

# **Multidimensional Tunneling in Hydrogen Transfer Reactions**

**INAUGURAL - DISSERTATION**

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## Erklärung

Hiermit versichere ich, dass ich die vorliegende Arbeit mit den angegebenen Hilfsmitteln selbständig angefertigt habe.

Berlin, den 16. September 2005