Appendix B

Coherent states

For a harmonic oscillator with mass m and frequency ω the creation and annihilation operators are given by:

$$a^\dagger = \sqrt{\frac{\gamma}{2}}\left(\hat{q} - \frac{i}{\hbar\gamma}\hat{p}\right), \tag{B.1}$$

$$a = \sqrt{\frac{\gamma}{2}}\left(\hat{q} + \frac{i}{\hbar\gamma}\hat{p}\right), \tag{B.2}$$

where $\gamma = m\omega/\hbar$ and, respectively, $\hat{q}$ and $\hat{p}$ are the position and momentum operators. Non-normalized eigenstates are generated by successively applying the creator $a^\dagger$ to the ground state $|0\rangle$ of the harmonic oscillator. A coherent state is defined by [32]:

$$|z\rangle = \exp\{-|z|^2/2\}\ \exp\{za^\dagger\}\,|0\rangle, \tag{B.3}$$

with an arbitrary complex number z. The coherent states $|z\rangle$ ($\langle z|$) are right (left) eigenfunctions of the annihilation (creation) operator with eigenvalue z (z^*), i.e.,

$$a\,|z\rangle = z\,|z\rangle, \quad \langle z|a^\dagger = \langle z|z^*. \tag{B.4}$$

The straight forward proof uses the expression of the exponential according to $e^x \equiv \sum_{n=0}^{\infty} x^n/n!$. The prefactor $\exp\{-|z|^2/2\}$ ensures normalization. The coherent states fulfill a completeness relation of the form [32]:

$$\int \frac{dz}{\pi}|z\rangle\langle z| = 1, \tag{B.5}$$

but two individual states are not orthonormal. This behavior has been termed *overcompleteness*: one can express any normalized state by coherent states, but the expression is non-unique. It should be noted that the overcomplete state basis depends on the parameter γ; if this is important, it will be notified in the form $|z;\gamma\rangle$.

A coherent state can be characterized by the expectation value of position q_e (corresponding to the operator $(a + a^\dagger)/\sqrt{2\gamma}$) and momentum p_e (corresponding to the operator $\sqrt{\gamma}\hbar(a - a^\dagger)/i\sqrt{2}$). One observes:

$$z = \sqrt{\frac{\gamma}{2}} \left(q_e + \frac{i}{\hbar\gamma} p_e \right) . \tag{B.6}$$

The position state representation of the coherent states is given by Eq. (2.84). In the main the simplifi ed notation $\mathbf{z} = (\mathbf{p}_e, \mathbf{q}_e)$ is used for convenience.