Chapter 1

Introduction

This thesis is devoted to the study of infinite ensembles of ordinary differential equations with common monotonicity properties. Such ensembles arise in the research about sustainable development, which aims to meet current human needs while maintaining the environment and natural resources for future generations (WCED 1987). In the domain of sustainability science, uncertainties about dynamic social-ecological systems pose major challenges, and typologies of such systems are an important research field.

I present qualitative differential equations (QDEs) and differential inclusions as important methods for this task and introduce the general framework of *model ensembles* which includes both methods as special cases. They are complemented with concepts from viability theory to combine reasoning about dynamics with the evaluation of possible development paths. Due to the general nature of infinite ensembles, they do not produce unique trajectories but a large set of solutions. I develop several new techniques to evaluate and simplify such solutions sets in this thesis. The strengths of the new methods, as well as the strengths of QDEs in general, are demonstrated with several applications from natural resource management.

There are many instances where natural resources degrade even if they are managed in some way. There is an urgent need to assess management regimes and design sustainable options, as an important goal of sustainability science which “seeks to understand the fundamental character of interactions between nature and society. Such an understanding must encompass the interaction of global processes with the ecological and social characteristics of particular places and sectors” (Kates et al. 2001).

Syndrome Research and Management of Renewable Resources

The methods I develop and present in this thesis are motivated by and applied to problems from natural resource overuse by agriculture, fishery and water management.

Soils and water are crucial for agriculture. However, approximately 15% of the terrestrial surface is covered by soils which are degraded by human activities. These results of intensive industrial agriculture or overuse of marginal land – probably under the additional pressure of climate change – are most visible in the poor regions of the world (WBGU 1994). Management tries to change land-use practices and the forces driving overuse, for example by development programs.

Marine fish stocks are under extreme pressure worldwide. Approximately 50% of the world's fish resources are fully exploited, 20% overexploited, and 10% depleted (FAO 2004). Even though over-fishing has been a fact since historical times (Jackson et al. 2001), the problem has gained a new quality due to the industrialisation of commercial fishing, which has reduced community biomass by 80% within 15 years of exploitation by highly capitalised fishing fleets (Myers and Worm 2003). Common management approaches include catch quotas and effort limitations.

Fresh water is essential for basic human needs, health, and food production. It is also an indispensable element to maintain biodiversity and ecosystems, which, in turn, provide fresh water. Human withdrawal of water is mainly for agriculture (69%), but also for industrial uses (23%) and households (8%) (WBGU 1997). Management efforts are centred around the task of sustaining water quantity and quality, the latter being, among other influences, threatened by eutrophication.

Patterns of non-sustainable use emerge from interactions between society and ecosystems. Although they may be coupled via global processes, they occur mostly on a regional or local scale. However, they can be observed at many places on Earth in a more or less similar way. This qualifies them as so called *syndromes of global environmental change* (Schellnhuber et al. 1997). The syndrome approach seeks to analyse the interactions of the most relevant processes of global change while preserving local and regional peculiarities by considering case studies. To provide a global view of local and regional dynamics of environmental degradation, it aims at the identification of typical functional patterns of social-ecological dynamics (Lüdeke et al. 2004). This poses various research challenges. A typology of patterns requires an adequate notion of similarity between instances of problems, and strategies to mitigate or prevent syndromes have to be developed (Petschel-Held et al. 1999).

Methodological Challenges

Causal loop diagrams and mathematical models are important tools to classify and understand syndromes (Schellnhuber et al. 2002). Once a set of variables which describes interconnected relevant processes in a particular case study is chosen, these diagrams are a well-established tool to formulate their interactions (Petschel-Held et al. 1999; Stave 2002). In its simplest form, a causal loop diagram is a directed graph with marked edges. Each vertex represents a variable, and each edge an influence of the source variable on the target variable which can be marked as positive or as negative (for an example, see Fig. 1.1). The meaning of the edges is vague at this stage, but needs to be made explicit for modelling purposes (see below). In traditional systems dynamics modelling (Forrester 1968; Sterman 2000), the causal loop diagram is a starting point to develop a quantitative model, usually in the form of an ordinary differential equation (ODE). In contrast, within the context of syndrome research we do not "refine" the diagram to that point, so that it still subsumes a broad range of systems which share only the structure expressed by the diagram. It is an "archetype" for a generalised pattern of global environmental change.

In addition to finding generalised patterns, modelling in the domain of syndrome research and sustainability science has to face various challenges, including the following (Eisenack

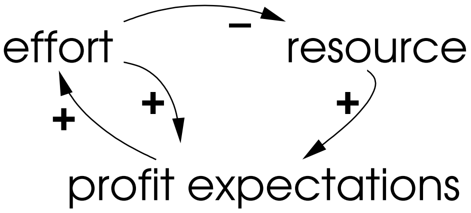

Figure 1.1: Example for a causal loop diagram: simplified version of the Overexploitation Syndrome (Cassel-Gintz and Petschel-Held 2000; Kropp, Eisenack, and Scheffran 2006).

et al. 2002):

- Generality: models should provide insights for single applications, but should also apply to a broader set of cases with general features in common. Such communalities between different cases are important to obtain a global overview, to classify different instances, and are the base for transferring best practices.
- Uncertainty: many interactions of social-ecological systems are not known quantitatively, knowledge about the processes is often limited, there are data gaps or unpredictable future influences, e.g. depending on strategic political choice. Under such conditions, the modeller cannot discriminate between reasonable alternatives, but if urgency to solve a problem is high, the analysis and the management strategies should be robust.
- Complexity: social-ecological systems tend to be composed of many tightly coupled, non-linear subsystems and interactions which are often difficult to disentangle.
- Normativity: it is necessary to classify patterns of interactions as problematic or desirable, e.g. to distinguish syndromes from other functional patterns. This involves value judgements which cannot be made by science. However, normative knowledge has to be considered in a transparent way and research can contribute to the assessment and development of management practices.
- Non-quantitative knowledge: to understand social-ecological systems, knowledge from different disciplines and with different degrees of quantification has to be integrated.

Qualitative Reasoning with Model Ensembles

By considering not a single model but a whole ensemble of models, some degree of generality can be obtained and a variety of possible system configurations which we can think of under uncertainty can be covered. In contrast to so called ensemble runs, where solutions for a finite set of parameterisations of a model are computed to determine, e.g. sensitivity to parameters (e.g. Stainforth et al. 2005), we will deal with infinite ensembles. I introduce the concept of a model ensemble as a (possibly infinite) set $\mathcal{M}$ of functions, where each $f \in \mathcal{M}$ constitutes an ordinary differential equation $\dot{x} = f(x,t)$. It is clear that such an approach does not yield unique solutions in general. The task is to describe $\mathcal{M}$ in a concise way which only considers knowledge which is certain to a high degree and which allows for an aggregated computation of the resulting (infinite) solution set.

This systematic framework can be used to define differential inclusions, QDEs and traditional methods on a common base. Differential inclusions represent an important approach to account for generality and uncertainty (Aubin and Cellina 1984), since contingent dynamics can be computed even if no probabilistic knowledge is available. In contrast, qualitative differential equations (Kuipers 1994) are deterministic but subsume a broad set of possible configurations. For the assessment of large solution sets, viability theory provides a powerful way to consider normative issues in a formal way by investigating whether solutions satisfy prescribed state constraints (Aubin 1991). Thus, some of the new methods to deal with model ensembles I develop in this thesis draw on viability theory.

A causal loop diagram is a third way to define a model ensemble. I will present a precise definition which specifies a criterion for an ODE to be consistent with a causal loop diagram. Since the diagram only contains qualitative information, there is an infinite number of such ODEs for a given diagram. Hence, $\mathcal{M}$ is defined to cover every consistent ODE, because *all* of them should be considered as possible realisations.

We will see that QDEs provide all solutions of such a model ensemble. In the graph theoretical terminology I propose, they allow for the computation of a state-transition graph which covers all trajectories consistent with a causal loop diagram. More complex model ensembles can also be treated. In many cases relevant conclusions can be drawn from a state-transition graph. However, due to the generality of the diagram, common qualitative reasoning has its limitations because we usually obtain a very large state-transition graph. Dealing with this problem is the main content of this thesis. I pursue two general strategies:

- Automated detection of structures in large state-transition graphs which are relevant for sustainability issues.
- Simplification of state-transition graphs by posing additional model assumptions without restricting the model ensemble to a singular ODE.

Outline

The thesis is motivated by the following hypotheses:

- Current qualitative reasoning methods can be substantially improved for applications.
- Model ensembles are the common root of methods operating under uncertainty and generality, subsuming QDEs, differential inclusions and causal loop diagrams. They meet central challenges of sustainability science.
- Even under uncertainty and generality, robust properties of social-ecological systems can be found. This contributes to the management of natural resources.
- QDEs can be used to design viable control strategies.

In Chapter 2 I begin with the formalisation of the idea of model ensembles. Using this concept, QDEs are introduced and partially re-formulated using graph theory. An overview is given of common strategies to handle large state-transition graphs from a new and systematic perspective which draws on model ensembles and the graph theoretical formulation. This perspective is also used in later chapters. Finally, basic results from differential inclusions and viability theory are summarised. In Chapter 3 I present new techniques to analyse and simplify state-transition graphs of large QDEs by introducing new types of formal assumptions which are motivated by problems from sustainability science. The first technique identifies normative relevant subgraphs by transferring concepts from viability theory to graph theory. The second eliminates edges of limited importance from the state-transition graph. The third refines model ensembles by making assumptions on the order of the coefficients of the Jacobian of a system. Finally, I combine QDEs, differential inclusions and viability theory to investigate how knowledge about intervals which enclose partial derivatives yields stronger results. All these methods are applied in Chapter 4 to problems of natural resource management. At first, interventions which may stop the so called impoverishment-degradation spiral which can be observed in developing countries are presented. Then, I investigate several problems of marine fisheries: the problematic capital dynamics in the unregulated case, and conditions under which a participatory process to determine catch restrictions is effective. Finally, options for a lake manager to prevent eutrophication are compared. Chapter 5 concludes the thesis with a summary of the scope and limits of the methods developed in the first chapters with respect to the experience from Chapter 4.