Model Ensembles for Natural Resource Management: Extensions of Qualitative Differential Equations Using Graph Theory and Viability Theory

Dissertationsschrift
vorgelegt von

Klaus Eisenack

Berlin, 30. September 2005

Fachbereich Mathematik und Informatik
der Freien Universität Berlin

Betreuer und erster Gutachter:
Prof. Dr.-Ing. Rupert Klein, Freie Universität Berlin

Zweiter Gutachter:
Prof. Dr. Patrick Saint-Pierre, Université Paris-Dauphine

Disputation am 13. Februar 2006

For Gerhard

Preface

Anecdote Concerning the Lowering of Productivity

A fisherman is dozing in the sun in his rowing boat. A tourist asks: “The weather is great and there’s plenty of fish, so why are you lying around instead of catching more?”
The fisherman replies: “Because I caught enough this morning.”
“But just imagine,” the tourist says, “you could go out three or four times a day and bring home three or four times as much fish! And then you know what could happen?” – The fisherman shakes his head.
“After a year you could buy yourself a motorboat, and after two years a second one. One day you might be able to build a freezing plant.” – “And then?” asks the fisherman.
“Then you could spend your time at the harbour, dozing in the sun and looking at the beautiful ocean.”
“But that is exactly what I am already doing,” says the fisherman. (based on Böll 1986)

This work is motivated by concerns about the way our society interacts with its natural environment. The story about the fisherman illustrates how social processes shape the state of natural resources. I’m convinced that mathematical methods can help designing sustainable futures – if they are used with a respectful attitude towards other domains of knowledge. I hope my work contributes to this task.

The thesis was written at the Department of Integrated Systems Analysis at the Potsdam Institute for Climate Impact Research. Parts were funded by the Deutsche Forschungsgemeinschaft (German Research Association, project PR 1175/1-1 “Viabilität nicht-linearer dynamischer Systeme unter Unsicherheit”) and by the EU (Research Training Network HPRN-CT-2002-00281). I would like to thank my supervisor R. Klein for his open-mindedness and support. Further thanks go to my second reviewer, P. Saint-Pierre, and to J.-P. Aubin, which both introduced me to viability theory. In particular, I wish to thank J. Kropp and M. Lüdeke for general support, proofreading and numerous discussions which advanced my work. For various discussions and assistance I would like to thank N. Bauer, J. Scheffran, B. Knopf, T. Kleinen and further colleagues, in particular from the Eureca and Synapse teams. R. Holmes helped me by carefully proof-reading the English text. Last but not least, I’m grateful to Gerhard Petschel-Held whose life came to a sudden end some days ago. He inspired my scientifc thought, backed me – and we had great fun in designing and playing games. My thoughts go to his wife Ulrike Held. My deepest gratitude goes to Anja Wirsing and my parents for keeping me grounded and for extensive encouragement.

Klaus Eisenack
Potsdam, September 2005

Contents

Notations and Concepts

$\mathbb{R}, \mathbb{R}_+, \mathbb{R}_-$	real numbers, non-negative and non-positive real numbers
$\mathbb{N}$	natural numbers (including 0)
$X \subseteq \mathbb{R}^n$	state space
$C(X,Y), C^1(X,Y)$	space of continuous and continuously differentiable functions from X to Y
$x(\cdot), (x_i)$	trajectory or solution on $\mathbb{R}_+$, sequence
$\dot{x}(\cdot) = \frac{d}{dt}x(\cdot)$	derivative with respect to time
$D_x f = \frac{\partial f}{\partial x}, D_j f_i = \frac{\partial f_i}{\partial x_j}$	partial derivatives
$\mathcal{J}(f), \mathcal{P}(X)$	Jacobian of a function f, power set of a set X
D^c	complement of a set D
$\langle x_1, x_2, \ldots, x_n \rangle$	ordered set
$(x_1 \, x_2 \, \ldots \, x_n)^t$	transpose of a vector

Model Ensembles

$\mathcal{M}$	model ensemble
$\mathcal{E}$	space of admissible trajectories
$\mathcal{M}(\Sigma)$	monotonic ensemble (DEF. 1, p. 22)
$\mathcal{M}(\mu, C)$	monotonic landmark ensemble (DEF. 4, p. 29)
$\mathcal{S}_{\mathcal{M}}(\cdot)$	set-valued solution operator

Qualitative Differential Equations

$\mathcal{A}$, $\mathcal{A}_*$	domain of signs and of extended signs $[-], 0, [+], [?]$
$\sigma, \sigma_i, \sigma_{i,j}$	(extended) sign
$[\cdot], [\cdot]_\lambda$	sign operators, extended to vectors and matrices (p. 22, 28)
$\approx$	consistency of signs, sign vectors or sign matrices (p. 22)
Σ	matrix of extended signs
v, w	sign vectors, qualitative states or vertices of a state-transition-graph
$\tilde{x}$	(landmark) abstraction of a reasonable function $x(\cdot)$ (DEF. 2, p. 23, DEF. 5, p. 29)
$\mathcal{S}_{\mathcal{M}(\Sigma)}(\cdot)$	solution operator of a monotonic ensemble (p. 22)
λ, Λ	landmark, landmark vector (p. 28)
Q, S	quantity space, qualitative state space (p. 28)
$\mathrm{qmag}(v), \mathrm{qdir}(v), \mathrm{qval}_i(v)$	qualitative magnitude, qualitative direction, qualitative value of a qualitative state v (Eq. 2.8, Eq. 2.10, Eq. 2.6, p. 28)
$\mathcal{S}_{\mathcal{M}(\mu,C)}(\cdot)$	solution operator of a monotonic landmark ensemble (p. 29)
μ	map from a quantity space $Q \to \mathcal{A}_*^{n \times n}$
$C = \{C_1, \ldots, C_m\}$	set of constraints on a qualitative state space
$a_\Lambda(x, \dot{x})$	state abstraction with respect to a landmark vector Λ (p. 28)
$Z_0(v)$	vanishing indices of a sign vector v (p. 24)
$v \wedge w$	intermediate state of qualitative states v, w (p. 25, 30)
$\pi_I(G)$	simple projection of a state-transition graph G with respect to an index set I (DEF. 11, p. 39)

Differential Inclusions and Viability Theory

$F : X \rightsquigarrow Y$	set-valued map from X to the power set $\mathcal{P}(Y)$ (p. 42)
$\mathrm{Dom}(F), \mathrm{Graph}(F)$	domain and graph of a set-valued map F (p. 42)
$\mathcal{S}_F(\cdot)$	solution operator of a differential inclusion given by a set-valued map F (p. 43)
$\mathrm{Viab}_F(K, C)$	viability kernel of K with target C (under a set-valued map F, DEF. 13, p. 46)
$\mathrm{Inv}_F(K, C), \mathrm{Abs}_F(K, C)$, $\mathrm{Capt}_F(K, C)$	invariance kernel, absorption basin and capture basin of K with target C (DEF. 13, p. 46)

Graph Theory

G	directed graph
$V(G), E(G)$	set of vertices and edges of a graph G
v, w and e, f	vertices and edges
$v_1, \ldots, v_m$	path in a graph
$\Gamma(v), \Gamma^*(v)$	successors of vertex v in G and in its transitive closure G^*
G^*, G^{-1}	transitive closure, reversal of a graph G
G^-	bi-directed subgraph of G (p. 38)

For a directed graph G with **vertices** $V(G)$, $E(G) \subseteq V(G) \times V(G)$ is the set of **edges**. Since we deal only with undirected graphs, we simply call them graphs. For an edge $e = (v, w)$, v is called the **source** of e, and w its **target**. The set of **successors** of a state $v \in V(G)$ is denoted by $\Gamma(v) := \{w \in V(G) \mid (v, w) \in E(G)\}$. The number $|\Gamma(v)|$ is the **degree** of the vertex v. A graph G is **loop free** if $\forall v \in V(G) : (v, v) \notin E(G)$. A finite sequence of vertices in $V(G)$, written $v_1, \ldots, v_m$, is called a **path** of length $(m-1)$ if $\forall i = 1, \ldots, m-1 : v_{i+1} \in \Gamma(v_i)$, and no state occurs more than once in the sequence. The only exception to the last restriction is if $v_m = v_1$, when the path is called a cycle. The restriction to finite sequences is sufficient for this thesis. The **transitive closure** G^* of G is defined by $V(G^*) := V(G)$ and

$$E(G^*) := \{(v, w) \in V(G) \times V(G) \mid \text{ there is a path } v, \ldots, w \text{ in } G\}.$$

A graph H with $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$ is a **subgraph** of the **supergraph** G. A graph H is induced by a subgraph H' of G if $V(H) = V(H')$ and

$$E(H) = \{(v, w) \in E(G) \mid v, w \in V(H')\}.$$

A subgraph H of G is **strongly connected**, if for all $v, w \in V(H)$, $v \neq w$, there is a path $v, \ldots, w$ and a path $w, \ldots, v$. It is called a **strongly connected component** if it is a maximal strongly connected subgraph with respect to inclusion of vertices. For two graphs G, H, the **intersection** $G \cap H$ is defined by $V(G \cap H) = V(G) \cap V(H)$, $E(G \cap H) = E(G) \cap E(H)$ and the **union** $G \cup H$ by $V(G \cup H) = V(G) \cup V(H)$, $E(G \cup H) = E(G) \cup E(H)$. For an edge $(v, w) \in E(G)$, its reversal is $(w, v) \in V(G) \times V(G)$. The **reversal** G^{-1} of G is obtained by reversing all edges, i.e. $V(G^{-1}) := V(G)$,

$$E(G^{-1}) := \{(v, w) \in V(G) \times V(G) \mid (w, v) \in E(G)\}.$$

An edge $e \in E(G)$ is **bi-directed** if $E(G)$ also contains its reversal.