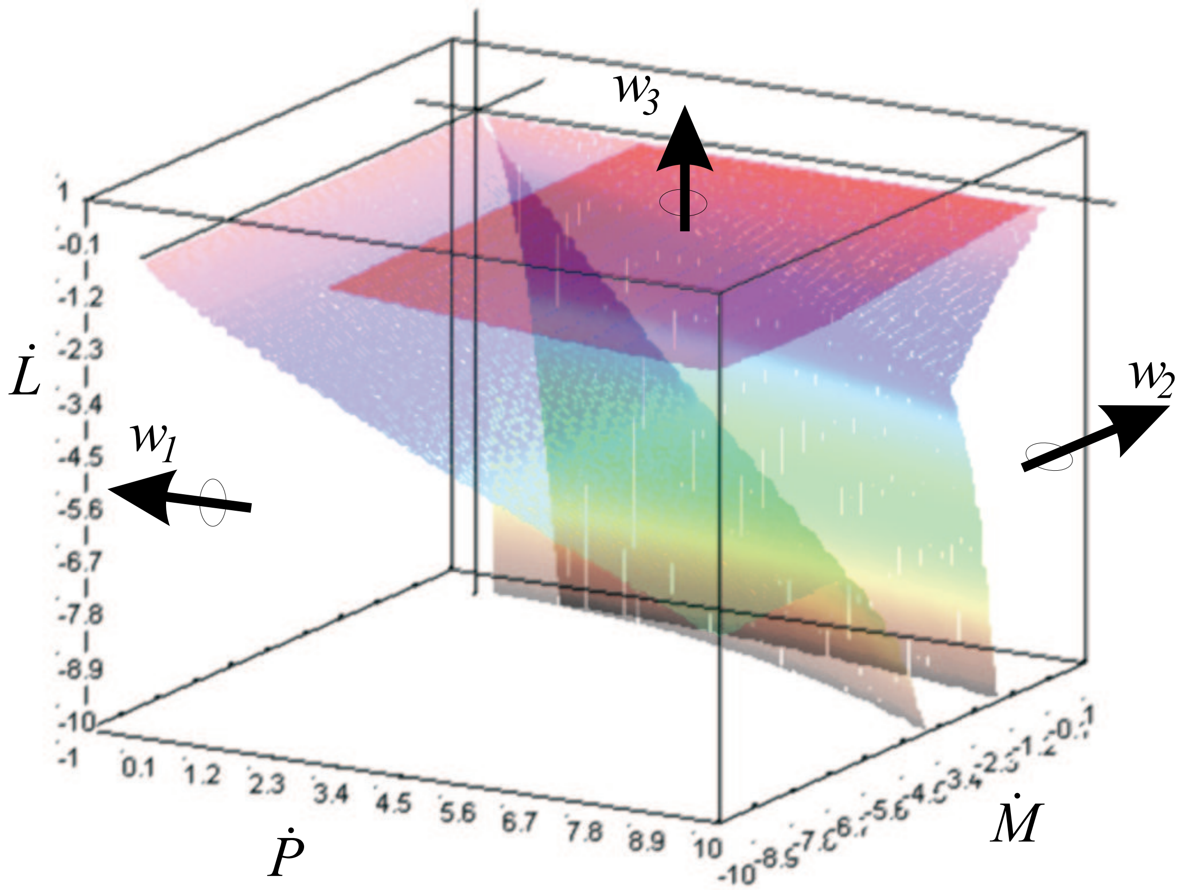

Figure 4.12: The absorption basins for successor states of v_1 for $\beta \in [0.2, 0.4]$.

phosphorus concentrations necessarily decreases. However, none of the three successors can be generally excluded. The volume of $\mathrm{Abs}_F(\bar{K}(v_1), \bar{K}(v_1) \cap \bar{K}(w_2)) \cap Q$, where the phosphorus content in the sediment will begin to increase again is the smallest compared to the other absorption basins. This is different from the state v_2 denoted by (B) in Fig. 4.9, where $\mathrm{qmag}_P(v_2) = \{\lambda_2, \lambda_{eu}\}$, $\mathrm{qdir}_M(v_2) = [+]$, and $\mathrm{qdir}_P(v_2) = \mathrm{qdir}_L(v_2) = [-]$. For one successor w_4, P begins to increase and there is the risk of entering the invariant set with unavoidable eutrophication. There is only one other combination of qualitative directions of the successor states of v_2, which is denoted by w_5: M begins to decrease. Again we use the parameter ranges given in Tab. 4.4, with one exception: since in v_2 unambiguously $\phi(P) = 1$, one obtains

$$U = \begin{pmatrix} [-1.00, -0.70] & [0.0150, 0.0250] & 1 \\ [0.60, 0.80] & [-0.0265, -0.0155] & 0 \\ [0.56, 1.20] & [-0.0300, -0.0120] & [-1.2, -0.8] \end{pmatrix},$$

yielding the absorption basins as indicated in Fig. 4.11. Here, $\mathrm{Abs}_F(\bar{K}(v_2), \bar{K}(v_2) \cap \bar{K}(w_4))$ is significantly smaller than the other absorption basin – we conclude that in state v_2 the linear-interval control strategy performs very well. It is also possible to assess how absorption basins change if the management strategy is modified. Assuming a smaller value for $D_P u$, e.g. $\beta \in [0.2, 0.4]$, the absorption basins associated to v_1 are depicted in Fig. 4.12. In contrast to Fig. 4.10, the volume of the absorption basin leading to an increasing M or L (restricted to Q) becomes much more smaller. There is a large absorption basin of initial velocities from which every trajectory enters the preferable invariant set. Thus, a less "sensitive" lake management is profitable under the conditions of state v_1.

We conclude from this section that making ordinal assumptions can substantially improve the structure of the state-transition graph and that the linear-interval version of the qualitative model gives clear hints for the tendency of alternative system changes in critical states. Both proved to be useful tools for the design of resource management strategies. As in the previous section, the search for a preferable qualitative control is performed on a finite set of possible alternatives. Ordinal assumptions provide further degrees of freedom for management design. Although the search space for management options becomes infinite when using linear-interval differential inclusions, it can be used to refine results already obtained by qualitative reasoning. Thus, the two methods complement each other.