Monetary and Fiscal Policy Interaction in the Enlarging European Economic and Monetary Union

Essays on Business Cycles and Welfare

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Zusammenfassung

Das Euro-Währungsgebiet steht vor neuen Herausforderungen zu Beginn des zweiten Jahrzehnts der gemeinsamen Währung. Auch vor dem Hintergrund des einheitlichen Binnenmarktes sowie einer fortschreitenden Finanzmarktintegration ist der konjunkturelle Gleichlauf zwischen Mitgliedsländern im Bruttoinlandsprodukt, der Konsumentenpreisinflation und im privaten Verbrauch noch unvollkommen. Inflationsdifferentiale zwischen den Mitgliedsstaaten sind seit 2000 wieder angestiegen und führen zu zunehmenden Unterschieden in der nationalen Wettbewerbsfähigkeit. Eine einheitliche Geldpolitik erfordert daher weiterhin einen hohen Anpassungsbedarf im Zuge von länderspezifischen und Euro-Raum weiten makroökonomischen Schocks. Strukturelle Heterogenitäten in der Organisation von Faktor- und Produktmärkten, im Offenheitsgrad und im Preissetzungsverhalten der Firmen führen jedoch dazu, dass diese Schocks (die ggf. wünschenswerte Reallokationen von Arbeit und Kapital darstellen) unterschiedlich schnell und unterschiedlich effizient verarbeitet werden können. Vor diesem Hintergrund sind lang anhaltende Wachstumsdifferentiale zwischen Ländern sowie ineffiziente Relativpreisstarrheiten möglich, die zu Wohlfahrtsverlusten führen. Die Osterweiterung des Euro-Raums in den kommenden Jahren könnte diese Effekte noch verstärken. Aufgrund der Aufholprozesse in diesen Ländern könnte jedoch auch die Erfüllbarkeit der Maastrichtkriterien und insbesondere der Test der Stabilität der Währungen im Wechselkursmechanismus II (ERM II) weiterhin kritisch bleiben.

Die vorliegende Arbeit untersucht den Beitrag von Geld- und Fiskalpolitik zur Begrenzung dieser konjunkturellen Kosten, die zum einen durch die Wechselwirkung zwischen der einheitlichen Geldpolitik und nationalen Fiskalpolitiken im Währungsgebiet und zum anderen durch die Beschränkung dieser Instrumente im Zuge der Euro-Einführung in den neuen EU-Mitgliedsstaaten Mittel- und Osteuropas entstehen. Die Fragestellungen werden im allgemeinen dynamischen Gleichgewicht im Rahmen der Neuen Makroökonomik Offener Volkswirtschaften (New Open Economy Macroeconomics, NOEM) ausgewertet. Besonderer Wert wird hierbei auf eine empirisch relevante Modellierung gelegt, welche die Entstehungsseite des BIP besser abbilden kann, als die in der verwandten Literatur weit verbreiteten Ein-Sektoren Modelle. Durchweg

werden die Länder daher als Volkswirtschaften mit zwei Produktionssektoren betrachtet, die Waren ('handelbare Güter') und Dienstleistungen ('nicht handelbare Güter') herstellen. Hierdurch gewinnt die Einbeziehung sektoraler Ursachen aggregierter nominaler und realer Rigiditäten (der aggregierten Preissetzungsdauer) in einzelnen Ländern an Bedeutung, die empirisch von Belang ist. Weiterhin werden Übertragungseffekte bei Schocks in einzelnen Ländern auf andere Länder bzw. bei gemeinsamen Schocks wesentlich davon bestimmt, welcher Sektor einer konjunkturellen Hoch- bzw. Schwächephase ausgesetzt ist. Die Untersuchung konzentriert sich auf drei wesentliche Aspekte, die in jeweils eigenen Kapiteln behandelt werden:

Im Kapitel 2 werden die Wohlfahrtseffekte anhaltender struktureller Heterogenität im gegenwärtigen und erweiterten Euro-Raum untersucht. Hierzu wird ein Modell einer Zwei-Länder Zwei-Sektoren Währungsunion entworfen und alternative Möglichkeiten geldpolitischer Steuerung ausgewertet. Die offizielle geldpolitische Strategie des Euro-Systems ist auf die Wahrung der Preisstabilität auf Basis des aggregierten harmonisierten Verbraucherpreisindex (HVPI) ausgerichtet. Das Gewicht eines Landes im aggregierten HVPI bestimmt sich anhand des Anteils der monetären Konsumausgaben der inländischen privaten Haushalte an den Gesamtausgaben aller Haushalte des Euro-Raums. Es besteht also eine direkte Korrespondenz zwischen Ländergröße und Gewicht in der einheitlichen Geldpolitik.

Im Modell zeigt sich jedoch, dass optimale Geldpolitik, die sich an der Wohlfahrt des durchschnittlichen Haushalts des Währungsraums ausrichtet, Heterogenität zwischen Mitgliedsstaaten in der Setzung des Zinssatzes mit einbeziehen würde. Neben der Ländergröße ist für das Gewicht eines Landes im Aggregat auch das Preissetzungsverhalten der Firmen, die Zusammensetzung der Entstehungsseite des BIP und des privaten Verbrauchs von Bedeutung. Die Zerlegung der Entstehungsseite des Bruttoinlandsprodukts in handelbare und nicht-handelbare Güter (bzw. in Industriegüter und Dienstleistungen) führt weiterhin dazu, dass die nominale Rigidität eines Landes im Hinblick auf Preissetzungsdauer und Inflationspersistenz zur Beurteilung des optimalen Gewichts eines Landes im aggregierten HVPI nicht mehr ausreicht. Wertschöpfung im Bereich der Dienstleistungen führt vielmehr dazu, dass neben entstehungsseitigen Faktoren auch die Gewichte der einzelnen Gütergruppen im harmonisierten Konsumentenpreisindex auf Mitgliedslandebene zusätzlich zur Ländergröße miteinbezogen werden sollten. Dieser Zielkonflikt zwischen angebots- und nachfrageseitigen Faktoren kann unter Umständen dazu führen, dass ein Land mit höherer preislicher Flexibilität vorrangig behandelt wird. Es kann also ein höheres Gewicht im aggregierten Index erhalten als durch die Ländergröße angezeigt.

Strukturelle Heterogenität führt im kalibrierten Modell für den Euro-Raum (aufgeteilt in Deutschland und andere große Mitgliedsländer) zu Wohlfahrtsverlusten pro Quartal in Höhe von rund 0,5% des langfristigen Konsumniveaus. Bei Konzentration auf die größten Mitgliedsländer, Deutschland und Frankreich, entstehen ähnlich hohe Kosten. Ein möglicher Anstieg der durchschnittlichen Verluste aufgrund der Euro-Einführung in den Mitgliedsländern der Europäischen Union aus Mittel- und Osteuropa ist jedoch nicht zu erwarten, obwohl die Heterogenität des Währungsgebietes durch den Beitritt weiter ansteigen würde. Berücksichtigt die gemeinsame Geldpolitik die Ursachen struktureller Heterogenität zwischen Mitgliedsländern, lassen sich Wohlfahrtsverluste aufgrund von ineffizienten Konjunkturzyklen in allen betrachteten Ländergruppierungen um über 55% reduzieren. Schließlich werden strukturelle Reformoptionen diskutiert die geeignet sind, die Anpassungsfähigkeit der Mitgliedsländer zu erhöhen. Es zeigt sich, dass die Erhöhung der preislichen Wettbewerbsfähigkeit das vorrangige Reformziel ist. Nationale fiskalische Stabilisierungspolitiken tragen stattdessen nicht zu einer wesentlichen Ergebnisverbesserung bei.

Im Kapitel 3 wird die Erfüllbarkeit der monetären Maastricht-Kriterien durch die neuen mittel- und osteuropäischen Mitgliedsländer der Europäischen Union untersucht. Im Gegensatz zu einigen älteren Mitgliedsländern der EU mit weiterhin eigener Währung ist die Euro-Einführung nach Erfüllung der Kriterien vorgeschrieben. Die Literatur weist vielfach auf die Spannungen in der gleichzeitigen Erfüllbarkeit des Inflations- und des Wechselkurskriteriums hin vor dem Hintergrund anhaltender preislicher und realwirtschaftlicher Konvergenzprozesse. Die Stabilisierung des nominalen Wechselkurses in Ländern mit fixem Wechselkurs (Bulgarien, Estland, Lettland, Litauen) kann die Erfüllung des Inflationskriteriums gefährden. Bei flexiblem Wechselkurs (Polen) kann zwar die inländische Inflation stabilisiert werden, hierbei ist aber mit einer nominalen Aufwertung zu rechnen. In Ländern mit Wechselkursregimen zwischen diesen Fällen (die restlichen Visegradstaaten und Rumänien) könnte sich die Erfüllbarkeit beider Kriterien als besonders schwierig erweisen. Zur Analyse wird ein Modell einer kleinen offenen Volkswirtschaft entworfen und die Erfüllbarkeit der Kriterien untersucht. Um empirisch sinnvoll interpretierbare Ergebnisse zu erhalten, wird das Modell für jedes der neun betrachteten Länder bayesianisch mit Daten der vierteljährlichen volkswirtschaftlichen Gesamtrechnungen geschätzt, wobei eine Vielzahl struktureller Schocks auf der Angebots- und Nachfrageseite zugelassen wird. Die Analyse zeigt, dass das Einhalten der monetären Kriterien unter Einbezug eines lang anhaltenden Wachstums in der totalen Faktorproduktivität in der Industrieproduktion in der Mehrzahl der Länder langfristig (nur) im Mittel gewährleistet ist. Hohe konjunkturelle Volatilität entlang dieses Konvergenzpfades verhindert allerdings, dass die gleichzeitige Erfüllbarkeit aller Kriterien als sehr wahrscheinlich angesehen werden kann. In einem zweiten Schritt wird daher der Beitrag der inländischen Geldpolitik untersucht, die Unsicherheit in der Erfüllbarkeit der Kriterien zu reduzieren. Es zeigt sich, dass auch bei aktiver Stabilisierungspolitik alle Länder weiterhin Schwierigkeiten ausgesetzt sind, die Kriterien zu erfüllen. In einem dritten Schritt werden strukturelle Ursachen für dieses Ergebnis beleuchtet. Es zeigt sich, dass ein nachhaltiges, also wiederholbares, Bestehen der Kriterien wesentlich vom Rückgang der Volatilität in strukturellen Schocks abhängig ist. Wird optimal eingesetzte Geldpolitik durch restriktive Fiskalpolitiken begleitet, könnte die Wahrscheinlichkeit des Einhaltens der Kriterien deutlich erhöht werden.

Im Kapitel 4 wird die Schockverarbeitungskapazität der Volkswirtschaft Bulgariens bei konjunkturellen und strukturellen Störungen untersucht. Im Mittelpunkt steht hierbei, wie 'reif' Bulgarien für den Beitritt in den Währungsraum ist. In der Modellierung wird insbesondere auf die Darstellung des Currency-Board-Mechanismus Wert gelegt. Im Unterschied zu der Darstellung im vorangegangenen Kapitel kommt der Geldmenge zur Beschreibung der geldpolitischen Transmission eine wichtigere Rolle zu. Es werden die Effekte transitorischer und permanenter Schocks in der totalen Faktorproduktivität auf die sektoralen Produzentenpreisinflationsraten und die Verbraucherpreisinflation untersucht. Von Bedeutung ist zudem, welche Auswirkungen anhaltende Leistungsbilanzdefizite haben und welche Implikationen für den internen realen Wechselkurs hieraus abgeleitet werden können. Die Auswirkung expansiver Fiskalpolitik auf die Handels- und Leistungsbilanz bei fixierter Geldpolitik ist weiterer Gegenstand der Analyse.

Es zeigt sich, dass Bulgarien temporäre Angebotsschocks gut verarbeiten kann. Expansive Fiskalpolitik im Hinblick auf die Nachfrage nach handelbaren oder nicht handelbaren Gütern führt zu einem Anstieg der Preise im handelbaren Sektor und verbessert somit das Austauschverhältnis zwischen Importen und Exporten. Besteht vor allem Nachfrage nach nicht handelbaren Gütern, so ist zudem mit einer Verschlechterung der externen Wettbewerbsfähigkeit zu rechnen. Abschließend wird reale Konvergenz untersucht. Ein fortschreitender und anhaltender Anstieg in der totalen Faktorproduktivität des handelbaren Sektors führt unter der Annahme eines Zuwachses der Produktivität von 10% bis 30% zu einer langfristigen realen Aufwertung im Bereich von 2% bis gut 5%.

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List of Symbols[1]

Endogenous variables	
P_t, P_t^*	consumer price indices
E_t	external real exchange rate
Q_t, Q_t^*	internal real exchange rates
T_t	terms of trade
i_t	short term interest rate
C_t, C_t^*	overall consumption baskets
$C_{T,t}$, $C_{T,t}^*$	tradables consumption baskets
$C_{N,t}$, $C_{N^*,t}^*$	nontradables consumption baskets
C_H^*, C_F	exports of Home, Foreign
$W_{J,t}$	gross wage in sector J
$MC_{J,t}$	real marginal cost in sector J
$Y_{J,t}$	gross value added in sector J
C_t^U	union consumption basket
Y_t^U	union gross value added
M_t, M_t^*	money stocks
M_t^U	union money stock
$A_{J,t}$	factor productivity in sector J
CA_t, CA_t^*	current accounts
$K_{J,t}$	capital input in sector J
$L_{J,t}$	labour input in sector J
$G_{J,t}$	fiscal expenditures in sector J
π_t, π_t^*	consumer price inflation rates
$\pi_{T,t}$, $\pi_{T,t}^*$	cons. price inflation rates T goods
$\pi_{N,t}$, $\pi_{N^*,t}^*$	cons. price inflation rates N goods
$\pi_{J,t}$	prod. price inflation rate in sector J
π_t^U	union consumer price inflation rate

Exogenous structural shocks	
$S_{J,t}$	supply shock in sector J
$g_{J,t}$	demand shock in sector J
$S_{T,t}$	area wide supply shock in T goods
Innovations to structural shocks	
$\epsilon_{S_J,t}$, $\epsilon_{Y_J,t}$	innovation in S_J, A_J in sector J
$\epsilon_{S_T,t}$	innovation in common supply shock
$\epsilon_{g_J,t}$, $\epsilon_{G_J,t}$	innovation in G_J in sector J
η_t^i	innovation in monetary policy
Structural parameters	
ρ, ρ^*	relative risk aversion
n	country size of the Home country
$1/\eta_J$, $1/\kappa$	labour supply elasticity in sector J
ϖ_J	share of non-optimising firms in sector J
ρ_J, ρ_J^*	elasticity of substitution in sector J
$1-\theta_J$	price-resetting probability in sector J
γ, γ^*	share of tradables cons. in C_t, C_t^*
ν, ν^*	share of H goods in $C_{T,t}$, $C_{T,t}^*$
Monetary and fiscal policy parameters	
r_i, k	interest rate smoothing elasticity
r_π, ϖ_π	union inflation rate elasticity
$r_{\triangle\pi}$	union inflation speed limit elasticity
r_Y, ϖ_{GDP}	(union) output elasticity
$r_{\triangle Y}$	union output speed limit elasticity
ϖ_S	exchange rate elasticity
ρ_{G_J}	persistency of fiscal spending in J
a_{Y_J}	fiscal stabilisation of output in sector J

[1] Essential selection. Variables with an asterisk apply for the foreign economy. $J = \{H, F, N, N^*\}$ where H, F Home/Foreign production of tradables, N, N^* non-tradables production, respectively. T, T^* is tradable consumption. Variables superscripted with U indicate union-wide variables.

Chapter 1

Introduction

The Eurosystem just celebrated the tenth anniversary of the European Central Bank (ECB) which was founded on the 1st of June 1998 after the European Council had decided that 11 member states had fulfilled the conditions for adopting the euro. The euro area has been enlarged since then and will expand further in the coming years where euro adoption in new European Union (EU) members of the 2004 and 2007 accession waves is on the horizon. The launch of the euro in January 1999 is widely seen as a success story. Average annual inflation rates in the member countries were significantly higher than in the euro area over the last ten years and 15 million new jobs were created. In consequence, the unemployment rate in the end of 2007 was at its lowest level since the early 1980s.[1]

Nevertheless, the ECB and the euro area face challenges as they enter their second decade. The introduction of a single currency has tied together participating countries in their monetary and exchange rate policies. Consequently, prevailing national real and nominal rigidities on factor and product markets as well as country-specific shocks cannot be absorbed by nation-specific monetary stabilisation and/or nominal exchange rate realignments. In addition, heterogeneity between members might increase with further enlargement making a 'one-size-fits-all' monetary policy less likely. Research questions along various dimensions emerge from these challenges: How can members cope with asymmetric shocks and the asymmetric transmission of common shocks? Do national characteristics in labour and factor markets matter for the stabilisation properties of supranational monetary policy? Is there a welfare loss from disregarding country-specific factors? Should there be an active contribution to business cycle stabilisation by national fiscal policies? How will new EU members cope with price pressures from convergence in income and price levels aiming at a soon euro adoption at the same time? Will the exchange rate regime matter for the fulfilment of Maastricht criteria by new EU

[1] European Central Bank (2008) provides an overview of the first ten years of the ECB where these figures are taken from.

members from Central and Eastern Europe?

This dissertation approaches these questions within dynamic general equilibrium in three self-contained - yet interlinked - chapters in an empirically plausible way oriented at deriving policy recommendations. Taking a dynamic general equilibrium perspective in assessing challenges for the current and the enlarged euro area is not an end in itself. Compared to earlier open economy macroeconomic frameworks like the Mundell-Fleming (1962) and Dornbusch (1976) model, dynamic stochastic general equilibrium models (DSGEs) are based on optimising behaviour of economic agents that form rational (forward-looking) expectations, possibly to a varying extent. Therefore, systematic policy changes in monetary and fiscal policies and rationally expected shocks will not cause deviations from optimal plans and robustness to the Lucas (1976) critique is obtained. Their stock-flow consistency concerning the role of the evolution of nominal wealth and the physical capital stock yields a well defined description of macroeconomic equilibrium within each period and in the long run (the steady state) determined by utility maximisation of households and profit maximisation of firms.[2]

Early approaches, summarised under the Real Business Cycle paradigm mainly initiated by Kydland and Prescott (1982), focused nearly exclusively on nonmonetary factors as determinants of business cycles and emphasised the role of technological innovation in determining macroeconomic activity. Unexpected shocks to the level of technology were assumed to be the main cause of fluctuations in GDP, private consumption and hours worked. According to that research programme, fluctuations about trend are Pareto efficient and leave no role for fiscal and monetary stabilisation policies as business cycles represent the optimum response of the economy following technological innovations. Though microfounded, these models were challenged by their inability in explaining observed inertia and persistence in aggregate nominal time series like price levels on the producer and consumer stage that lead to muted adjustment in economic activity following shocks.

A stream of empirical work in the late 1980s has made the case that monetary policy is capable of significantly influencing the short-term course of the real economy (Clarida et al., 1999, p. 1661). The following incorporation of market imperfections in goods and labour markets (real rigidities) and nominal rigidities of various sorts allowed to model fluctuations about trend in a second best world. Under nominal rigidities, nominal variables can affect real outcomes such that there arises a non-trivial role for monetary policy in coping with macroeconomic fluctuations. Imperfect competition between firms caused by heterogeneity in production is a key ingredient in these new models where monopoly power permits the

[2] See DeJong (2007) for a textbook treatment of building and estimating modern structural macroeconometric models. Wickens (2008) provides a general overview of dynamic macroeconomic theory.

explicit analysis of pricing decisions (Lane, 2001, p. 235). As equilibrium prices are set above marginal cost by monopolistically-competitive price setters, output can be rationalised as being demand determined in the short run. As the presence of less than full competition implies that equilibrium production falls below the social optimum, there arises a distortion that potentially can be corrected by activist monetary policy intervention (Lane, 2001, p. 235). Coupled with rigidity in nominal variables rationalised by the inability of firms to reset prices every period, producer and consumer prices will respond with inertia following cost-push or demand-pull shocks. This reasoning reinvented the Phillips curve in a microfounded sense as a relationship between expected real marginal cost of firms and output price inflation. Price-stickiness thereby assigns a non-trivial role to monetary policy in eliminating the inefficiency in the price formation process across time and in increasing the information content of the price system. Whereas in the absence of price inflexibility the policy rate set by the central bank reflects the natural rate of real interest (i.e. the yield on physical capital investments) directly, price stickiness assigns monetary policy control over the ex ante real interest rate such that monetary policy can affect real activity by influencing consumption-saving decisions of households and investment decisions of firms.

As DSGE models are derived from microfounded behaviour, responses of key macroeconomic quantities like real GDP and consumer price inflation following economic shocks for given economic policies represent the aggregated economic decisions of the private sector (households and firms). Therefore, causal links and transmission channels from shocks hitting the economies to endogenous variables can be established thereby allowing for 'story-telling' and policy recommendations based on economic behaviour. The recent advances in the Bayesian estimation of macroeconomic systems allow to incorporate actual empirical information from macroeconomic time series in a meaningful way. System-wide estimation of structural parameters that guide the behaviour of the private sector and the government is made possible.

Analysing interactions between supranational monetary policy and fiscal policies on the one hand and interactions between monetary and fiscal policy settings of new EU members in their course towards euro adoption on the other, requires a detailed look at the sources of fluctuations on both the expenditure and production side of GDP. Concerning the supply side, this thesis develops frameworks that take into account that modelling gross value added should feature both the production of goods (a proxy for industry production including construction) and services (a proxy for wholesale and retail trade, business related and financial services, hotel stays and restaurant visits, and public services). We also deem this a reasonable approach to align models more closely with reasoning of policymakers at central banks where sectoral developments in prices and economic activity are featured prominently in monthly reports. Concerning the demand side, we highlight the role of fiscal spending shocks that might fall on national (sectoral) produce only. Regarding the sectoral price formation process, we allow for endogenous inflation persistence caused by non-optimising firms in all settings which can vary across countries and sectors. In order to allow for differences in production structures in driving aggregate activity and prices, we need to move beyond New Keynesian type interpretations of DSGE models to inspect the interactions in an empirically insightful way.[3] As a consequence, we spell out relationships between production sectors explicitly and allow households to have access to consumption baskets that comprehend both domestic and imported traded goods as well as services produced only domestically. As a consequence, it will be possible to distinguish between the evolution of producer and consumer price inflation and to assess the sources of aggregate persistency. In detail, this dissertation is based on the following three chapters:

Chapter 2: How costly is lasting structural heterogeneity of euro area member states? Welfare results from a two-region two-sector DSGE model. Lasting structural heterogeneity of euro area member state economies poses a challenge to joint monetary policy that targets union-wide developments only. Our main contribution is to address and quantify business-cycle related welfare costs that arise from differences in macroeconomic structures and shock exposure between member states evaluated within in a two-region two-sector sticky-price currency union model. Our novel framework takes into account that a large share of regional gross value added is not traded, proxied by services, whereas tradables, proxied by (industry) goods production, often account for less than thirty percent of overall domestic production. Highlighting the role of sectoral heterogeneity in explaining regional and union-wide business cycle fluctuations is supported by empirical evidence for the euro area where price changes occur more often in food and energy production and less frequent in services. Also, there is sizeable dispersion of harmonised index of consumer price (HICP) inflation rates across member states, most of it originating in the service category. The role of the decomposition of gross value added for monetary policy, albeit of high relevance empirically, seems surprisingly under-researched so far. This holds true for welfare implications. Our model aims at filling this gap.

The official monetary policy stance is targeted at maintaining price stability in the union as a whole where stability is indicated by small, but positive changes in the union-wide HICP. Consumer price developments in each member state contribute according to the share of domestic

[3] The New Keynesian model condenses the analysis of macroeconomic activity and the evolution of inflation to a forward-looking 'IS' curve derived from intertemporal optimisation of households coupled with a forward-looking Phillips curve derived from profit maximisation of firms. The model is closed by an instrument rule where the central bank has control over the short run policy rate (the Euler rate adjusted for expected inflation). A detailed analysis of the New Keynesian Model can be found in the monograph of Woodford (2003) and in Clarida et al. (1999).

monetary household expenditures in total expenditures in the area leading to a one-to-one correspondence between economic size and weight in union monetary policy setting. Accordingly, other factors than region size will not influence interest rate setting. In our model, targeting aggregates only will however not provide the welfare-maximising policy based on the utility of the average euro area consumer. Optimal monetary policy will address heterogeneity between member states explicitly. Besides economic size, also sectoral price setting of firms and the composition of GDP from its production and expenditure side will turn out to be crucial. The recommendation in Benigno (2004) and Benigno and López-Salido (2006) - optimal monetary policy should be more concerned about the region with the higher aggregate nominal rigidity - will however not be sufficient in the two-sector case. Higher importance of one group of goods for households in the consumption basket might outweigh the lower weight that would be attached by the policymaker if these goods exhibit higher price flexibility. In total, monitoring price developments in the sector with higher price flexibility can be beneficial. Monetary policy might even be more concerned about the region with overall lower price duration which establishes a channel for 'self-enforcing' structural reforms in member states aimed at increasing price flexibility.

We then assess business-cycle related welfare costs from heterogeneity comparing the current monetary strategy (proxied by an interest-rate setting rule) with optimal strategies, i.e. the optimal interest-rate setting rule and the optimal full commitment policy. The evaluation of welfare is based on scenarios for the aggregate euro area (the union split in Germany and other major members), the Large Member Area (Germany and France), and the Eastward Enlarged Area (the current euro area and new EU members from the east). Aggregate structural parameters and shocks are calibrated using estimates from Rumler (2007) and Smets and Wouters (2003). As comparable sectoral estimates are not available on the member state level, we calibrate sectoral values in a way such that structural shock decompositions of euro area aggregates could be broadly matched and the aggregate values remain intact.

Our results indicate that about 0.5% of long-run consumption is foregone per period for the average euro area citizen from inefficient business cycle dynamics. Losses could be lowered by more than 55% if regional heterogeneity would be taken into account in monetary policy setting. With regard to structural reform options, highest priority should be given to flexibilising price setting in all scenarios, followed by moderation in shocks that originate from both the supply and the demand side. A more prominent role for countercyclical national fiscal policies to assist monetary policy in dampening inflationary pressures will not yield lower losses than under the status quo.

Chapter 3: Can convergence really be blamed? Assessing sustainable compliance with monetary Maastricht criteria in EU9. Accession to the euro area is on the horizon for new EU members from Central and Eastern Europe. All countries plan to adopt the euro within the next five to ten years and there is no opt-out clause provided in case Maastricht criteria are fulfilled. There is however an often voiced concern that real and nominal convergence processes (convergence in real income and price levels to the EMU average) will hinder compliance with the monetary Maastricht criteria. The appreciation of the external real exchange rate is assumed to deliver a trade-off between compliance with the nominal exchange rate and the inflation rate criterion. Stabilising the nominal exchange rate under a fixed exchange rate regime (in case of the Baltic States and Bulgaria) might prevent fulfilment of the inflation criterion whereas under flexible regimes (Poland), the inflation rate could be stabilised at the cost of pronounced nominal appreciation. Intermediate Regimes (the other Visegrad States and Romania) might face pressures from both sources.

We set up a medium-scale small open economy DSGE framework in order to assess tensions in passing the criteria in the short and long run. As throughout the dissertation, a two-sector production structure is assumed that accommodates potentially different developments on the supply side in each country. Hence, it is taken into account that the production sector open to international trade (i.e. industry production) in new EU member states is assumed to experience prolonged factor productivity growth whereas advances in non-tradables (services) might be rather limited. In order to have a reasonable setup for policy analysis, the model is estimated by Bayesian methods conditional on the empirically prevailing exchange rate regime for each country in turn based on quarterly national accounts data. Sources of business cycle dynamics are attributed to real domestic and foreign shocks as well as monetary shocks.

Taking the current macroeconomic situation in new EU members and the respective exchange rate regime as initial conditions for an forecasting exercise where real convergence is accounted for, we find that in the longer run the majority of countries would meet the monetary criteria on average. However, a too wide range of possible paths for values of criteria triggered by volatile developments in underlying forecasted time series makes fulfilment a low probability event in general. We therefore argue that real convergence should not be blamed exclusively for hindering compliance with Maastricht criteria.

In consequence, average developments are not considered sufficient to judge whether criteria could be actually passed in the near future. As the average outcome is assessed per se conditional on the recent economic situation, criteria might have been only temporarily met. We thus argue that sustainable fulfilment of criteria requires that they need to be met with high probability. This also implies that a country will remain on the right track, after criteria have been passed (once), based on the average outcome. The analysis thus takes into account variability in economic aggregates that determines realisations of future values for criteria. As policy can potentially limit fluctuations by appropriate stabilisation policies, a role for macroeconomic policies in affecting outcomes can be introduced.

Assessing the scope of monetary policy in reducing uncertainty of meeting criteria for each country in turn, we find that risks in complying with the inflation criterion remain even when policies are selected optimally thereby minimising deviations from policy objectives. Also, the interest rate criterion is still hard to meet with high probability, whereas the exchange rate criterion is met by most countries. As monetary stabilisation policies are not sufficient in making euro adoption more likely in the model, we explore the role of the composition of markets, sources of inflation dynamics, and a different policy mix between domestic monetary and fiscal policies in more detail. We find that domestic real and foreign shocks need to mitigate considerably before monetary Maastricht criteria can be met sustainably. Adjustments in market structure or stabilisation policies alone (the optimal selection of monetary and fiscal policies) will not do. As these conclusions apply for all countries, we argue that the current exchange rate regime in place is not crucial for sustainable fulfilment of the investigated criteria.

Chapter 4: How will Bulgaria cope with shocks on its way to euro adoption? A microfounded model for an economy under the currency board. We present a medium-scale two-sector open economy model of the Bulgarian economy with the objective to explore the effects of transitory and permanent economic shocks a new EU member country like Bulgaria is currently exposed to. As before, gross value added is composed of traded and non-traded goods production and firms also face real rigidities in form of physical capital adjustment costs. Nominal rigidities are introduced by backward and forward-looking price setting that leads to hybrid inflation dynamics. We incorporate the currency board mechanism in the model in order to derive results that fit the institutional setting of Bulgaria. In contrast to chapter 3, the monetary stock takes centre stage for explaining monetary transmission. We explore the effects of temporary and permanent shocks to tradables productivity and consequences for the evolution of the sectoral producer price and overall CPI inflation rate. We then discuss current account sustainability and implications for the internal real exchange rate. The impact of fiscal policy on the trade balance and the current account is further analysed.

As main results we obtain that temporary supply or productivity shocks originating in the tradables sector are unlikely to cause severe inflationary pressures for the economy overall. A temporary increase in government expenditures however fuels home tradable inflation and improves the terms of trade. This result holds whether or not purchases fall on tradables or non-tradables. However, government expenditures that fall on non-traded goods rather than on traded goods contribute to the worsening in external competitiveness. Real convergence, simulated as a permanent increase in the level of technology in the industry sector in the range of 10% to 30%, is associated with an appreciation of the external real exchange rate in the range of 2.0% to 5.3%.

Chapter 2

How Costly is Lasting Structural Heterogeneity of Euro Area Member States? Welfare Results from a Two-Region Two-Sector DSGE Model

2.1 Introduction

The potential costs of structural differences between countries that share a common currency have already been highlighted in Mundell (1961) where arguments for the optimum domain of currency areas were laid out. In general, participation in a currency union can be considered beneficial for a country when gains like deepened trade integration and abandoning of the exchange rate risk, enhanced capital mobility and increasing foreign direct investments, reduced risk premia on interest rates, and lower overall inflation outweigh costs of loosing monetary and exchange rate policies as national economic stabilisation instruments.[1]

On the one hand, potential country-specific stabilisation costs materialise from the common short run interest rate set by the central bank. Different, potentially lasting, regional inflation rates cause different (ex ante) regional real interest rates that affect consumption and investment plans, and therefore domestic demand, heterogeneously. A country-specific shock might not affect union aggregates considerably, given that the economy is of small size and therefore of low weight in the union inflation rate. Joint monetary policy might remain unchanged, thereby widening even further the inflation and output growth differentials between that region and other regions. Stabilising a union wide cost-push shock (a shock to markups of firms or higher than productivity implied wage increases) that drives up aggregate inflation might impair all countries under consideration, however with different outcomes. Given that

[1] Following McCallum (1997, p. 15) one can therefore argue that "[...] these [considerations whether or not to adopt a fixed exchange rate] boil down to the question of whether the microeconomic (i.e., ressource allocation) advantages of an extended area with a single medium of exchange outweigh the macroeconomic (i.e., stabilisation policy) disadvantages of being unable to tailor monetary policy to local conditions".

domestic inflation is above the union average in one region whereas it is below in the other, union monetary tightening might not be sufficient to cool down the economy of the region with above average inflation. There might remain too low or even negative ex-ante real interest rates. Investment and current consumption are then still more profitable if undertaken today than in the future such that aggregate demand increases and inflation is fuelled further. In the other region with below average inflation, the interest rate increase might put even more stress on the economy and contract activity and decrease prices further. Hence, inflation differentials build up again and monetary policy is acting in a destabilising way.

On the other hand, nominal exchange rates can no longer absorb domestic inflation from external cost pressures and the external real exchange rate directly resembles countries' relative CPI inflation rates. Real exchange rates therefore reflect external competitiveness of firms directly. Consequently, a real appreciation puts downward pressure on equilibrium real wages and therefore also affects labour market outcomes by the income generated from supplying labour. Movements in the real exchange rate (and the terms of trade respectively) between euro area members are expected to offset the widening in inflation differentials between countries. Higher than average inflation in one region is accompanied by a loss in external competitiveness vis-à-vis the region with below average inflation in any period via the trade channel. The former region will loose external demand. Output and inflation will decrease subsequently whereas the latter region will experience a surge in demand which will raise output and demand accordingly. Whether the latter competitiveness channel dominates the former real interest rate channel is an open question and depends on the degree of price flexibility. In the longer run, the competitiveness effect might dominate as gains or losses in competitiveness will accumulate over time (level effect) whereas the real interest rate effect is determined anew (first difference effect). Consequently, lasting inflation differentials might be an equilibrium outcome.

The market forces that ensure self-adjustment of member state economies to these shocks are impaired by potential inertia in the response of prices such that persistency in inflation is created thereby reducing the ability of prices to signal relative scarcities. Persistency implies that past inflation developments have a big impact on how inflationary expectations are formed. When inflation differentials are lasting, i.e. prices are adjusting slowly to changing economic conditions and differently across regions, the stabilisation efforts by joint monetary policy can have even detrimental effects on regional business cycles. Transmission lags are introduced such that monetary tightening or expansionary policy may affect economies when adjustment in relative prices has already taken place.

Potentially lasting inflation differentials that are accompanied by asymmetries in cyclical output and consumption movements across members have to be seen against the backdrop of the union-wide perspective of monetary policy. Monetary policy has the ultimate goal to ensure union-wide price stability as laid down in article 105 (1) of the Maastricht treaty. The treaty thereby establishes a clear hierarchy of objectives for the Eurosystem. It assigns overriding importance to price stability in the conduct of monetary policy over the medium term for the euro area as a whole. Therefore "[t]he Treaty makes clear that ensuring price stability is the most important contribution that monetary policy can make to achieve a favourable economic environment and a high level of employment"[2]. Further weight on (union) real economic performance should only be given without interfering with the ultimate goal of maintaining price stability for the union as a whole. The main argument for stability in the aggregate price level is straightforward. Stability improves the transparency of the relative price mechanism, thereby alleviating from distortions that arise from rigidities and persistencies that cause inefficient market results. In consequence, stable prices - up to a positive but low growth rate implied by the targeted inflation rate - help to ensure that the market allocates resources efficiently, i.e. with minimal departure from the first best allocation across time and uses. The Governing Council of the ECB has provided a nominal anchor associated with this objective in 1998 by clarifying that price stability means "[...] a year-on-year increase in the Harmonised Index of Consumer Prices (HICP) for the euro area of below 2%".[3] Price stability is to be maintained over the medium term. The quantitative target for the inflation rate was refined in 2003 when it was stressed that stability in prices is given when inflation is below, 'but close to' 2%.[4]

This view has been further substantiated in the Monthly Bulletin of May 2005 where it is emphasised that the ECB "[...] does not seek to address questions of relative prices or inflation differentials". The ECB thereby denies a more prominent role for member state heterogeneity and its impact on inflation differentials across regions on its course of policy. Nevertheless, the role of sectoral developments in affecting inflation differentials and inflation persistence is (informally) overseen to some degree.[5] The same bulletin makes clear, that monitoring national and sectoral developments is considered a fundamental element of the ECB's assessment of the risks to price stability in the euro area. It is acknowledged that if service prices are indeed characterised by a systematically longer adjustment process, perhaps on account of some

[2] See `http://www.ecb.int/mopo/intro/html/objective.en.html`, as of 29.04.2008.

[3] See `http://www.ecb.int/press/pr/date/1998/html/pr981201_3.en.html`, as of 13.07.2008.

[4] See `http://www.ecb.europa.eu/press/key/date/2003/html/sp030612.en.html`, as of 13.07.2008.

[5] Inflation persistence is identified here in line with the definition used by the Inflation Persistency Network, namely "[...] the tendency of inflation to converge slowly towards its long-run value following a shock which has led inflation away from its long-run value", see Altissimo et al. (2006, p. 9). Therefore, inflation persistency is referred to as the lack of inflation responsiveness to changing economic conditions (triggered by supply, demand, monetary, foreign shocks and policy shocks). This means that the growth rate in prices does not change immediately when economic activity changes.

intrinsic features of the price-setting mechanism, significant and persistent inflation divergence could be generated (ECB Monthly Bulletin as of May 2005, p. 68). This view is however not part of the official policy stance.

The (purely) aggregate perspective is also enshrined in the construction of the HICP on the member state and union level. Assessing union price developments is based on the aggregation of expenditures by households in each member state on a harmonised reference basket of goods and services. Categories included are identical across members such that country-specific goods are removed and based on the classification of individual consumption by purpose (COICOP).[6] Consumption weights of single categories (item weights) in this scheme are member-state specific and derive from expenditures on single items as share of the euro value of the reference basket, i.e. each member state's 'household final monetary consumption expenditure'. Country weights used in the aggregation for compiling the union index are then calculated as ratio of final expenditures in a member state to the sum of all member states' final household consumption expenditures.[7] Aggregation therefore resembles country size but disregards potential heterogeneity in the price formation processes of goods and services consumed in each member state. Asymmetries in the price formation process across members might however contribute substantially to aggregate inflation in the union as a whole. This missing link between disaggregate sources of rigidities and the aggregate monetary policy can become crucial for the welfare effects of the common monetary policy stance along the business cycle, as will be argued below.

Against the backdrop of these arguments, our main objective is to determine and evaluate the welfare costs that arise from monetary policy in a union where structural heterogeneity between member states is present along various dimensions. The goal is to determine how union monetary policy should respond - if at all - to observed differences in nominal and real rigidities between regions and if there are gains to be achieved by taking into account heterogeneity in economic structures more formally. This approach allows to determine the costs from monetary policy that has its focus on the aggregate GDP and inflation rates only. Policy that takes into account asymmetries in activity and inflation rates across member states might at the same time create new trade-offs: Considering disaggregated information on price rigidity, output and inflation persistence might help alleviate from inefficiencies and inertia in adjustments in one country but at the same time increase it in another. These tensions can especially arise in case country-specific shocks just hit one sector of gross value added (say the industry sector, but not services) or in case of common shocks that only affect industry production. Welfare-

[6] See `http://epp.eurostat.ec.europa.eu/pls/portal/url/page/SHARED/PER_ECOFIN`, section 'Prices'.
[7] For details, see the manual by the Office for Official Publications of the European Communities (2004).

maximising policy will be shown to optimally distribute these tensions across member states such that adjustments following shocks are less inefficient compared to the first best world.

2.1.1 Stylised Facts and Related Literature

Table 2.1 on page 16 summarises stylised facts regarding heterogeneity that are expected to affect macroeconomic activity and price developments in current euro area members (upper panel) and prospective members from Central and Eastern Europe (lower panel). The table displays economic size (country gross value added as share of area gross value added), co-movement in (the cyclical component of) output and consumption of each country with the area aggregate as a whole, and sectoral composition of GDP in each country. Furthermore, external competitiveness measured by the effective real exchange rate between a member and its trading partners[8] is presented. Also, the decomposition of domestic HICPs in tradable and non-tradable consumption (that also serves as a measure of openness) and the degree of nominal rigidity in a region is covered.[9]

Current Euro Area Member States

One main source of heterogeneity between members directly stems from differences in the relative economic size of euro area countries, $n_i = \frac{Y_i}{Y^U}$. Dispersion in the share of national gross value added Y_i relative to aggregate gross value Y^U added implies that the magnitude of spillovers triggered by country-specific (idiosyncratic) shocks to other members via the trade and competitiveness channel vary according to which country is hit.[10] Even so, we observe from column three, that correlation between the output gap (the deviation from trend) in a euro area country and the euro area as a whole displays a high degree of homogeneity. This result can be attributed to the 'great moderation', namely that the variance of output has decreased everywhere in the last century, in accordance with the argument in Giannone and Reichlin (2006, p. 10). Giannone and Reichlin (2006) explore trends and cycles in the euro area in real activity and real consumption more formally in order to determine how much heterogeneity exists and if policy makers should worry about it. The authors find that, since 1970, gaps in levels of per capita income have been persistent but business cycle characteristics measured by

[8] Real Effective Exchange Rate E_t (deflator: consumer price indices - 12 trading partners), quarterly data. Source: Eurostat.

[9] All indicators shown in the table will have a model counterpart and are thus meaningful for the model based analysis. Hence we try to rationalise the empirically observed heterogeneity condensed to the measures shown and the welfare implications thereof, which we deem is a reasonable way of approaching the issue.

[10] Co-movement between any country and the euro area cycle is calculated by the deviation about trend, the latter being determined by the Hodrick and Prescott (1997) procedure for quarterly data. Note that the correlation between gaps is calculated instead of using the correlation in quarter on quarter growth rates. As a consequence, one does not need to cope with steady state growth, which is present in the latter measure.

comovement in levels in output have been very similar across countries. Accordingly, correlation between euro area countries and the euro area average is found to be high, in line with our measure of output correlation.[11] They further find that output variance can be mainly explained by common shocks with similar propagation mechanism while idiosyncratic shocks, which are found to be persistent, are small. The authors support the official ECB policy stance to take an area wide perspective and to focus on common characteristics of the European cycle only thereby abstracting from regional developments.

Afonso and Furceri (2007) support our points that cyclical comovement in GDP is high and that a sectoral view matters. Business cycle synchronisation in the enlarged EU is analysed on a sectoral basis using annual data for the period 1980-2005. Overall, the sectors that provide the most relevant contributions to variability are industry, building and construction, as well as agriculture, fishery and forestry. In contrast, the services sector, the largest one in terms of value added share, shows a relative low business cycle synchronisation and volatility. The authors conclude, that services contribute only marginally to the aggregate output business cycle synchronisation. These results emphasise that persistent developments in the services sector may have a large impact on aggregate price inertia that is not visible when looking at comovements in sectoral and overall gross value added across members (as the contribution to overall volatility is muted).

A suitable measure when interested in welfare consequences of potentially disparate business cycle movements in euro area members is provided by the contemporaneous correlation in (the cyclical component of) real consumption of a region with (the cyclical component of) euro area average consumption, $\mathsf{Corr}(\hat{C}_t^i, \hat{C}_t^U)$. The measure can be considered as indicating the success of risk sharing and consumption smoothing between euro area citizens. Under fully integrated financial markets and in the absence of credit rationing (all households are able to participate in markets) we would expect that country-specific risk to the level of consumption is pooled in the euro area completely such that movements in real consumption levels should be independent of country-specific heterogeneity. In contrast, it becomes visible from column 4 that correlation of regional with union consumption is very disparate across regions. Correlation of member state real consumption with the euro area private sector consumption cycle is lowest, and even negative in case of Finland (-0.16), and highest in Germany (0.77), closely followed by Belgium (0.76) and Austria (0.74).[12] In consequence, member state consumption is affected by

[11] Volatility is measured by the unconditional variance of the growth rate of PPP adjusted per capita income of a country. Correlation is expressed as the correlation between this growth rate and the average growth in the euro area as a whole.

[12] We are aware of the potential endogeneity problem that arises from that measure, as any country also is part of the euro area aggregate. It is common to assess correlation in this way, see also Giannone and

idiosyncratic risk to income that is not pooled across countries such that heterogeneity in real consumption fluctuations is present.[13]

The next column of table 2.1 presents the sectoral decomposition of gross value added (the production side of GDP) into tradable goods (proxied by total industry production excluding construction according to NACE classification C_D_E) and services (overall gross value added less of total industry). As emphasised above, we deem a closer look at the composition of GDP worthwhile, as the size and organisation of sectors can become crucial for determining costs from heterogeneity across regions. Rigidity in the development of relative prices between goods baskets produced within a country (the price of industry goods in terms services) might then have considerable consequences for price rigidity and persistence in the aggregate index. A high share of tradables production implies that external competitiveness is crucial for GDP growth. Also the aggregate price index will to a large extent be influenced by price and wage developments in the tradable sector. A low share of tradable production implies instead that non-tradable prices might have a large effect on external competitiveness, despite the fact that these goods are not exported. Turning to actual figures, one observes that the composition of gross value added is heterogenous across members as well: The share of industry production in gross value added[14] is highest in Finland with 34.5%, followed by Germany with 25.4% and lowest in Greece with 15.7%.[15]

The cumulative change in the (log of the) real effective exchange rate E_t, $\sum_t \triangle \ln E_t$ (the overall growth rate in E_t over all quarters considered) in the next column shows the increase (a negative value) or decrease (a positive value) in external competitiveness vis-à-vis other members of the euro area since 1999. One observes that the gap between member states has widened considerably over the past years which points to large heterogeneity regarding price competitiveness across members. Germany (DE), Austria (AT), France (FR) and Finland (FI) could increase their competitiveness compared to 1999 indicated by a real depreciation led by Germany (-5.79) followed by Finland (-3.53), France (-2.09) and Austria (-1.94). The difference in price levels between least competitive countries (IR and ES) and most competitive ones (AT,

Reichlin (2006). Excluding the country under investigation from the aggregate makes the aggregate no longer comparable across countries. Also, the correlation measure is a direct equivalent to the model-based measure, as shown below. Using more sophisticated methods than descriptive statistics, Giannone and Reichlin (2006) find that risk sharing has increased in the last decade in the countries forming the euro area.

[13] We state co-movement in output and consumption. When risk sharing is efficient, there can be high real consumption correlation albeit the individual countries might be prone to country-specific shocks to output. Under complete risk-sharing in real consumption (the terms of trade move in order to offset any movement in consumption), all idiosyncratic risk can be hedged against, such that only systematic risk (common shocks to the euro area) will affect country-specific consumption. As a consequence, there is perfect comovement in consumption across regions.

[14] We average the share for the years 2000 to 2006 and employ gross value added in volume.

[15] Considering gross value added in absolute value, industry production is strongest in Germany.

Euro	GDP	Cyclical activity		Gross value added		REER	Trad.	Nom. rigidity
area	$\frac{Y_i}{Y^U}$	$(\hat{Y}^i, \hat{Y}_t^U)$	$(\hat{C}_t^i, \hat{C}_t^U)$	$\frac{Y_H}{Y}$	$1-\frac{Y_H}{Y}$	$\sum_{1999q2}^{2006q4} \triangle \ln E_t$	γ	$\frac{1}{1-\varpi_i}\frac{1}{1-\theta_i}$
AT	3.4	0.64	0.74	24.6	75.4	-1.94	37.3	4.7
BE	4.0	0.62	0.76	23.1	76.9	0.31	42.3	4.6
DE	33.3	0.91	0.77	25.4	74.6	-5.79	39.7	12.8
ES	9.3	0.81	0.59	21.5	78.5	8.20	38.7	4.1
FI	2.1	0.75	-0.16	34.8	65.2	-3.53	37.6	4.4
FR	22.0	0.75	0.66	19.9	80.1	-2.09	39.4	6.8
GR	2.0	0.50	NA	15.7	84.3	4.45	37.6	3.5
IT	14.8	0.92	0.68	22.5	77.5	2.73	40.1	7.0
NL	6.2	0.80	0.55	20.1	79.9	4.14	41.5	3.8
EA	100.0	1.00	1.00	22.9	77.1	-	39.6	6.7
EU7	GDP	Cyclical activity		Gross value added		REER	Trad.	Nom. rigidity
	$\frac{Y_i}{Y^{U+EU7}}$	$(\hat{Y}^i, \hat{Y}_t^U)$	$(\hat{C}^i, \hat{C}_t^U)$	$\frac{Y_H}{Y}$	$1-\frac{Y_H}{Y}$	$\sum_{1999q2}^{2006q4} \triangle \ln E_t$	γ	$\frac{1}{1-\varpi_i}\frac{1}{1-\theta_i}$
CZ	0.7	0.41	-0.27	35.5	64.5	29.99	38.9	3.7
EE	0.1	0.15	-0.18	25.1	74.9	12.90	41.5	3.7
HU	0.7	-0.16	-0.52	30.3	69.7	23.94	42.0	3.7
LV	0.1	0.35	0.13	22.8	77.2	8.99	38.9	3.7
LT	0.1	-0.48	-0.15	28.6	71.4	20.60	40.2	3.6
PL	2.2	0.41	0.20	30.4	69.6	19.36	44.0	3.7
SK	0.3	-0.04	-0.13	36.8	63.2	49.47	42.7	3.6
EU7	4.2	0.39	-0.32	31.4	68.6	-	41.2	3.7

Table 2.1: Stylised facts on heterogeneity for current and enlarged euro area member states. Source: Own calculations based on quarterly, seasonally and working-day adjusted data provided by Eurostat.

DE, FI, and FR) has considerably widened. Besides the large and increasing heterogeneity in competition we observe a large similarity in the degree of openness where openness is measured by the weight of industry goods in the consumer price index in each member country, γ_i.[16] Openness can also be measured by the relative economic size of a country, n_i. When there is no home bias in consumption, n_i can be considered a measure for the share of home produced tradable goods in the tradable consumption basket. Hence, one expects tradable prices in smaller economies like AT, BE, FI, GR, and NL to be substantially determined by price developments in other (larger) members such that heterogeneity is 'imported' in these countries.

Finally, in the last column of table 2.1, we report the average nominal rigidity that indicates the price adjustment capabilities of each economy following shocks. The measure is based on the Phillips curve paradigm of the hybrid New Keynesian Framework.[17] Price rigidity θ_i and intrinsic inflation persistence ϖ_i affect the responsiveness of member states' business cycles following real shocks and following monetary policy signals via the Phillips curve that links real marginal cost developments to the inflation rate. Price rigidity (the inability to change

[16] γ_i is derived from the COICOP classification, category IGOODS and averaging over the years 2003 to 2007.

[17] Hybrid dynamics imply that besides forward-looking optimising firms also the existence of backward-looking, price-indexing, firms is taken into account.

prices in any period) for firms in the same sectors across regions and across sectors within a region causes distortions in relative prices. Firms cannot respond optimally to changing economic conditions leading to price dispersion such that the price of a firm's output differs from the average price set within the sector. Optimal production plans by firms and consumption plans by households are affected and distorted supply-demand relations will lead to efficiency losses that reduce welfare. Inflation persistence ($\varpi_i > 0$) implies that prices are reset in an non-optimal way by adjusting for gross sectoral inflation only instead of taking into account expected real marginal cost.[18] Accordingly, the measure presented in the last column reports the average aggregate price duration of price contracts in quarters, $\frac{1}{1-\theta_i}$, adjusted for inflation inertia. The higher is nominal rigidity $\frac{1}{1-\varpi_i}\frac{1}{1-\theta_i}$, the longer it will take for optimising firms to readjust prices, the higher is the efficiency loss.

We present measures for rigidity compiled from Phillips curve estimates for quarterly data 1980Q1-2003Q4 in Rumler (2007).[19] Large discrepancies in average nominal rigidity across the big four euro area economies, Germany, France, Italy, and Spain are visible. Nominal rigidity is highest in Germany with 12.8 quarters, way above the euro area average of 6.7 quarters. France is in line with the euro area average, so is Italy. Spain exhibits rather low overall rigidities of about 4 quarters. Intrinsic inflation persistence ϖ_i contributes to the variation in nominal rigidity substantially. It is highest in France, indicating that more than 50 percent of firms are non-optimising in each quarter but behave in a rule-of-thumb fashion instead, followed by Italy with 49 percent. Germany's share is 40 percent and again Spain does best with 18 percent. As a result, inflation differentials between regions seem to be lasting and thereby impair the effectiveness of monetary policy in stabilising the cycle where inertia causes impact lags. Whereas Germany is rather less inflation persistent compared to its immediate neighbours, the high degree of nominal rigidity mainly stems for its high price rigidity $\theta_{DE} = 0.87$ which is highest compared to all other countries considered. θ_i is found to be higher in large members with rather closed economies than in smaller members with rather open economies, with the exception of Belgium, see Rumler (2007, p. 439). The suggested relation between size and openness on the one hand and the relation between size and estimated degree of price rigidity is found to be statistically significant (only) in case of the latter. Larger economies therefore seem to have higher duration of price contracts. This points to the fact that Germany cannot be considered an outlier, but just an extreme case of the found relationship.

Our measures of inflation persistence and price rigidity corroborate findings by the ECB

[18] Accordingly, efficiency losses from price rigidity are aggravated and relative prices remain even longer away from optimal values (i.e. values that would materialise if prices would be fully flexible).

[19] The overall inflation rate π_t is proxied by the (log change) in the output deflator. See Galí et al. (2001) for a related exercise for the aggregate formed by euro area members estimated for 1970-1998.

in its Monthly Bulletin of May 2005 where differences in headline inflation rates between EMU member countries were found to be prolonged. Also, Bulir and Hurnik (2008) find that member states' inflation rates in the EU-15 have stopped converging in recent years. Employing a two-step generalised method of moments panel regression of a reduced-form model, they find that developments in the EU-15 inflation rates during 1996-2005 have been associated mostly to the variability of the output gap and structural reform variables (product market deregulation), while the price-level convergence variable has not been statistically significant. Empirical evidence that a sectoral view regarding price developments is worth exploring is provided in Vermeulen et al. (2007). They find significant sectoral heterogeneity in the degree of price stickiness. Altissimo et al. (2005) detect that the main source of dispersion in countries' headline inflation rates is in those components of the HICP where non-traded goods are more intensely represented. The dispersion in the service sector has been almost always higher than the overall dispersion of regional to euro area HICP rates and its contribution has been increasing over time.[20] The authors conclude that sizeable dispersion of HICP inflation rates across euro area countries becomes visible which is in line with our stylised facts regarding nominal rigidity.

New EU Member States

Facing euro area enlargement to the east, heterogeneity is about to increase. Countries of the 2004 EU accession wave as well as Bulgaria and Romania, which joined in 2007, are still in the process of restructuring their economies such that the composition of gross value added faces large swings. GDP per capita is much lower than the average in the EMU, so are price levels.[21] Hence, these countries are and will be in a phase of convergence (both of real and nominal nature) towards income and price levels of their neighbours for a long period of time. Higher factor productivity growth in the production of tradable goods leads to 'catch-up' inflation as wage increases also in sectors that do not experience higher productivity growth (Balassa-Samuelson effect).[22]

[20] The authors assess the inflation differential between subindex J of the HICP of country i and the euro area aggregate by $\delta_{J,i,t} = \pi^{i}_{J,t} - \pi^{ea}_{J,t}$. The dispersion of inflation in the subcomponent j is measured as $\triangle_{J,t} = \left(\frac{1}{10}\sum_{i=1}^{10}\delta^2_{J,i,t}\right)^{1/2}$.

[21] Data on economic size n_j comes from Eurostat and refers to 2001-2006 averaged rounded yearly data. The economic weight is calculated as GDP of country j (in constant prices) as share of the enlarged euro area GDP $Y^U + Y^{EU7}$.

[22] According to the Balassa-Samuelson effect, it is expected that countries that experience higher productivity growth in the tradable sector will also exhibit higher consumer prices (leaving the role of the exchange rate regime aside). Higher productivity growth in the tradable sector (e.g. ongoing permanent shocks to the level of technology) translates into wage rises in this sector where it is assumed that the price level of tradables is determined in the world market, see Balassa (1964) and Samuelson (1964). Due to labour mobility within a country, wages will also rise in the other sectors that did not experience productivity gains. Hence, product

These developments become visible when looking at effective real exchange rates, as the lower panel of table 2.1 illustrates. A marked appreciation has accumulated in the last 40 quarters where developments were especially strong in the Slovak Republic, the Czech Republic and Hungary. Nevertheless and despite these 'low-frequency' (trend-related) price pressures that might pose an obstacle to fulfilment of the monetary Maastricht criteria, all new EU members have intentions to join the euro area within the next five to ten years. There seems to be wide disagreement of policymakers that adopting the euro should be linked to convergence being completed which could defer euro adoption by 20 to 30 years, according to projections reported in Lewis (2007).

Breitung and Eickmeier (2006) explore the business cycle synchronisation between new EU member states and the euro area within a structural factor model. However, results are rather mixed.[23] Correlations between new EU members and the euro area are on average lower than between individual members of the current EMU as is supported by our results reported in table 2.1, but they are higher than for some small peripheral EMU countries. Their result is confirmed by variance shares of output and inflation explained by common euro-area factors. The propagation of shocks that arise in the euro area and transmit to new members does not differ significantly from the propagation of shocks to EMU countries in most cases. The authors further find considerable heterogeneity across new member states, implying that for some countries, accession to EMU would be more costly than for others. Poland, Slovenia, Hungary and Estonia are found to be more suitable EMU candidates than other countries.

Turning to high frequency price developments, we report the measure for nominal rigidity in the last column of table 2.1 where values are based on estimations of hybrid New Keynesian Phillips curves with Bayesian methods in chapter 3 of this dissertation.[24] Compared to results for the current euro area, new EU members feature a rather low duration of inefficient price contracts with an average nominal rigidity of 3.7 quarters that mainly stems from lower price rigidity θ_i.[25] This finding is in line with the point made above that small, open economies might experience a lower degree overall price rigidity. Our findings mirror those of Arratibel et al. (2002). They examine inflation dynamics in the - then - EU accession countries in Central

prices need to increase in order to compensate for increased factor costs such such that the firms in the less productive sector are willing to produce the same amount of output. Eventually, the overall price level in the country will increase. The Balassa-Samuelson effect therefore concerns trend behaviour and will not be adressed in this chapter. We assess welfare results arising from inefficient fluctuations only.

[23] General arguments for euro adoption in new EU members are summarised in Schadler et al. (2005).

[24] The value for the EU7 is based on a weighted geometric average of country specific values, where $\frac{1}{1-\varpi}\frac{1}{1-\theta} = \Pi_{i=1}^{7}\left(\frac{1}{1-\varpi_i}\frac{1}{1-\theta_i}\right)^{w_i}$ with the weight $w_i = \frac{Y_i}{\sum_{i=1}^{7} Y_i}$.

[25] We acknowledge that estimates might be highly prior driven, where priors where set at estimates of the hybrid Phillips curve for Hungary. Detailed estimation results are illustrated in table 3.5 on page 169.

and Eastern Europe (2004 entrants and Bulgaria and Romania). They provide estimates for an inflation dynamics equation (a hybrid Phillips curve) for headline, tradable, and non-tradable inflation within the New Keynesian framework. Real convergence is taken into account as well as the potential impacts of differing exchange rate regimes. Domestic factors seem to have a systematically stronger impact upon non-tradable goods inflation whereas international factors have a stronger impact over tradable goods inflation. Headline inflation, tradable as well as non-tradable inflation is inertial and the Balassa-Samuelson effect is found to be only a nonprominent factor behind the current experience of dual inflation in these countries.[26] Franta et al. (2007) support the point made here that price rigidity might be lower, but inertia is nevertheless high in new EU members. They find that backward-looking behaviour in price-setting (a prominent role of lagged inflation in the Phillips curve) may be a more important component in explaining inflation dynamics in new member states than in euro area countries.[27]

The Welfare Consequences of Heterogeneity in Structural Models

In order to highlight the welfare consequences of the observed heterogeneity across current and future members, structural frameworks are increasingly employed. Benigno (2004) investigates how monetary policy should be conducted in a two-region one-sector general equilibrium model of a currency union with monopolistic competition and price stickiness. As is common in these frameworks, the welfare criterion to assess the loss under various monetary policy settings arises from the utility of the consumer, in this case the representative average union household. In a framework where all goods are traded and all price-setting is purely forward-looking, it is found that optimal monetary policy is given by targeting a weighted average of the regional inflation rates, given both regions share the same degree of nominal rigidity. These weights coincide with the economic sizes of the region. The result is therefore in favour of actual ECB policy where the focus is on the aggregate inflation rate as stressed above. However, given that nominal rigidity differs (e.g. assuming that one region is flexible and thus fast in shock absorption whereas the other region is price-sticky), an inflation targeting policy that gives higher weight to the region with higher nominal rigidity is nearly optimal compared to the full commitment policy. This result is intuitive. Given that price dispersion across firms in one region causes large welfare losses in the union, the central bank will stand by the region that cannot respond to shocks properly as prices are rigid whereas the other region manages shock absorbtion on its own, which will work as prices adjust fast. However, the result can be criticised as the more

[26] The hybrid Phillips curve is found to be not an appropriate short-run inflation dynamics model in case of Poland and Slovakia.

[27] Lendvai (2005) corroborates this finding in case of Hungary.

flexible country has to lift the burden of adjustment following shocks, whereas the central bank steps by the rigid country. This policy might create incentive problems and cast doubt on the benefits of joint monetary policy as 'being rigid and large' is rewarded. We later on show that this finding not general enough when allowing for within country heterogeneity regarding the composition of consumption and production that is made possible by exploring dynamics in a two-sector framework.

Benigno and López-Salido (2006) extend the results of Benigno (2004) by allowing for price-indexation in one region. Product price contracts are indexed to past period's inflation rate The analysis is motivated by the existence of heterogeneity in inflation dynamics across euro area countries where Germany seems to be more 'forward-looking' than other large euro area members in that framework. Based on the estimation of New Keynesian Phillips curves for five major countries of the euro area for the time span 1970-1998, they find that there are two different zones within the area: inflation in Germany had a dominant forward-looking component prior to the third stage of EMU, while in the other group of countries inflation showed a significant inertial (backward-looking) behaviour. The authors recommend that optimal monetary policy should eliminate, or at least mitigate, the distortions in inter-regional relative prices that might arise because of the difference in the degree of adjustment in inflation rates to terms of trade shocks. This result is a straightforward extension of Benigno (2004) where higher nominal rigidity has the same qualitative effects as higher price duration only. A region with a combined higher degree of price rigidity and backward-looking behaviour (and hence higher overall nominal rigidity) should receive higher weight in an optimal inflation targeting policy.[28] In light of the measures for nominal rigidity presented in table 2.1, monetary policy should be mainly concerned about Germany and less about Greece where our measures for Germany include more recent data and upturn the authors' findings for rigidity in the area.

Amato and Laubach (2003) provide stronger microfoundations for intrinsic (endogenously generated) inflation persistence compared to Benigno and López-Salido (2006). Any firm can switch from backward to forward-looking price setting instead of assuming that the same firms are always price-indexing. Rule of thumb pricing leads to endogenous persistence in output and inflation and alters the policymaker's welfare objective. The authors derive the microfounded loss-function in the presence of rule-of-thumb firms and evaluate optimal monetary policy. The main finding is that highly inertial policy is optimal regardless of what fraction of agents occasionally follows a rule of thumb. When intrinsic inflation persistence plays a non-negligible role ($\varpi > 0$), optimal policy will also have a look at the growth rate of inflation, i.e. so-called

[28] For a further extension of the Benigno model, see Lombardo (2006). The author allows for differing degrees of competition besides differing degrees of nominal rigidity between the members of a monetary union.

'speed-limit' terms which is also true in the Benigno and López-Salido (2006) model.

Brissimis and Skotida (2008) investigate the gains from taking into account heterogeneity between members within a dynamic general equilibrium framework. The authors calculate optimal monetary policy within a two-region New-Keynesian framework relying on a non-microfounded policy objective. Conducting welfare analyses within the model estimated for two euro area countries (Germany and France), they show that there are gains to be achieved by the ECB taking into account the heterogeneity of economic structures. This finding appears to be robust under alternative weights given by the central bank to the stabilization of the target variables. The claim that monetary policy should follow a union-wide perspective (only) is also cast in doubt in Berger and Mueller (2007). The authors present evidence that national economic considerations play at least some role in the voting behaviour in the council such that the political weight in council decisions might differ from the relative economic size of the region (the weight in the union HICP).[29] Within a comparative-static framework, the optimal weight of a region is both a function of country-specific (idiosyncratic) economic shocks and preference shocks. Preference uncertainty allows the inflation objective of a member country to differ from the union objective. It turns out that optimal voting weights reflect two opposing forces, the wish to insulate common monetary policy from changing preferences at the national level, and the attempt to avoid an overly active or passive reaction to idiosyncratic national shocks. If preference shocks were sufficiently similar, it would always be optimal to overrepresent small countries and under-represent large countries such that a perfect match between economic size and voting rights is rarely optimal.

2.1.2 Research Setup

In order to assess welfare consequences of union monetary policy in an environment of member state heterogeneity we present a micro-founded general equilibrium framework. In general equilibrium, actions and decisions of firms, households, and the governments are mutually consistent within a period and across periods and result from optimising, forward-looking (as well as backward-looking) behaviour. Decisions are optimal by taking into account budget constraints and the structure of the economy in general equilibrium such that policy actions are not vulnerable to the Lucas (1976) critique. In such a framework, a natural measure of welfare is provided by household's utility (e.g. the present discounted value of life-time utility of the average euro area citizen).

Welfare losses evaluated in this chapter arise from the variability and persistence in con-

[29] See also Heinemann and Huefner (2004) for a related argument.

sumption, output and prices along the business cycle influenced by the factors of heterogeneity presented in table 2.1. Risk-averse citizens prefer sure consumption streams to volatile ones with same expected value but uncertain actual outcome. If risks to real consumption could be pooled across regions perfectly, there would arise no welfare costs and consequently there would be no role for monetary policy in stabilising economic outcomes.[30] Nominal rigidities that induce inflation differentials and asymmetries in cyclical output developments add to these costs from business cycles as they cause inefficient fluctuations in the goods and labour market outcomes. Stylised facts presented in table 2.1 indicate that consumption risks and nominal rigidities prevail in the euro area and in new EU members and are also different across countries. Monetary policy that targets aggregates only will not be able to cure inefficiencies in consumption, saving and labour supply decisions within and across periods and welfare losses from heterogeneity result.[31]

In order to determine the magnitude of these losses and potential solutions, our general equilibrium framework allows to disentangle factors that drive real GDP within each region and to detect whether inefficiencies in business cycle dynamics derive from the tradables (goods) or non-tradables (services) sector. We deem this a major advantage over related work. We can assess the relative contributions of rigidities in intranational relative price developments (the price of goods in terms of services) jointly with developments in interregional prices (the price of foreign goods in terms of home goods) in driving aggregate price developments. We make the model results empirically plausible by calibrating deep parameters with estimates for current and future euro area members and the union as a whole based on scenarios. We allow for different shares of industry production in gross value added across members as well as sectoral differences in nominal rigidities. All aspects presented in table 2.1 will therefore be rationalised within the model such that there is an analogy between model implied and actual empirical results.[32] The analysis is structured in three main experiments:

Experiment I: General Welfare Implications of Member State Heterogeneity

Within experiment I, we explore the welfare implications of structural differences between union member states for business cycle dynamics in member states and in the union in general. We ask the following questions:

[30] However, distortions in relative prices would still move the households away from first best choices.

[31] It also becomes clear why in such a setup asymmetries in real consumption correlation are of much higher concern than asymmetries in cyclical GDP developments. This property cannot be accounted for in frameworks without microfoundations.

[32] All measures shown in table 2.1, could therefore in principle be replicated in the model by taking the model as data generating process.

- *Should monetary policy address macroeconomic heterogeneity between euro area member states regarding openness, competitive structure, shock exposure and sectoral composition of GDP instead of targeting union aggregates only?*

- *Are structural asymmetries always welfare reducing? Does an increase in competition in goods markets, increased flexibility in price-setting, and reduced shock exposure come as a 'free lunch' for a member state?*

We reach the following findings: Monetary policy should address structural heterogeneity. Using aggregate price rigidity and inflation persistence in member states as guidelines for the optimal conduct of monetary policy, as proposed in the related literature, can however be misleading. Taking into account the role services have in the composition of gross value added and in domestic consumer price indices, optimal monetary policy will not necessarily be more concerned about the region with the higher aggregate price rigidity. What matters besides aggregate nominal rigidity in a region is the expenditure weight in the domestic HICP of goods produced in price rigid sectors. We find that monetary policy aiming at maximising utility of the average household in the area generally increases the weight of items that have a high expenditure share in the member state's harmonised consumption basket. At the same time, more price rigid items will receive higher weight also to reduce the distortions created by the respective sector through its inability to adjust to changing economic conditions: A trade-off materialises.

In case there are no services such that all goods are traded, as assumed in Benigno (2004) and others, there is a one-for-one relationship between nominal rigidity, country size and concern in policy. In the presence of services, this link breaks down: A large preference weight of less price rigid goods in the basket can overcompensate the weight of items that are indeed more rigid but of lower weight for the consumer. Further, policy will face a trade-off in stabilising the relative price between regions (the terms of trade) and the relative prices within regions (the price of manufacturing goods in terms of services). Overall, it can be welfare-optimal to increase the weight of the member state with the lower overall price rigidity. As services comprise more than half of gross value added in euro area member states we deem this effect important. The mechanism established in our model opens a 'corridor' for enacting structural reforms that are aimed at delivering *higher* price flexibility in one region. Reform effort can be rewarded by *higher* concern in union monetary policy (e.g. a higher share of voting rights than implied by relative economic size) such that fostering flexibility can come as a 'free lunch' for a member state. The 'incentive effect' implied by rewarding higher price rigidity of a region with higher policy concern in stabilisation policy, as argued in the related literature, can be resolved under certain conditions.

Experiment II: The Optimal Course of Monetary Policy under Heterogeneity

Having investigated the main workings of heterogeneity in the framework calibrated with aggregate euro area data, we present the role monetary policy could have in coping with member state heterogeneity. Current policy which targets aggregates only is proxied by an interest-rate setting rule that responds to deviations of levels and growth rates of activity and union inflation from targets. This is a simplification of actual ECB policy that is based on a two-pillar approach to the analysis of risks to price stability. Organising, evaluating and cross-checking information provided by the economic (short to medium term) analyis and the monetary (longer-term) analysis form the foundations of monetary policy decisions by the governing council.[33] However, the suitability of the monetary pillar has been questioned in recent times (Woodford 2003). Also, relying exclusively on the economic analysis seems to be in line with financial markets that appear to no longer put appreciable weight on ECB communication relating to the monetary analysis (Berger et al., 2008, p. 3). Current policy modelled this way is contrasted to policies that take account of heterogeneity in a welfare-maximising way for three scenarios we deem useful: the Current Area Scenario, where Germany is grouped versus other major euro area countries; the Large Member Area (focussing on Germany and France); and the eastward enlarged euro area, where current members are grouped versus new eastern EU members. Using our welfare framework, we explore the following questions:

- *How costly is lasting structural heterogeneity in the current and the enlarged euro area when monetary policy targets aggregates only?*
- *Are there ways of minimising welfare losses under the given policy?*
- *How should members be represented in joint monetary policy in case of lasting structural differences?*

The following results are obtained: Heterogeneity of member states is costly. About 0.47% of household's per period steady state consumption flow is foregone compared to the first best (i.e. in the absence of nominal and real rigidities) in the Current Euro Area Scenario when monetary policy targets aggregates only. About 0.33% would be lost when heterogeneity would be absent, i.e. when losses solemnly derive from equally distributed rigidities found

[33] See `http://www.ecb.int/mopo/strategy/html/index.en.html`, as of 08.10.2008.

for the aggregate euro area. Taking heterogeneity across members into account when setting monetary policy in a welfare-maximising way would make losses negligible. As this result requires implementation of the full commitment plan through time which might be hard to communicate, we also analysed the scope of monetary policy that continues to follow the rule that describes current policy but can select weights on components in an optimal way. Targeting a union harmonised index of consumer prices that respects observed (sectoral) degrees of nominal and real rigidities, efficiency losses could be reduced by more than 63% compared to targeting aggregates only.

In the union formed by Germany and France, average per period losses amount to 0.40% under the current policy. Losses are in the same range as for the aggregate union which indicates that aggregate losses might be driven mainly by inefficient fluctuations and spillovers between the area's two largest members. Again, implementation of the full commitment policy would make losses from heterogeneity negligible. Further, taking the current policy rule as given but choosing weights in a welfare-maximising way allows to reduce losses from 0.40% to about 0.18% of steady state consumption flows. Accordingly, losses could be more than halved when respecting heterogeneity in the current formulation of policy. This policy would increase the weight of France compared to Germany slightly. As Germany exhibits higher nominal rigidities according to our stylised facts presented in table 2.1, relying on recommendations obtained in one-sector frameworks in the related literature would lead to an increase of the weight on Germany as sectoral effects are ignored. However experiment I in this chapter made clear why increasing the weight of France actually presents the welfare-maximising policy.

Considering euro area enlargement, monetary policy faces the task of optimally taking into account macroeconomic characteristics of a number of countries that differ from current members along various dimensions as presented in table 2.1. Targeting aggregates only produces losses of about 0.45% such that losses are slightly lower on average than experienced in the current union. The result can be aligned with the lower nominal rigidity for prospective members such that the average degree of nominal rigidity in the union as a whole decreases by enlargement. Full commitment policy in the enlarged union would allow for a major reduction in losses, as also found in preceding scenarios. Choosing the current policy rule optimally leads to the underrepresentation of new EU members. In that case the 'rigidity effect' dominates the 'consumption weight effect' and the weight of less rigid members is decreased. This policy leads to losses that are about 70% lower than those experienced by targeting aggregates only.

Experiment III: Structural Reform Priorities for Reducing Heterogeneity

In this experiment, we discuss reform options to eliminate structural heterogeneity in an efficient way. We assume that the full commitment policy cannot be implemented due to the difficulties associated with communicating an intertemporally optimal plan. Policy targeting aggregates only is compared to current policy that selects weights on member state contributions optimally. We answer the following questions:

- *Should reform priorities focus on increasing price flexibility, increasing market competition, or the mitigation of structural shocks?*
- *What should be the contribution of national fiscal authorities, if any, in mitigating economic differences across regions and dampening inflationary pressures?*

We make the following points: Policy priorities in removing observed dissimilarities between regions should be targeted on fostering sectoral price flexibility and forward-looking price setting. Compared to other reform options like enhanced product market competition, a more active role for domestic fiscal policies, and the mitigation in country specific shocks, the absence of price flexibility and the presence of inflation persistence provide the largest contribution to welfare costs in the scenarios considered. Domestic fiscal policies seem less efficient in reducing welfare losses. In all scenarios, a more prominent role for fiscal policies in coping with heterogeneity made possible by active countercyclical stabilisation of sectoral activity will not provide a wide difference to the status quo. We also observe that the ranking of policy priorities is largely invariant across scenarios under both the aggregate policy rule and the rule where weights are chosen optimally. This makes clear that the impact of reform measures is quite effective over the range of nominal and real rigidities considered.

2.1.3 Main Contribution to the Literature

The outline of our main results together with the presented literature allows to condense our contribution to the literature to seven points: First, as became clear by now, the literature on assessing the optimal course of monetary policy in case countries exhibit structural asymmetries is overwhelmingly based on frameworks where all goods are traded, an assumption that cannot be aligned with the actual composition of gross value added. When tradable goods are proxied by industrial production, only about 30% of total gross value is tradable and therefore subject to interregional price movements. We highlight the importance relative price movements within countries might have for explaining aggregate price rigidity and inflation differentials across countries. The chapter finds that monetary policy should not only try to eliminate distortions in movements in the real exchange rate by stabilising the terms of trade but also care about developments in region specific relative prices. Therefore, optimal monetary policy might face a trade-off between stabilising internal real exchange rates, the external real exchange rate and the terms of trade. If union monetary policy targets aggregates only, the costs from heterogeneity in intranational developments increase the more disparate these prices develop. Hence, welfare costs from the common monetary policy stance might be largely understated when focussing on developments in tradables exclusively.

Second, unlike as Benigno and López-Salido (2006), empirical estimates for Phillips curves suggest that it is empirically plausible to allow for backward-looking price setting in all member states to be considered in the model. As table 2.1 revealed, inflation persistence is a prominent feature in all big euro area regions according to recent estimates of hybrid Phillips curves (that are also based on data including the third stage of EMU). Especially sectoral inflation dynamics across union members in services might be disparate, whereas industry inflation dynamics might exhibit a more common pattern across members (e.g. due to globalisation forces). Third, the cited work compares optimal policy in the presence of nominal rigidities with policy rules like optimal inflation targeting as well as rules for optimal output gap targeting and HICP targeting. Accordingly, real activity could be affected directly and there is no role for the monetary transmission process. In contrast, actual monetary policy by the ECB can be well approximated empirically by an extended Taylor (1993) instrument rule estimated in Smets-Wouters (2003). Thereby actual policy can be approximated by flexible inflation targeting (allowing for both a role for inflation and the output gap) including 'speed' limit terms on growth inflation rate and the growth in the output gap. Hence, we deem it a more empirically relevant approach to modeling heterogeneity also by featuring the use of a policy instrument that could in principle be chosen optimally in actual policy. It becomes possible here to contrast the fully optimal policy with results from optimising the Smets-Wouters rule and compare results to a rule aimed at targeting euro area aggregates only.

Fourth, we argue that the empirical evidence provided by factor models like Eickmeier (2006) also indicates that we should account for a wider range of sectoral supply and demand shocks and policy shocks in order to mirror sources of fluctuations in macroeconomic time series in the euro area more closely. Whereas Benigno (2004) concentrates on terms of trade movements (relative shocks to supply and demand across union members) as the main source of fluctuations, we take into account that sectors adjust differently to country-specific shocks and transmission of common shocks might differ. Consequently, price movements will affect aggregate producer price and CPI price levels differently and besides terms of trade movements also internal real exchange rate movements will have consequences. We therefore acknowledge that there are no economic forces visible why shocks to services should become more correlated as the euro area deepens further in exchange of goods and assets.

Fifth, our findings indicate that the results in Benigno (2004) and Benigno and López-Salido (2006) are not general. Accordingly policy recommendations, namely that policy should target the rate of the CPI in the region with the higher nominal rigidity, are cast in doubt. We will show in detail that their results are not welfare optimal in cases where one allows for a role of services and where services characteristics differ across members. As a result, we find that there is no more one-to-one relationship between nominal rigidity and optimal country weight in policy when rigidity derives from inertial price setting in services. Our results point to a potential solution to the incentive problem of consolidating structural inefficiencies inherent in the Benigno (2004) proposition. Sixth, as our framework allows for a two-sector setup, it can be suitable employed for studying eastward enlargement of the euro area. Especially developments in the non-tradables sectors might be a source of increased heterogeneity in the enlarged area. Albeit we find that nominal rigidity might in fact be lower in new EU members, there is evidence that structural shocks are more volatile in new members such that costs from fluctuations are increased. Eventually , we provide policy recommendations. We identify sources of distortions that should receive highest priority by policy makers and assess the potential gains from removing them, based on empirically plausible calibrations of the model using our well-defined welfare measure.

Our contributions are also subject to limitations. Employing a micro-founded framework makes necessary the calibration of a wide range of structural parameters that guide the behaviour of private sectors and governments as well as the response of monetary policy. For assessing inflation differentials, estimates on nominal rigidity (the last column in table 2.1) are crucial. However, estimates on sectoral hybrid Phillips curves based on time-dependent Calvo (1983) pricing are so far not available in the literature on a comparable basis.[34] We solve this issue by calibrating sectoral values such that structural shock decompositions of euro area aggregates could be broadly matched and the aggregate values implied by nominal rigidity reported in table 2.1 could be reproduced by weighing sectoral values. Also, welfare results depend critically on the magnitude of volatility induced by structural shocks where sectoral estimates are hardly available. Below, it will be shown how we cope with these data limitations in order to still obtain interpretable results that match the aggregate characteristics presented

[34] Micro-evidence in case of Germany is provided in Stahl (2005).

in table 2.1.

In order to substantiate these results, we proceed as follows. In the next section we set up the structural model. Its log-linear approximation is presented under 2.3. In section 2.4 we calibrate the framework for scenarios to be evaluated in the three main experiments. The empirical validity of the model is secured by comparing time series generated by the model with empirical series for countries considered in the scenarios. The main workings of heterogeneity along the business cycle are investigated by means of impulse responses to supply and demand shocks in member states as well as to common shocks that hit the union as a whole. Main experiments as outlined in the introduction are presented and evaluated then. In experiment I in section 2.5, we highlight welfare implications of member state heterogeneity and its main sources in general. We first derive the micro-founded loss function for the currency union that guides the analysis throughout experiments. The role of member state size, nominal and real rigidities, and the composition of the domestic HICP are highlighted then. Experiment II in section 2.6 determines the optimal course of monetary policy under heterogeneity. Experiment III is evaluated in section 2.7. Structural reform options for mitigating structural differences between union regions are researched there. Conclusions follow.

2.2 The Two-Region Two-Sector Currency Union Model

In order to answer above posed questions in dynamic general equilibrium, we develop a sticky-price currency union framework composed of two members (two groups of countries). Two countries[35], Home and Foreign form a currency union such that the nominal exchange rate is irrevocably fixed between them. Both economies are characterised by a two-sector production structure where goods within a sector are imperfect substitutes to each other which gives rise to monopolistically-competitive price setting. Goods produced in the tradables sector of the home region can be exported to the foreign region at no cost and vice versa. The union is closed such that all tradable produce needs to be either consumed at Home or Foreign. Tradable goods are assumed to represent industry production whereas non-tradables summarise services (total gross value added less of industry production).[36] Labour is the only variable input factor to produce non-tradable goods Y_N and tradable goods Y_H at home and non-tradables Y_{N^*} and tradables Y_F at foreign. Therefore sectoral gross value added is given by Y_J, where $J = H, F, N, N^*$. Accordingly, physical capital is assumed to be fixed in production

[35] The terms 'countries', 'regions', 'nations' will be used interchangeably in the text.

[36] According to the NACE ('Nomenclature générale des activités économiques dans les Communautés Européennes') classification, industry production as defined here refers to the category C_D_E with main components manufacturing, energy, food production.

as the focus is on business cycle dynamics, i.e. on the short to medium run where capital input does not vary much. The model is microfounded in the sense that it belongs to the New Open Macroeconomics suite of models, see Lane (2001) for an overview. As discussed above, heterogeneity will be assessed regarding shock exposure (union-wide versus asymmetric), shock propagation (different response of consumption, inflation and output to a certain shock), regarding the domestic structural set-up of factor and product markets, as well as price rigidities and inflation persistence.

2.2.1 Households

Preferences

The two country currency union is populated by a continuum of households (citizens) that are monopolistic consumer-producers where an agent is a home agent if $j \in [0, n)$, and a foreign one $j \in [n, 1]$. A home agent j maximises life-time utility

$$U_t^j = \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left\{ \mathcal{U}\left(C_s^j\right) - \mathcal{V}\left(y_{J,t}^j, A_{J,t}^j\right) \right\} \tag{2.1}$$

conditional on all information that is available at the beginning of period t. As consumer, the household enjoys utility from consuming a consumption basket C_t^j which yields $\mathcal{U}(C_t^j)$.[37] The household experiences disutility from producing a differentiated good y_J^j in sector J given by $\mathcal{V}(y_{J,t}^j, A_{J,t})$ where $\mathcal{V}$ is a convex function that is increasing in y_J^j. $A_{J,t}$ is a disturbance affecting the disutility of work, which will be interpreted as a labour productivity shock in sector $J = H$, N (more generally, a shock to the level of production technology as interpreted in Beetsma and Jensen (2005b)). The higher is $A_{J,t}$, the lower is disutility of producing one unit of $y_{J,t}^j$, the higher is per period utility. We refrain from introducing any explicit assumptions about functional forms for $\mathcal{U}$, $\mathcal{N}$, and $\mathcal{V}$ that guide the behaviour of households and firms, stressing that elasticities are sufficient for summarising results for business cycles and costs thereof. We therefore obtain some generality of our results.

Our specification of production decisions is analogous to Aoki (2001). There, households as firms decide between production in the flexible-price sector (homogenous good sector) and the sticky price (differentiated goods sector). In our case, household j as producer decides on producing a single good, either in the industry sector ($J = H$) or in the services sector

[37] Note that unlike as in the following chapter, utility from liquidity services of holding real money balances $\mathcal{N}(\frac{M_t^j}{P_t}, \xi_t)$ is not included in U_t^i. As the nominal interest rate (the three month money market rate) serves as monetary policy instrument, the money market equilibrium is of no role for the dynamics in the model, see also Beetsma and Jensen (2005b) and Woodford (2003). In the following chapter, money is included as the balance sheet of the central bank is modelled explicitly.

$(J = N)$. Goods production is differentiated in both sectors such that goods are imperfect substitutes within a sector which gives rise to monopolistically-competitive price setting. As households are consumer-producers, the mass of households producing in the industry sector is given by the economic weight of industry goods in overall gross value added of the Home region, $\frac{Y_H}{Y}$, and accordingly for the services sector $\left(1 - \frac{Y_H}{Y}\right)$. Average disutility of giving up leisure for work in region Home is given by

$$\frac{1}{n}\int_0^n \mathcal{V}\left(y^j_{J,t}, A^j_{J,t}\right) dj = \frac{1}{n}\left[\int_0^n \mathcal{V}(y_{H,t}(h), A_{H,t})dh + \int_0^n \mathcal{V}(y_{N,t}(h), A_{N,t})dh\right] \tag{2.2}$$

$$= \frac{1}{n}\left[\frac{Y_H}{Y}\mathcal{V}(Y_{H,t}, A_{H,t}) + \left(1 - \frac{Y_H}{Y}\right)\mathcal{V}(Y_{N,t}, A_{N,t})\right] \tag{2.3}$$

where $Y_{H,t}$ and $Y_{N,t}$ denote real gross value added in the industry sector and services sector respectively. We therefore straightforwardly extend the analysis in Benigno (2004) to the two sector case. In that contribution, non-tradables production is absent, $\int_0^n \mathcal{V}(0, A_{N,t})dh = 0$ such that for a generic household $y^j_{H,t} = y_{H,t}(h)$ and the subscript H can be omitted.

The overall consumption basket C^j_t is a Dixit-Stiglitz aggregator composed of tradable goods (consumption), $C^j_{T,t}$, and non-tradable goods (consumption) $C^j_{N,t}$

$$C^j_t = \frac{(C^j_{T,t})^\gamma (C^j_{N,t})^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \tag{2.4}$$

where γ denotes the share of total expenditure allocated to the consumption of traded goods. γ can therefore be proxied by the measure presented in table 2.1 on page 16, namely the weight of consumption expenditure on industrial goods in household's consumption basket. The Cobb-Douglas form for C^j_t derives from the assumption that the elasticity of substitution between the goods bundles C_T and C_N is 1 as in Benigno and Thoenissen (2003) which serves as a simplifying but common assumption here. Empirical estimates are not available for euro area member states, older estimates in Stockman and Tesar (1994) point to values for the elasticity below one. The aggregate price index measured at the consumer price level is obtained from minimising expenditures for obtaining one unit of C^j_t, such that

$$P_t = P^\gamma_{T,t} P^{1-\gamma}_{N,t} = (\frac{P_{T,t}}{P_{N,t}})^\gamma P_{N,t} = Q^\gamma_t P_{N,t} \tag{2.5}$$

We introduced the internal real exchange rate $Q_t = \frac{P_{T,t}}{P_{N,t}}$ as the relative price of tradables $P_{T,t}$ in units of non-tradables consumption $P_{N,t}$. Tradable consumption $C^j_{T,s}$ is by itself a composite of home produced tradable goods consumption $C^j_{H,t}$ and foreign produced tradable (hence imported) consumption $C^j_{F,t}$

$$C^j_{T,t} = \frac{(C^j_{H,t})^{\nu}(C^j_{F,t})^{1-v}}{\nu^{\nu}(1-\nu)^{1-\nu}} \tag{2.6}$$

ν denotes the share of home produced tradable goods in tradable consumption. $v > n$ would imply that there exists home bias in tradable consumption. Again, the assumptions about consumption preferences imply that the intratemporal elasticity of substitution between the bundles C_H and C_F equals unity.[38] Goods within the basket are imperfect substitutes to each other. Consequently the (absolute value of the) price elasticity of demand is $\infty > \rho_H > 1$ in case of the choice between elements in the home tradable basket C_H and accordingly $\infty > \rho_F > 1$ for imported items C_F. The price index for tradables $P_{T,t}$ is then

$$P_{T,t} = P^v_{H,t}P^{1-v}_{F,t} = T_t^{-v}P_{F,t}$$

where we introduced the definition of the terms of trade as the price ratio of imported goods in terms of exports $T_t = \frac{P_{F,t}}{P_{H,t}}$.

Overall consumption at foreign C^{j*}_t and tradable consumption $C^{j*}_{T,t}$ are accordingly

$$C^{j*}_t = \frac{(C^{j*}_{T,t})^{\gamma^*}(C^{j*}_{N^*,t})^{1-\gamma^*}}{\gamma^{*\gamma^*}(1-\gamma^*)^{1-\gamma^*}}, \qquad C^{j*}_{T,t} = \frac{(C^{j*}_{H,t})^{\nu^*}(C^{j*}_{F,t})^{1-v^*}}{v^{*\nu^*}(1-v^*)^{1-v^*}}$$

where a superscripted asterisk generally denotes a variable that applies for a foreign household. Price indices are accordingly

$$P^*_t = \left(P^*_{T,t}\right)^{\gamma^*}\left(P^*_{N^*,t}\right)^{1-\gamma^*}, \qquad P^*_{T,t} = \left(P^*_{H,t}\right)^{v^*}\left(P^*_{F,t}\right)^{1-v^*}$$

As P_t and P^*_t contain the same categories of goods, the price indices are harmonised in line with the empirical counterparts (the domestic HICPs). We assume that the law of one price (LOP) holds for any good that is shipped from home to foreign and vice versa. One condition that LOP will hold in the tradable sector is that the share of expenditures on home produced tradables in the overall consumption basket is same across regions. This assumption requires that $v = v^*$ such that there must not be home bias in consumption of $C^j_{T,t}$ and $C^{j*}_{T,t}$. Accordingly, home and foreign must have the same preferences about H and F consumption and hence the same composition of these goods in their overall basket. This assumption is in line with Obstfeld and Rogoff (2000) and Benigno (2004). It can be argued that home bias in consumption might be

[38] As a consequence, relative price changes $\frac{P_F}{P_H}$ relate to changes in relative quantities $\frac{C_H}{C_F}$ one-for-one.

less of an issue for countries within a currency union than between arbitrarily selected countries.

The individual consumption baskets are bundled according to

$$
\begin{aligned}
C^j_{H,t} &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\rho_H}} \int_0^n c_{H,t}(h)^{\frac{\rho_H-1}{\rho_H}} dh\right]^{\frac{\rho_H}{\rho_H-1}}, \quad C^{j*}_{H,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\rho^*_H}} \int_0^n c^*_{H,t}(h)^{\frac{\rho^*_H-1}{\rho^*_H}} dh\right]^{\frac{\rho^*_H}{\rho^*_H-1}} \\
C^j_{F,t} &= \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\rho_F}} \int_n^1 c_{F,t}(f)^{\frac{\rho_F-1}{\rho_F}} dh\right]^{\frac{\rho_F}{\rho_F-1}}, \quad C^{j*}_{F,t} = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\rho^*_F}} \int_n^1 c^*_{F,t}(f)^{\frac{\rho^*_F-1}{\rho^*_F}} dh\right]^{\frac{\rho^*_F}{\rho^*_F-1}} \\
C^j_{N,t} &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\rho_N}} \int_0^n c_{N,t}(h)^{\frac{\rho_N-1}{\rho_N}} dh\right]^{\frac{\rho_N}{\rho_N-1}}, \quad C^{j*}_{N^*,t} = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\rho^*_{N*}}} \int_n^1 c^*_{N^*,t}(f)^{\frac{\rho^*_{N*}-1}{\rho^*_{N*}}} dh\right]^{\frac{\rho^*_{N*}}{\rho^*_{N*}-1}}
\end{aligned}
$$

where the left panel shows consumption baskets consumed by household j at Home and the right panel lists baskets consumed at Foreign by household j^*. The notation is such that subscripts refer to the type of goods category, such that $J = H, F, N, N^*$. Therefore, goods consumed at Home or Foreign are produced either in the industry sectors at home or abroad, H or F, or in the service sectors, N or N^*.[39] A superscripted asterisk indicates that the destination of consumption is Foreign, Home otherwise. $\infty > \rho_K > 1$ is the elasticity of substitution across goods, where $K = H, F, N$ for Home and $\infty > \rho^*_K > 1$ with $K = H, F, N^*$ in case of Foreign. The non-tradable consumption basket in (2.4) is defined as aggregate consumption over all non-tradables produced at home, where economic size of the non-tradable sector and the country size coincides. The sectoral (producer price) indices (PPI) that are derived from minimising consumption expenditures for obtaining one unit of H, F, N at Home and H, F, N^* at foreign are accordingly

$$
P_{H,t} = \left[\frac{1}{n}\int_0^n (p_{H,t}(h))^{1-\rho_H} dh\right]^{\frac{1}{1-\rho_H}}, \quad P^*_{H,t} = \left[\frac{1}{n}\int_0^n (p^*_{H,t}(h))^{1-\rho^*_H} dh\right]^{\frac{1}{1-\rho^*_H}} \tag{2.7}
$$

$$
P_{F,t} = \left[\frac{1}{1-n}\int_n^1 (p_{F,t}(f))^{1-\rho_F} df\right]^{\frac{1}{1-\rho_F}}, \quad P^*_{F,t} = \left[\frac{1}{1-n}\int_n^1 (p^*_{F,t}(f))^{1-\rho^*_F} df\right]^{\frac{1}{1-\rho^*_F}} \tag{2.8}
$$

$$
P_{N,t} = \left[\frac{1}{n}\int_0^n (p_{N,t}(h))^{1-\rho_N} dh\right]^{\frac{1}{1-\rho_N}}, \quad P^*_{N^*,t} = \left[\frac{1}{1-n}\int_n^1 (p^*_{N^*,t}(f))^{1-\rho^*_{N*}} dh\right]^{\frac{1}{1-\rho^*_{N*}}} \tag{2.9}
$$

$P_{H,t}$ is the PPI for home goods industry H, $P_{N,t}$ is the PPI for home services N. $P^*_{F,t}$ and $P^*_{N^*,t}$ are the respective measures at foreign. $P_{F,t}$ denotes the producer price of foreign produced goods sold at home (the import price index) and $P^*_{H,t}$ is the index for home exported goods (the import price index at foreign).

Private sector demand for a generic tradable good h for an agent at Home and tradable

[39] For example $C^{j*}_{H,t}$ defines home exports. The subscripted H indicates that goods are produced in the home tradable sector and the superscripted $j*$ denotes goods consumed by a foreign household.

good f for an agent at Foreign is given by

$$
\begin{aligned}
c^j_{H,t}(h) &= \frac{1}{n}\left(\frac{p_{H,t}(h)}{P_{H,t}}\right)^{-\rho_H} C^j_{H,t}, \qquad c^{j*}_{H,t}(h) = \frac{1}{n}\left(\frac{p^*_{H,t}(h)}{P^*_{H,t}}\right)^{-\rho^*_H} C^{j*}_{H,t} \\
c^j_{F,t}(f) &= \frac{1}{1-n}\left(\frac{p_{F,t}(f)}{P_{F,t}}\right)^{-\rho_F} C^j_{F,t}, \qquad c^{j*}_{F,t}(f) = \frac{1}{1-n}\left(\frac{p^*_{F,t}(f)}{P^*_{F,t}}\right)^{-\rho^*_F} C^{j*}_{F,t}
\end{aligned}
$$

For example, $c^j_{F,t}(f)$ denotes the demand for a foreign produced good f by a home household j and $c^{j*}_{H,t}(h)$ denotes demand for an exported good h by household j^* at Foreign. The respective price elasticities of demand are $\frac{d\ln c^j_{F,t}(f)}{d\ln \frac{p_{F,t}(f)}{P_{F,t}}} = -\rho_F$ and $-\rho^*_H$, respectively. Accordingly for non-tradables

$$
c^j_{N,t}(h) = \frac{1}{n}\left(\frac{p_{N,t}(h)}{P_{N,t}}\right)^{-\rho_N} C^j_{N,t}, \qquad c^{j*}_{N^*,t}(h) = \frac{1}{1-n}\left(\frac{p^*_{N^*,t}(f)}{P^*_{N^*,t}}\right)^{-\rho^*_{N^*}} C^{j*}_{N^*,t}
$$

As households derive utility from consuming an aggregate of differentiated goods, supplies face a downward-sloping demand schedule for their product, see also Amato and Laubach (2003, p. 795) and Blanchard and Kiyotaki (1987).

Risk Sharing

Benigno (2004) assumes that asset markets are complete within a region and incomplete across regions. Hence within each region, households can perfectly insure against all household-specific consumption fluctuations across time and states. For each contingency occurring there is a pay-off available that eliminates the drop in consumption such that consumption is the same in all states and at all dates, and for any history. Trade in interregional assets is in a non-state contingent union currency denominated nominal bond.[40] Benigno shows that in a model where GDP is completely tradable (there are only tradable goods in the model), perfect risk sharing in *real* consumption will materialise, i.e. consumption in growth rates will be identical across regions (and also in levels given that initial endowments are equally distributed across agents). Further, the interregional bond is redundant.[41]

Contrary to Benigno (2004), we need to extend the asset market structure as in the presence

[40] The return implies that the pay-off is independent of an event occuring and certain, it is therefore riskless. Especially, the return will not depend on next period's consumption that might materialise above or below trend. There might be a large discrepancy between the pay-off of this risk-less asset and the actual drop in consumption occuring. Hence incomplete markets will generally imply welfare losses for the household.

[41] This result is obtained by the assumption of unitary elasticity of subtitution between home and foreign consumption bundles. Then the terms of trade will immediately adjust and absorb any consumption fluctuations. Therefore the current account is independent of the terms of trade and terms of trade movements will not induce wealth effects.

of non-tradables international tradability of assets will no longer be redundant. An additional relative price, the internal real exchange rate that prices tradable in terms of non-tradable consumption, will influence the external real exchange rate between regions. Further, one key characteristic (and benefit) of a monetary union is financial asset market integration which makes it worthwhile to include that property in the model. We thus move beyond the incomplete interregional markets assumption and assume completeness of asset markets on the interregional level. Accordingly, households can hold a portfolio of shares of firms in the H, N, F, N^* sector. Thus, as in Galí and Monacelli (2005) the households have access to a complete set of contingent claims, traded across the union.

As non-tradable goods can not be shipped across borders by nature of these type of goods, a loss in non-tradable consumption at Home or Foreign cannot be smoothed by shipping real consumption bundles C_N to foreign.[42] Accordingly there is no arbitrage possible and regional price levels for non-tradables, P_N and $P^*_{N^*}$ will not adjust in order to equalise consumption across countries. As marginal rates of substitution in real tradable consumption and real non-tradables will not equalise across regions, the resulting equilibrium allocations will not be first-best, even in the absence of nominal and real rigidities. Hence international risk sharing will not be perfect *in real terms* (consumption will not equalise in levels and growth rates across regions), but nominal income risk can be fully insured against.[43] Therefore, if a drop in N^* consumption occurs at foreign, a dividend is paid by home households. The claim of a foreign household on home non-tradables consumption baskets C_N can be converted to tradables consumption by using its internal equilibrium relative price $Q = \frac{C_T}{C_N}\frac{\gamma}{1-\gamma}$ such that the payment $C_T\frac{\gamma}{1-\gamma}$ results which can be shipped to abroad. The foreign household uses the income stream to purchase tradable consumption baskets C^*_T until he receives the same period utility $\mathcal{U}\left(C^j_s\right)$ as before the drop in non-tradable consumption. Hence allowing for complete markets allows to establish risk sharing which will still be perfect *in nominal terms*, but not in real terms.

Budget Constraint

The budget constraint of a household j at home reads

$$(1-\tau^J)p_{J,t}(j)y_{J,t}(j) + Q^{\#j}_t + M^j_{t-1} + D^j_{t-1,t} \geq P_tC^j_t + M^j_t + \mathcal{E}_t\left\{V_{t,t+1}D^j_{t,t+1}\right\} \tag{2.10}$$

[42] Hence in the presence of non-tradables, a country's overall consumption growth $\ln C_t - \ln C_{t-1}$ could be stronger correlated with domestic output growth than with union consumption growth even in the presence of a full set of state-contigent claims on future tradables, simply because it is infeasible for countries to pool nontraded consumption risks directly. See also Obstfeld and Rogoff (1996, chapter 4).

[43] A benevolent planner would potentially find a way of equilibrating non-traded consumption across borders, which denotes a Pareto improvement as it lifts utility of at least one household without lowering utility of others.

The budget constraint compares nominal income (left hand side) with period expenditures (right hand side). $D^j_{t-1,t}$ denotes the nominal pay-off of a portfolio held up to the end of period $t-1$ (which includes shares in local and foreign firms of both sectors) and pays off at beginning of period t. Income is used to finance current consumption $P_t C^j_t$ as well as money holdings M^j_t and to purchase the portfolio $D^j_{t,t+1}$ for savings at the end of period t. $D^j_{t,t+1}$ pays off in union currency at the beginning of $t+1$. The inverse of the short term gross interest rate $\frac{1}{1+i_t}$ can then be defined as the date t price of the portfolio $D_{t,t+1}$ which delivers one unit of union currency in all states and dates that can occur in the next quarter. $V_{t,t+1}$ is the stochastic discount factor for one-period ahead nominal pay-offs (the nominal contingent claims price of portfolio D_{t+1}).

The value of an household's net accumulation of assets on date t (agent's total wealth) given by money holdings M^j_t and shares $\mathcal{E}_t \left\{ V_{t,t+1} D^j_{t,t+1} \right\} - D^j_{t-1,t}$ must equal the difference between its income from producing output, receiving transfers and holding her past period's money stock, and consumption expenditures

$$(1-\tau^J) p_{J,t}(j) y_{J,t}(j) + Q^{\#j}_t + M^j_{t-1} - P_t C^j_t = \underbrace{M^j_t + \mathcal{E}_t \left\{ V_{t,t+1} D^j_{t,t+1} \right\} - D^j_{t-1,t}}_{\text{total period } t \text{ nominal wealth}} \tag{2.11}$$

The constraint binds as an efficiency condition (such that the budget constraint is exhaustive). As an arbitrage condition between holding bonds or shares

$$\frac{1}{1+i_t} = \mathcal{E}_t \left\{ V_{t,t+1} \right\} \tag{2.12}$$

$\frac{1}{1+i_t}$ therefore denotes the risk-free return and is the date t price of a one-period zero coupon bond. Due to arbitrage, on average, the pay-off on the portfolio cannot exceed the pay-off on the riskless bond.[44]

External and Internal Relative Prices

There are four real exchange rates in the model. The CPI based external real exchange rate compares the consumer price of a representative basket for each country across countries expressed in currency of the home country

$$E_t = S_t \frac{P^*_t}{P_t} = \frac{P^*_t}{P_t} \tag{2.13}$$

[44] Under incomplete markets, only the average consumption loss can be insured against. Thus, consumption can be inefficiently low in some states occuring in $t+1$.

where the nominal exchange rate $S_t = S$ is irrevocably fixed in the currency union and set to one for convenience.[45] The basket is not representative across regions, as its composition will differ according to consumption preferences agents have for T versus N consumption which becomes clear from (2.4). E_t is the direct model analogon to the measure of external competitiveness presented in table 2.1 on page 16. In order to abstract from the effect of non-tradables on the external real exchange rate we can use the *trade based* external real exchange rate as a measure of external competitiveness, i.e. the terms of trade. The terms of trade are defined as the price of a country's exports divided by the price of its imports, again expressed in domestic (H) currency

$$T_t = \frac{P_{F,t}}{P_{H,t}} \tag{2.14}$$

Hence an increase in T_t denotes a worsening in the terms of trade: One unit of home currency buys less imported goods than before such that the purchasing power of the home currency decreased and home assets are worth less when evaluated in imported items. A decrease in T_t indicates that imported goods have become relatively cheaper and the terms of trade therefore improved such that the purchasing power of Home increased.[46] We assume that the LOP holds for the tradables basket. Therefore one euro buys the same representative basket composed of tradable goods in the home and foreign region. Hence we implicitly assume that both countries have the same preferences over tradables which results in the same composition of a basket for that sort of goods. According to the LOP, the domestic currency price of a home produced tradable basket sold in the export market $P^*_{H,t}S$ must have the same price as when sold at home directly. Analogously in case of the imported good. Therefore

$$P^*_{H,t}S = P_{H,t}, \qquad P^*_{F,t}S = P_{F,t}$$

Note that due to product differentiation also in the tradables sector, the price of the produce is not determined in the world market (and hence not exogenous). Therefore the producer has pricing power over its product both at home and abroad. We assume that the firm has no ability to use its pricing power to discriminate prices across borders, such that $p_H(h) = p^*_H(h)$ and $p^*_F(f) = p_F(f)$. Consequently, there is no possibility of local currency pricing. Using the definition for the price index for tradables for home and foreign yields that then price levels in tradables in each country fluctuate one for one

$$P^*_{T,t} = P_{T,t} \tag{2.15}$$

[45] Any value for S will leave the fluctuations about the trend unaffected.

[46] Alternatively, we can refer to the terms of trade as the 'nominal exchange rate' for tradables.

As a prerequisite we need to assume that home bias in tradable consumption in each region is absent, such that $v = v^*$. We also assume that the share of home produced exportable goods corresponds to the country size, such that $v = v^* = n$. We can use these results in the terms of trade expression

$$T_t = \frac{P_{F,t}}{P_{H,t}} = \frac{P^*_{F,t} S_t}{P^*_{H,t} S_t} = \frac{P^*_{F,t}}{P^*_{H,t}} \tag{2.16}$$

The law of one price guarantees that the terms of trade adjust such that the relative price of tradables remains constant at all dates. In other words, $T \equiv P_F/P_H$ adjusts such that one unit of the union currency buys the same tradables basket in each country.[47] Although the LOP holds in tradables, purchasing power parity (PPP) does not hold, i.e.

$$P_t \neq P^*_t \tag{2.17}$$

due to different shares of non-tradables in overall consumption (different tastes of households in each region) determined by the share of income spent on tradable goods $\gamma \neq \gamma^*$. Consequently, a different weighting in the regional CPI will be present and price levels will not equalise.

The framework implies a third measure of international competitiveness, namely the domestic real exchange rate which expresses the price of tradables in terms of units of non-tradables:

$$Q_t = \frac{P_{T,t}}{P_{N,t}}, \qquad Q^*_t = \frac{P^*_{T,t}}{P^*_{N^*,t}} \tag{2.18}$$

The external real exchange rate can be decomposed in contributions from the terms of trade and contributions from internal real exchange rates

$$E_t = \frac{P^*_t}{P_t} = \frac{\left(P^*_{T,t}\right)^{\gamma^*}\left(P^*_{N^*,t}\right)^{1-\gamma^*}}{P^{\gamma}_{T,t} P^{1-\gamma}_{N,t}} = \frac{\left(P^*_{N^*,t}/P^*_{T,t}\right)^{1-\gamma^*}}{\left(P_{N,t}/P_{T,t}\right)^{1-\gamma}} \frac{P^*_{T,t}}{P_{T,t}} = \frac{Q_t^{1-\gamma}}{Q_t^{*1-\gamma^*}} \frac{P^*_{T,t}}{P_{T,t}} = \frac{Q_t^{1-\gamma}}{Q_t^{*1-\gamma^*}} \tag{2.19}$$

using that by the LOP $P^*_{T,t} = P_{T,t}$. E_t is hence a composite of the internal real exchange rates that are determined by the region specific relative price of industry goods in terms of services.[48] A real appreciation in Q_t triggered by an increase in non-tradables prices will cause a real appreciation in E_t where the magnitude depends on the consumption weight of nontradables in the overall index, given by $1 - \gamma$ and $1 - \gamma^*$. As movements in non-tradables affect E_t,

[47] By the LOP, inflation rates in tradables are perfectly linked as otherwise one country's increase in tradables inflation leads one for one to an increased demand for the other country's tradables. An important difference between the flexible and fixed exchange rate case concerns the role of price stickiness. In the currency union, S is fixed which implies that stickiness in P_H translates into stickiness in import prices in the other region P_{H^*}. Under flexible exchange rates, fluctuations in S_t under the assumption of the LOP will break this link.

[48] Home bias and market segmentation are absent in our specification, unlike to Benigno and Thoenissen (2003), equation 22.

both regions cannot fully insure against fluctuations in total real consumption and overall real consumption in levels and growth rates will not be fully correlated across regions. However nominal consumption growth will fluctuate one-for-one as will be derived in the following section.[49]

Intertemporal Consumption Decisions

Any household at home and foreign decides on allocating income between consumption and savings within each period and across periods. In order to move consumption across time, the household saves by buying one period portfolios of state-contingent claims on profits of firms in all sectors in the union. The household therefore chooses consumption C_t^j subject to the budget constraint (2.10) such that

$$\beta^{s-t}\mathcal{U}_C(C_t^j) - \lambda_t P_t = 0$$

and decides on portfolio holding $D_{t,t+1}$

$$-\lambda_t V_{t,t+1} + \beta\lambda_{t+1} = 0$$

The complete markets assumption implies that the preceding equation holds in any state of nature in period t and $t+1$ and any preceding history. Hence conditional expectations can be omitted as the information set at the beginning of period t is not relevant for making choices. Hence in any state, the intertemporal consumption-savings decision follows

$$\mathcal{U}_C(C_t^j) = \beta V_{t,t+1}^{-1}\mathcal{U}_C(C_{t+1}^j)\frac{P_t}{P_{t+1}} \tag{2.20}$$

where the stochastic discount factor is therefore determined from consumption asset pricing.

$$V_{t,t+1} \equiv \beta\frac{\mathcal{U}_C(C_{t+1}^j)}{\mathcal{U}_C(C_t^j)}\frac{P_t}{P_{t+1}} = m_{t+1}\frac{P_t}{P_{t+1}} \tag{2.21}$$

$V_{t,t+1}$ (the pricing kernel for evaluating nominal income) provides a price for intertemporal consumption in any state that is possible and m_{t+1} denotes the pricing kernel for evaluating real income streams measured in total consumption units. Consumption in period $t+k$ is

[49] Purchasing power parity would hold, given that the share of service goods in consumption is same across regions, $1-\gamma = 1-\gamma^*$ and there is no home bias, i.e. $\nu = \nu^*$. Further, price levels in services between regions should equalise. However, there is no reason why this should be the case as there is no interregional equilibrating mechanism for $P_{N,t}$ and $P_{N^*,t}$.

accordingly evaluated as

$$V_{t,t+k} \equiv \beta^k \frac{\mathcal{U}_C(C^j_{t+k})}{\mathcal{U}_C(C^j_t)} \frac{P_t}{P_{t+k}} = m_{t+k} \frac{P_t}{P_{t+k}} \tag{2.22}$$

The relationship implies that discounted marginal rates of substitution between period t and period $t+k$, $\beta^k \frac{\mathcal{U}_C(C^j_{t+k})}{\mathcal{U}_C(C^j_t)}$, equal real contingent claims price ratios m_{t+k}. Taking conditional on period t expectations of (2.20) one can use that due to arbitrage $\mathcal{E}_t V_{t,t+1} = \frac{1}{1+i_t}$ where $\frac{1}{1+i_t}$ denotes the period t price of a one period zero-coupon bond that pays-off one unit of C in $t+1$ for sure. One arrives at the conventional Euler equation for consumption

$$\mathcal{U}_C(C^j_t) = \beta(1+i_t)\mathcal{E}_t \left[\mathcal{U}_C(C^j_{t+1}) \frac{P_t}{P_{t+1}} \right] \tag{2.23}$$

where $\mathcal{E}_t$ denotes the conditional expectations operator. At foreign, the Euler equation to evaluate average consumption is correspondingly given by

$$\mathcal{U}_{C^*}(C^{j*}_t) = \beta(1+i_t)\mathcal{E}_t \left[\mathcal{U}_{C^*}(C^{j*}_{t+1}) \frac{P^*_t}{P^*_{t+1}} \right] \tag{2.24}$$

Conventional in this sense means that this relationship can be obtained both under complete markets and incomplete markets (given there is available a riskless bond under incomplete markets). However, under incomplete markets this is the only equation for pricing consumption intertemporally. In that case, the household can insure only against the average shortfall in consumption which might not be a very suitable cushion against consumption losses that realise in a certain state, resulting in possibly large utility losses.

Under complete markets, the state-contingent portfolio is available in both regions and an analogous equation to (2.20) will hold in foreign as well.[50] Using this property, we can combine the intertemporal consumption-savings decision in each region in any state to link marginal utility across regions

$$\frac{\mathcal{U}_C(C^j_t)}{\mathcal{U}_{C^*}(C^{j*}_t)} E_t = \frac{\mathcal{U}_C(C^j_{t+1})}{\mathcal{U}_{C^*}(C^{j*}_{t+1})} E_{t+1} \tag{2.25}$$

Such a relationship will hold in any state of nature in period t and $t+1$ and any preceding history.[51] Normalising initial conditions $\frac{\mathcal{U}_C(C^j_0)}{\mathcal{U}_{C^*}(C^{j*}_0)} E = \kappa$ where κ depends on the initial endow-

[50] As utility functions and subjective discount factors are same across regions, one obtains that for $C^j_{t+1} = C^{j*}_{t+1}$ at any date $\mathcal{U}_C(C^j_{t+1}) = \mathcal{U}_{C^*}(C^{j*}_{t+1})$. Hence the stochastic discount factor $V_{t,t+1}$ will price a certain level of consumption in both regions in the same way as the same level of utility is derived, $\beta \frac{\mathcal{U}_{C^*}(C^{*j}_{t+1})}{\mathcal{U}_{C^*}(C^{j*}_t)} = V_{t,t+1}$.

[51] Note that such a relationship will not hold under expectations in case of incomplete markets, as $\mathcal{E}_t \left[\mathcal{U}_C(C^j_{t+1}) \frac{P_t}{P_{t+1}} \right] \neq \mathcal{E}_t \left[\mathcal{U}_C(C^j_{t+1}) \right] \mathcal{E}_t \left[\frac{P_t}{P_{t+1}} \right]$ and similarily at foreign. Hence under incomplete markets, one (also) cannot insure against the developments in the real exchange rate E_t.

ments C_0^j and C_0^{j*} and initial price levels P_0 and P_0^*, one obtains perfect risk sharing in *nominal* terms (evaluated at the current real exchange rate E_t)

$$\mathcal{U}_C(C_t^j)E_t = \kappa\mathcal{U}_{C^*}(C_t^{j*}) \tag{2.26}$$

Perfect means, that marginal utilities in *overall* consumption measured in union currency will equalise between regions in all states and all dates.[52] From the equation follows that differences between marginal utilities (and hence consumption growth) should be explainable by real exchange rate movements: If marginal utility at foreign $\mathcal{U}_{C^*}(C_t^{j*})$ is above home, the real exchange should appreciate (a decrease in E_t) such that foreign consumption becomes cheaper, such that C_t^{j*} increases and C_t^j decreases.[53]

Observe that generally, due to the presence of non-tradables, the consumer price index at home, P_t and foreign P_t^* will not equalise. As there is no relative price for non-tradables across regions, marginal utilities will not equate in real terms, as stressed above. In case non-tradable goods would be absent and given the law of one price still holds in the tradable sector, then also purchasing power would hold (an interregional reference basket exists and P_t and P_t^* buy this basket). Marginal rates of overall real consumption would then equalise and fluctuations in consumption would be exactly same at home and foreign.

Further, due to the presence of the external real exchange rate E_t in (2.26), nominal rigidities and real rigidities that derive from monopolistic competition will affect the level of real consumption in each region and correlation between consumption levels. Spillovers that can originate in inefficiencies or price level persistencies in the non-tradables sector can cause inefficient consumption fluctuations in the other region. Hence, in the presence of non-tradables, there arises a non-trivial role for monetary policy stabilisation of consumption. Monetary policy geared at eliminating nominal rigidities and aiming at maximising the utility of the household will thus try to increase the comovement between real consumption across regions by stabilising E_t.

Intratemporal Consumption Decisions

Given the optimal intertemporal choice on overall consumption and savings, the household selects the composition of the basket that meets his preferences and minimises expenditures within a period. The intratemporally optimal decision between tradable and non-tradable

[52] The proof is by induction: Show that the condition holds initially at date $t = 0$, and pick an arbitrary period. Then show that the condition also holds for the next period. As a consquence (2.26) holds for all periods.

[53] The de facto low correlation has been coined the consumption-real exchange rate anomaly first documented by Backus and Smith (1993).

consumption is obtained as

$$\frac{C^j_{T,s}}{C^j_{N,s}} = \frac{\gamma}{1-\gamma} Q_t^{-1} \tag{2.27}$$

$\frac{C^j_{T,s}}{C^j_{N,s}}$ is a function of the relative price of tradables to non-tradables and the expenditure shares on traded goods γ and non-traded goods $1-\gamma$. An appreciation (decrease) in the internal real exchange rate Q_t implies that tradable goods have become relatively less expensive which will tilt consumption away from non-tradables towards tradables and vice versa. Similarly at foreign

$$\frac{C^{j*}_{T,s}}{C^{j*}_{N^*,s}} = \frac{\gamma^*}{1-\gamma^*} \left(Q_t^*\right)^{-1} \tag{2.28}$$

Completeness in asset markets also has consequences for consumption choices of union members within each region, as for any two households within a region (a household j producing a good in sector H and another household j' producing an N good, say), a condition similar to (2.26) can be derived between the two households j and j'

$$\mathcal{U}_C(C^j_t) = \kappa' \mathcal{U}_C(C^{j'}_t) \tag{2.29}$$

As a result, sectoral employment risks can completely be insured against. As households can insure against idiosyncratic income risk across regions (the risk of country-specific averse supply shocks) and within regions (the risk of differences in labour income when producing in sector H instead of sector N at Home, or F instead of N^* at Foreign), yields that each household within a region faces the same intertemporal budget constraint . Together with the assumption of identical initial wealth ($C^j_0 = C^{j'}_0$, $C^{j*}_0 = C^{j'*}_0$), households within each region will choose identical consumption plans. As in Aoki (2001, p. 59) insurance contracts (the portfolio decisions) are assumed to be made before households know in which sector they produce and provide labour. The insurance contracts therefore make the marginal utility of nominal income identical across the households at any time t. We therefore can drop the household specific index j regarding consumption C^j_t and $C^j_{J,t}$, and refer to the average consumption level C_t (per citizen consumption) and $C_{J,t}$ instead. Thus

$$C^j_t \equiv C_t \qquad C^{j*}_t \equiv C^*_t \qquad C^j_{J,t} \equiv C_{J,t} \tag{2.30}$$

where $J = H, N, F, N^*$. C_t and $C_{J,t}$ therefore coincide with average consumption.

Labour Supply

Besides intertemporal and intratemporal consumption decisions, the household j as a producer determines its labour supply and demand for producing good h, where $j = h$. and $j* = f$ as each household produces exactly one differentiated good. For a generic household h, the labour supply equates the marginal rate of substitution of consumption of goods and services (the basket C_t^j) for leisure (implicit in the disutility of producing output $y_{J,t}(h)$) with its opportunity costs, i.e. the post-tax real wage

$$-\frac{U_{y_J,t}}{U_{C,t}} = \frac{\mathcal{V}_{y_J}(y_{J,t}(h), A_t^J)}{\mathcal{U}_C(C_t)} = (1-\tau^J)\frac{W_{J,t}}{P_t} \tag{2.31}$$

$J = H$, N and accordingly for Foreign. There are two sources of real rigidities that distort the labour supply decision and cause labour supply to be inefficiently low. One arises from the pricing power of monopolistically competitive firms, which causes *consumer* real wages $\frac{W_{J,t}}{P_t}$ $(\frac{W_{J,t}}{P_t^*})$, i.e. the nominal wage $W_{J,t}$ deflated by the cost of consumption on the consumer level P_t (P_t^*), to be lower than under full competition. Thus labour supply is inefficiently low. The labour supply shifts leftwards and a deadweight loss arises. The other derives from the (time-invariant) taxation of the real wage by τ^J, which is not lump sum, but has first order effects. We will later on argue that the role of this instrument can be reversed in order to cure the equilibrium distortion provided by monopolistically-competitive price-setting. The subsidy will then in fact be a real wage subsidy and leads households to provide labour as in the first best world under full competition. The role of fiscal policy in offsetting the distortion is important, as it prevents that utility-maximising monetary policy would be concerned with removing the equilibrium distortion.[54]

2.2.2 Firms

Production and Price Setting

Each consumer as firm produces a single differentiated good by employing (only his own) homogenous labour as input. For firm h in sector J we obtain

$$y_{J,t}(h) = f(L_{J,t}(h)) \tag{2.32}$$

as in Benigno (2004). All firms have access to the same technology $f(\cdot)$ where the level of total factor productivity is not addressed explicitly here. Capital is assumed to be given and fixed

[54] For the impact of indexed wage contracts on social welfare see Heinemann (2006). In that model, wage indexation reduces the inflation bias but may raise the variance of inflation rates.

in producing output. We exclusively focus on the short-run where capital adjustments will not vary much such that capital input can be assumed as fixed. The market is the whole union in case the firm produces a tradable good and restricted to the domestic economy in case the good is a non-tradable one. Limited substitutability of goods gives rise to pricing power of firms over their differentiated product such that firms behave monopolistically competitive. Price rigidity across time is introduced by assuming time-dependent pricing à la Calvo (1983). For any firm, there is only a fixed probability $1-\theta_J$ of resetting prices in a given period that is exogenous and does not depend on how long ago the last adjustment in price was. Also, θ_J is independent of the firm.[55] A limited probability of resetting prices induces price dispersion of goods' prices across time as not all firms are able to adjust prices following (supply and demand) shocks which distorts relative prices between any two firms in a sector and across sectors and regions.

In line with the arguments presented in the introduction, both price rigidity and intrinsic inflation persistence are prominent features in euro area inflation dynamics. We therefore also introduce costs involved in price-setting based on Amato and Laubach (2003) and Gali and Gertler (1999) that are different from menu costs or shoe leather costs. Gali and Gertler (1999) assume that there are two types of firms within each sector. A share $1-\omega_J$ of firms in sector J is forward-looking, hence performs in an optimising way, whereas ω_J firms set prices following a rule of thumb. Backward-looking firms base decisions completely on past realisations of variables and ignore the structure of the economy as well as monetary policy announcements completely, see also Ambler (2007, p. 22). Backward-looking price settings allows to introduce endogenous inflation persistence in the price setting scheme. Whereas Gali and Gertler (1999) and Benigno and López-Salido (2006) assume that the share of backward-looking firms and its composition is constant and firms' pricing behaviour therefore does not change through time (backward-looking firms never become optimising firms, and vice versa for optimising firms), we follow the interpretation of rule-of-thumb price setting introduced by Amato and Laubach (2003).

At the beginning of each period, those agents who are offered the opportunity to reset prices learn whether they are choosing a new price by solving their optimisation problem, or by using the rule of thumb instead (Amato and Laubach 2003, p. 798). The share of backward-looking firms is therefore not made up of the same firms through time, albeit the share is

[55] Accordingly, it is possible to derive the average duration of pricing contracts. In probability terms, Calvo-pricing can be described by a geometric distribution. With the probability of success given by $1-\theta_J$, the probability of being able to reset prices in period $t+s$ when deciding in period t is given by

$$\Pr(reset\ in\ t+s|t) = \theta_J^{s-1}\,(1-\theta_J)$$

The expected duration of price contracts for an individual firm is then $\frac{1}{1-\theta_J}$. As the expectation is independent of the individual firm, it also applies to all firms in the sector and delivers the average duration of price contracts.

constant. As each firm in any sector faces all possibilities of price-setting, the concept allows to stick with the representative household concept, which is relevant when performing welfare analyses below. Given the firm receives a signal and finds itself changing the price, which occurs with probability $1-\theta_J$, with probability $1-\omega_J$ it will set prices in an forward-looking and with probability ω_J in a backward-looking fashion. With probability θ_J, the firm's price will remain unchanged. Therefore, the Calvo-mechanism implies that the probability that a firm is selected to reset prices does not depend on whether the firm is optimising or non-optimising (backward-looking).

The objective of forward-looking (optimising) firms is to maximise the present discounted value of real profit streams that arise from selling their product to all households. In making their decision, firms have to take into account that the price set in t needs to remain optimal until a new possibility to reset prices arises in period $t+j$ and also that the change in price will affect the demand for its produce.[56] As firms are owned by households (households are consumer-producers), future profit streams arising from revenues of selling output are evaluated by making use of the consumption capital asset pricing model using the pricing kernel $m_{t,t+s} \equiv \beta^s \frac{\mathcal{U}_C(C_{t+s})}{\mathcal{U}_C(C_t)}$. A forward-looking firm in sector J eventually chooses $p^o_{J,t}(z)$ to maximise

$$\mathcal{E}_t \sum_{s=0}^{\infty} (\theta_J\beta)^s m_{t,t+s} \left(\frac{p^o_{J,t}(z)}{P_{J,t+s}} y_{J,t+s}(z) - MC^J_{t+s}(z) y_{J,t+s}(z) \right) \tag{2.33}$$

subject to the demand function for good z given by $y_{J,t+s}(z)$ and the evolvement of the aggregate price index $P_{J,t+s}$. $MC^J_{t+s}(z)$ denotes real marginal cost, i.e. the shadow value of increasing output $y_{J,t+s}(z)$ by one more unit. The optimally set price of a firm z then becomes[57]

$$\frac{p^o_{J,t}(z)}{P_{J,t}} = \frac{\rho_J}{\rho_J - 1} \frac{\mathcal{E}_t \sum_{s=0}^{\infty} (\theta_J\beta)^s \mathcal{U}_C(C_{t+s}) MC_{J,t,t+s}(z) y_{J,t,t+s}(z)}{\mathcal{E}_t \sum_{s=0}^{\infty} (\theta_J\beta)^s \mathcal{U}_C(C_{t+s}) \frac{y_{J,t,t+s}(z)}{P_{J,t+s}}} \tag{2.34}$$

This objective applies to a fraction $1-\omega_J$ of firms in sector J that are selected at random each period whereas a share of ω_J firms sets prices according to the rule of thumb.

Rule-of-thumb behaviour implies that a firm that is able to reset prices in t sets its price $p^b_{J,t}(z)$ attached to the past period's aggregate index of newly chosen prices $P^{\#}_{J,t-1}$ and adjusting for the sectoral gross inflation rate $\frac{P_{J,t-1}}{P_{J,t-2}}$. Both measures $P^{\#}_{J,t-1}$ and $\frac{P_{J,t-1}}{P_{J,t-2}}$ can be easily

[56] Remember that monopolistic-competitive firms face a downward sloping demand schedule.
[57] See appendix A.6.2 for details.

observed in period t.[58] In sector J, the price set by a backward-looking firm is therefore

$$P^b_{J,t} = P^{\#}_{J,t-1}\frac{P_{J,t-1}}{P_{J,t-2}} \tag{2.35}$$

We used that $p^b_{J,t}(z) = p^b_{J,t} = P^b_{J,t}$, $p^{\#}_{J,t}(z) = p^{\#}_{J,t} = P^{\#}_{J,t}$ as non-optimising firms will not differ in observing $P^{\#}_{J,t-1}\frac{P_{J,t-1}}{P_{J,t-2}}$ such that the firm-specific index can be omitted. When gross domestic inflation increased, backward-looking firms will move accordingly and increase prices as well, given that they received a 'Calvo-signal' in order to be allowed to do so. As all non-optimising firms behave accordingly, the price level composed of aggregating prices of non-optimising firms $P^b_{J,t}$ will increase.

Taken together, the aggregate sectoral price level evolves according to

$$P_{J,t} = \left[(1-\theta_J)P^{\#,1-\rho_J}_{J,t} + \theta_J P^{1-\rho_J}_{J,t-1}\right]^{\frac{1}{1-\rho_J}} \tag{2.36}$$

$P^{\#}_{J,t}$ contains newly set prices of all firms that reset prices in t, be they forward-looking (optimising, contained in $P^o_{J,t}$) or backward-looking (non-optimising, contained in $P^b_{J,t}$)

$$P^{\#}_{J,t} = \left[(1-\omega_J)P^{o,1-\rho_J}_{J,t} + \omega_J P^{b,1-\rho_J}_{J,t}\right]^{\frac{1}{1-\rho_J}} \tag{2.37}$$

Analogous relationships hold in sectors $J = N, F, N^*$. Due to the presence of backward-looking firms, average duration of price contracts in quarters, $\frac{1}{1-\theta_J}$, is no longer a measure of average nominal rigidity in the respective sector. It is instead given by the average duration of prices set by forward-looking firms $\frac{1}{1-\theta_J}\frac{1}{1-\varpi_J}$ as argued in the introduction.[59] The presence of rule-of-thumb firms decreases the probability that an optimising firm will be chosen to adjust prices. Thus there is a one-for-one link between inflation persistence and efficiency losses.

Under flexible prices $(\theta_J,\ \varpi_J \to 0)$, all firms can reset prices any period. Optimising firms

[58] Woodford (2003) models intrinsic inflation persistence by assuming price-indexing firms that attach their decision to past period's sectoral inflation rate. In that framework, θ_J determines prices that are indexed, see also Christiano et al. (2005). Firms in our setup instead use more information available at the end of the last period.

[59] The expected duration until an optimising firm will be selected in $t+s$, given that the firm was selected in period t for the last time, is given by

$$E\left(X_J\right) = \left(1-\varpi_J\right)\left(1 + 2\varpi_J + 3\varpi_J^2 + \dots\right) = \frac{1}{1-\varpi_J}.$$

As the price-resetting probability is independent from the firm being selected, the expected price duration of contracts set by forward-looking firms is on average given by

$$\mathcal{E}\left[X_J Y_J\right] \equiv Cov\left(X,Y\right) + \mathcal{E}\left[X_J\right]\mathcal{E}\left[Y_J\right] = \mathcal{E}\left[X_J\right]\mathcal{E}\left[Y_J\right] = \frac{1}{1-\theta_J}\frac{1}{1-\varpi_J}$$

The higher is nominal rigidity, the longer it takes for optimising price-contracts to be reset.

will choose the same optimal price such that real marginal cost is independent of the price level

$$\frac{p^o_{J,t}(h)}{P_{J,t}} = \frac{\rho_J}{\rho_J - 1} MC^J_t(h) = 1 \tag{2.38}$$

which becomes clear by using (2.34). Then in any period $P_{J,t} = P^{\#}_{J,t} = P^o_{J,t}$ by (2.36) from which follows that $P^b_{J,t} = P^o_{J,t-1}\frac{P_{J,t-1}}{P_{J,t-2}}$ by using (2.35). As the aggregate price level will not move (all firms are optimising each period and set the same price relative to the aggregate index), backward-looking firms also choose the optimal price and the preceding equation holds for all firms in a sector.

Labour Demand

Profit-maximising firms hire labour up to the point where their nominal marginal revenue product of labour (nominal marginal cost times physical marginal product of labour) equates its opportunity cost, i.e. the nominal wage $W_{J,t}$.

$$\underbrace{W_{J,t}}_{\text{nominal wage}} = \underbrace{P_{J,t}MC^J_t(h)}_{\text{nominal marginal cost}} \underbrace{f'\left(L_{J,t}(h)\right)}_{\text{marginal product of labour}} \tag{2.39}$$

$$= P_{J,t}MC^J_t(h)f'\left(f^{-1}(y_{J,t}(h))\right) \tag{2.40}$$

$MC^J_t(h)$ denotes real marginal cost. $\frac{W_{J,t}}{P_{J,t}}$ denotes the *producer* real wage, i.e. the factor cost relevant for the producer by deflating with the sectoral producer price index. One observes that consumer and producer real wages will differ. In the one-sector case, only openness will have an impact where foreign items will be included in the consumer price index but not in the producer price index. In the two sector case, producer real wages are sector-specific as well. Our setup does not incorporate any rigidities that arise from labour markets such that labour is fully homogenous and there is no wage differentiation. For the current euro area, this assumption is heroic. For labour markets in the EU9 there is however evidence that labour markets are quite flexible, see von Hagen and Siedschlag (2005). However note that in the modeling context, either wage or price rigidity will result in rigidity in the aggregate sectoral price indices $P_{J,t}$, such that one could easily concentrate on one source of rigidity. Homogeneity implies that the sectoral nominal wage is equal for all labourers $W^j_{J,t} = W_{J,t}$.

Under *competitive* markets, goods would be perfect substitutes $y_{J,t}(h) = Y_{J,t}$ and using the demand function one obtains that $p_{J,t}(h) = P_{J,t}$ for all firms. Then factor demand is independent of the firm and time invariant $MC^J_t(h) = MC^J$ such that the (sticky) price level serves as a numeraire. Also, real marginal cost would not be increasing in the level of production. One then can set $P'_{J,t} \equiv P_{J,t} MC^J$ as a new price level. In (2.39), efficiency is obtained as the marginal product of labour is again equated to the real wage. However following a shock, the price level will not move such that the result remains intertemporally inefficient albeit there is no price dispersion across firms. In case of *price flexibility*, the real wage is equated to the marginal product of labour as under competitive markets, but the real wage will be lower than under full competition which becomes clear from (2.38). Under both flexible prices and full competition, $MC_t^J(h) = MC_t^J = 1$ such that wages equal the marginal revenue product of labour in (2.39) $W_{J,t} = P_{J,t} f'(L_{J,t}(h))$. We stressed these differences in detail, as when discussing structural reform options later on, we will distinguish efforts aimed at increasing price flexibility from those aimed at increasing competition in markets.

Labour Market Equilibrium

Starting with the production function (2.32), one can substitute out labour $L_{J,t}(h)$ and express disutility of labour $\mathcal{V}(y_{J,t}(h), A_{J,t})$ in terms of output and productivity. Therefore $L_{J,t}(h) = f^{-1}(y_J, t(h))$. Following Woodford (2003, p. 150) and Benigno and López-Salido (2002) we can introduce the function $\tilde{\mathcal{V}}(\cdot)$ to express the marginal disutility of additional production effort as

$$\begin{aligned} \mathcal{V}(y_{J,t}(h), A_{J,t}) &\equiv \tilde{\mathcal{V}}(f^{-1}(y_{J,t}(h))) \\ \mathcal{V}_{y_J}(y_{J,t}(h), A_t^J) &= \tilde{\mathcal{V}}_{y_J}(f^{-1}(y_{J,t}(h))) \frac{1}{f'\left[f^{-1}(y_{J,t}(h))\right]} \end{aligned}$$

Combining labour supply given by (2.31) with labour demand (2.39) for $J = H$, the level of production of good $y_{H,t}(h)$ is determined from the optimality condition[60]

$$\frac{\mathcal{V}_{y_H}(y_{H,t}(h), A_{H,t})}{\mathcal{U}_C(C_t)} = (1 - \tau^H) \frac{P_{H,t}}{P_t} MC_{H,t}(h) \tag{2.41}$$

Aggregating (2.41) over all firms in sector H, one obtains the labour market equilibrium

$$\frac{\mathcal{V}_{y_H}(Y_{H,t}, A_{H,t})}{\mathcal{U}_C(C_t)} = (1 - \tau^H) \frac{P_{H,t}}{P_t} MC_{H,t} \tag{2.42}$$

[60] Plugging (2.41) in (2.34), one obtains an expression similarly to equation (14) in Benigno (2004), adjusted for the two sector case. One further implication is that labour income taxation (as followed here) leads to the same optimal price setting framework as when leaving labour income untaxed and levying a revenue tax on goods sales (a VAT). For the latter approach, see e.g. Ferrero (2005).

where MC_t^H denotes average real marginal cost in the Home tradable sector. The same derivation applies in the non-tradable sector such that

$$\frac{\mathcal{V}_{y_N}(Y_{N,t},A_{N,t})}{\mathcal{U}_C(C_t)}=(1-\tau^N)\frac{P_{N,t}}{P_t}MC_{N,t} \tag{2.43}$$

For Foreign, analogous relationships are obtained

$$\frac{\mathcal{V}_{y_F}(y_{F,t}(f),A_{F,t})}{\mathcal{U}_{C^*}(C_t^*)}=(1-\tau^F)\frac{P_{F,t}}{P_t^*}MC_{F,t},\qquad \frac{\mathcal{V}_{y_{N^*}}(Y_{N^*,t},A_{N^*,t})}{\mathcal{U}_{C^*}(C_t^*)}=(1-\tau^{N^*})\frac{P_{N^*,t}}{P_t^*}MC_{N^*,t} \tag{2.44}$$

where $MC_{F,t}$ and $MC_{N^*,t}$ denote average real marginal cost in sectors F and N^*, respectively.

2.2.3 Union Monetary Policy and National Fiscal Policies

Union Monetary Policy

The union harmonised consumer price index P_t^U is a weighted geometric average of domestic harmonised indices of consumer prices P_t and P_t^*, $P_t^U \equiv P_t^n P_t^{*(1-n)}$. n denotes the size of the Home region which coincides with its economic weight in union real GDP, $n=\frac{PY}{P^UY^U}$. PY and P^UY^U are measured with the respective price levels in steady state (i.e. at constant union currency).[61] Accordingly, the union inflation rate π_t^U can be written as the arithmetic average of home and foreign rates of inflation in the HICP indices

$$\pi_t^U = n\pi_t + (1-n)\pi_t^* \tag{2.45}$$

where $\pi_t=\ln\left(P_t/P_{t-1}\right)$, $\pi_t^*=\ln\left(P_t^*/P_{t-1}^*\right)$.[62] π_t^U provides the inflation objective, the central bank seeks to stabilise. We assume that union monetary policy commits to following the instrument rule presented in Smets and Wouters (2003) that extends a Taylor (1993) rule,

$$\frac{1+i_t}{1+r}=\frac{\left(\frac{1+i_{t-1}}{1+r}\right)^{r_i}\left\{\left(1+\bar{\pi}_t^U\right)\left(\frac{1+\pi_{t-1}^U}{1+\bar{\pi}_{t-1}^U}\frac{1}{1+\bar{\pi}_t^U}\right)^{r_\pi}\left(\frac{Y_t^U}{Y_t^{U,ss}}/\frac{Y_t^{U,flex}}{Y_t^{U,eff}}\right)^{r_Y}\right\}^{(1-r_i)}}{\left(\frac{1+\pi_t^U}{1+\bar{\pi}_t^U}/\frac{1+\pi_{t-1}^U}{1+\bar{\pi}_{t-1}^U}\right)^{r_{\triangle\pi}}\left(\left(\frac{Y_t^U}{Y_t^{U,ss}}/\frac{Y_t^{U,flex}}{Y_t^{U,eff}}\right)/\left(\frac{Y_{t-1}^U}{Y_{t-1}^{U,ss}}/\frac{Y_{t-1}^{U,flex}}{Y_{t-1}^{U,eff}}\right)\right)^{r_{\triangle Y}}\exp[\eta_t^i]} \tag{2.46}$$

In (2.46), the policy rate of the central bank (the minimum bid rate on main refinancing operations) directly influences the per period, i.e. per quarter nominal interest rate i_t (the three month money market rate), such that both rates coincide. i_t is set relative to the natural

[61] Empirically, n is determined from private households final consumption expenditures in each member state, as stressed in the introduction.

[62] The expression is valid for low inflation rates. Then $\ln(1+\pi_t)\simeq\pi_t$ provides a good approximation with no need to use the *exact* expression $\pi_t\equiv\frac{P_t}{P_{t-1}}-1$.

real rate of interest r (the real rate of return under fully flexible prices). $\bar{\pi}_t^U$ denotes the inflation objective which will be assumed to be equal to zero and time-invariant. Whereas an inflation target at zero seems to be too restrictive at first sight, it turns out that a zero-inflation objective will be welfare-maximising as shown below. Under this assumption, the policy regime guarantees that in steady state the policy rate tracks the natural rate of interest $i = r$. Thus policy respects optimum consumption-savings decisions of the private sector in the absence of nominal rigidities.[63]

$\frac{Y_t^U}{Y_t^{U,ss}} / \frac{Y_t^{U,flex}}{Y_t^{U,eff}}$ denotes the gap in union real GDP defined as the ratio of actual real GDP Y_t^U (GDP in the presence of nominal and real rigidities) compared to the period t steady state GDP $Y_t^{U,ss}$ (the natural rate of Y_t^U, i.e. including real rigidities) over the evolvement of GDP in the absence of any rigidities $Y_t^{U,flex}$ relative to non-stochastic potential output $Y_t^{U,eff}$. We will later on assume that fiscal policy can fully redistribute the profits from monopolistic competition to households such that efficiency is obtained in steady state, $Y_t^{U,ss} = Y_t^{U,eff}$ at all dates (the natural rate coincides with potential output at any date). Terms in the second row of (2.46) indicate 'speed limit' terms in the sense that monetary policy also reacts to accelerations in inflation and the output gap, see also Stracca (2007). Also, if potential output varies slowly through time, targeting real output growth $\frac{Y_{J,t}^U}{Y_{J,t}^{U,ss}} / \frac{Y_{J,t-1}^U}{Y_{J,t-1}^{U,ss}}$ comes close to targeting the true relative output gaps $\left(\frac{Y_t^U}{Y_t^{U,ss}} / \frac{Y_t^{U,flex}}{Y_t^{U,eff}}\right) / \left(\frac{Y_{t-1}^U}{Y_{t-1}^{U,ss}} / \frac{Y_{t-1}^{U,flex}}{Y_{t-1}^{U,eff}}\right)$, such that speed limit terms provide a cushion against decisions based on wrongly measured potential output, as argued in Walsh (2004). r_i represents the elasticity of changes in the current policy rate i_t in response to changes in the lagged rate i_{t-1} and introduces an interest rate smoothing objective. Hence it is taken into account that it is common practice by central banks to adjust interest rates gradually through time. Gradual adjustment in the policy rate is also a feature of the full commitment policy within the New Keynesian model, see Walsh (2003, chapter 11). $\exp[\eta_t^i]$ denotes a transitory monetary policy shock that summarises other sources of uncertainty and potential, exogenous policy mistakes.

[63] See (A.2) in the appendix for a discussion of the steady state.

National Fiscal Policies

Public expenditure in each member state only falls on the domestic produce such that

$$\begin{aligned} g_{H,t}(h) &= \left(\frac{p_{H,t}(h)}{P_{H,t}}\right)^{-\rho_H} G_{H,t}, \quad g^*_{F,t}(h) = \left(\frac{p^*_{F,t}(h)}{P^*_{F,t}}\right)^{-\rho^*_F} G^*_{F,t} \\ g_{N,t}(h) &= \left(\frac{p_{N,t}(h)}{P_{N,t}}\right)^{-\rho_N} G_{N,t}, \quad g^*_{N^*,t}(h) = \left(\frac{p^*_{N^*,t}(h)}{P^*_{N^*,t}}\right)^{-\rho^*_{N^*}} G^*_{N^*,t} \end{aligned}$$

As a novel feature compared to Benigno (2004) and related work, we describe government expenditures as following spending rules that are sector-specific and exhibit persistence[64]

$$1 + \frac{G_{J,t}}{Y_{J,t}} = \left(1 + \frac{G_{J,t-1}}{Y_{J,t-1}}\right)^{\rho_{g_J}} \left(\frac{Y^U_{J,t}}{Y^{U,ss}_{J,t}} / \frac{Y^{U,flex}_{J,t}}{Y^{U,eff}_{J,t}}\right)^{a_{g_J}} \exp[\varepsilon_{G_{J,t}}] \tag{2.47}$$

$G_{J,t}$ denotes a fiscal spending shock which is zero in steady state as then the innovation $\varepsilon^{ss}_{G_{J,t}} = 0$ at all dates. Fiscal rules of this type have been shown to come close to implicit rules implied by optimal policy as argued in Galí and Monacelli (2005) and Beetsma and Jensen (2005a). The rule allows for an explicit role of fiscal policy in targeting the sectoral output gap for $a_{g_J} \neq 0$ where $a_{g_J} > 0$ would imply pro-cyclical spending and $a_{g_J} < 0$ countercyclical spending, i.e. dampening activity if $Y^U_{J,t}$ is above the natural rate $Y^{U,ss}_{J,t}$. Targeting output will be of a role when discussing if national governments should engage in counteracting heterogeneity when assessing reform options later on. For the basic analysis however, $a_{g_J} = 0$, such that

$$1 + \frac{G_{J,t}}{Y_{J,t}} = \left(1 + \frac{G_{J,t-1}}{Y_{J,t-1}}\right)^{\rho_{g_J}} \exp[\varepsilon_{G_{J,t}}]$$

When the shock is not persistent, $\rho_{g_J} = 0$, one obtains the specification as in Benigno (2004). Then government spending occurs in the form of (unexpected) spending shocks. In all cases, fiscal spending is zero in steady state, such that GDP equals private sector spending in the long-run equilibrium, as all output is consumed by the private sector. Government spending does not provide utility for the household. The main purpose of including public spending is to allow for - intertemporally persistent - demand side fluctuations not explained by optimum decisions of the private sector. Government spending shocks might therefore add to the costs from business cycle dynamics to be investigated below.

[64] Note that again we used that $G^{J,t} = G_{J,t}$, which is for notational convenience only.

The spending rules obey the balance of the government given by

$$\begin{aligned} &\tau^H \tfrac{1}{P_t} \int_0^n p_{H,t}(j) y_{H,t}(j) dj + \tau^N \tfrac{1}{P_t} \int_0^n p_{N,t}(j) y_{N,t}(j) dj + \tfrac{M_t}{P_t} - \tfrac{M_{t-1}}{P_{t-1}} \tfrac{P_{t-1}}{P_t} \\ &= \tfrac{1}{P_t} \int_0^n p_{H,t}(j) g_{H,t}(j) dj + \tfrac{1}{P_t} \int_0^n p_{N,t}(j) g_{N,t}(j) dj + \tfrac{Q_t^{\#}}{P_t} \end{aligned} \tag{2.48}$$

Hence per period tax receipts from taxing income earned in the tradable production sector $\tau^H \int_0^n p_{H,t}(j) y_{H,t}(j) dj$ and in the non-tradable sector $\tau^N \int_0^n p_{N,t}(j) y_{N,t}(j) dj$ as well as seigniorage earnings (redistributed from the union monetary authority) are used to finance government expenditures for home tradables $\int_0^n p_{H,t}(j) g_{H,t}(j) dj$ and $\int_0^n p_{N,t}(j) g_{N,t}(j) dj$ as well as lump-sum social security spending $\frac{Q_t^{\#}}{P_t}$. Therefore, the government budget constraint has to be balanced every period and no government debt can build up. Notice that tax rates are sector-specific τ^J, which will be of importance for eliminating the sector-specific steady state distortion that arises from monopolistic competition between firms to be introduced below. Similar relationships hold in case of the foreign region.

2.2.4 Regional Current Accounts

The aggregate real external balance of the Home region is obtained from consolidating (2.10) with the real government balance (2.48). Using demand functions for individual goods, (2.53) and (2.54) in the household balance for each agent j, aggregating over all households and plugging in (2.48), one obtains the resource constraint for the economy[65]

$$CA_t = \underbrace{n \frac{i_t}{1+i_t} \mathcal{E}_t \frac{D_{t,t+1}}{P_t}}_{\text{real asset income}} + \underbrace{n Q_t^{1-\gamma} \left[C_{T,t}^U - C_{T,t} \right]}_{\text{net real exports}} \tag{2.49}$$

In deriving the latter expression, we used that the current account is defined as the change in net real foreign assets, $CA_t \equiv n \mathcal{E}_t \frac{D_{t,t+1}}{P_t} - n \frac{D_{t-1,t}}{P_t}$ and $n \mathcal{E}_t \frac{D_{t,t+1}}{P_t} \frac{1}{1+i_t} - n \frac{D_{t-1,t}}{P_t} = n Q_t^{1-\gamma} \left[C_{T,t}^U - C_{T,t} \right]$. We obtain that the current account (Home's external savings) is equal to the expected real return from the portfolio holdings and net real exports. We observe that the non-tradable price level has an impact on the current account via the internal real exchange rate Q_t as it evaluates net exports of home, given by $C_{T,t}^U - C_{T,t}$.[66] The terms of trade are of no explicit role

[65] For variables decided upon by a household, a union variable is obtained by summing the country aggregates, where country aggregates are obtained by weighing the average consumer by the country size n. Hence, for an arbitrary variable in levels that relates to household behaviour $X_t^U \equiv n X_t + (1-n) X_t^*$. $n X_t$ is the contribution of the home region, and $(1-n) X_t^*$ respectively of the foreign region. X_t is the variable chosen by the average consumer at home and X_t^* at foreign. Any relative variable is defined as $X_t^R \equiv X_t^* - X_t$. We proceed analogously for exogenous aggregate and relative shocks. Accordingly, union shocks are weighted averages of country shocks.

[66] Note further that in case of incomplete asset markets, when there is available a riskless bond B_t, that pays off the same amount irrespective of the shortfall in consumption that might occur following a shock and is traded across regions, we would replace $\mathcal{E}_{tt} \frac{D_{t,t+1}}{P_t} = B_t$. Further, the uncovered interest rate parity condition (UIP) becomes redundant. The UIP condition is however already redundant as the nominal exchange rate is

here which derives from our assumption of unitary elasticity of substitution between home and foreign goods and the LOP. The current account at foreign can be written as[67]

$$CA_t^* = (1-n)\,\frac{i_t}{1+i_t}\mathcal{E}_t\frac{D^*_{t,t+1}}{P^*_t} + (1-n)\,Q_t^{*1-\gamma^*}\left[C^U_{T,t} - C^*_{T,t}\right] \tag{2.50}$$

Under complete markets and using the assumption of unitary elasticity of home and foreign goods, the role of the current account is irrelevant for the dynamics in real consumption as there is a claim on tradable and non-tradable produce in the other region for any state. If a specific state occurs (a shortfall of production of a good z within the variety of goods in the H or N sectors which can no longer be consumed), output (of the F or N^*) sector in the other region will be converted to Euros and transferred to Home. Home records the payment as a positive income transfer, $n\frac{i_t}{1+i_t}\mathcal{E}_t\frac{D_{t,t+1}}{P_t} > 0$. In case the consumption drop was in a tradable good, the transfer is used to restore the level of utility before the drop. In case an N good is affected, the household can only increase H consumption, which will eventually yield the same level of utility as H and N goods are substitutable to some extent. In both cases C_T rises and net exports $C^U_{T,t} - C_{T,t}$ will decline accordingly. Hence under complete markets there are no current account dynamics that arise from risk sharing motives: Any decline in consumption will be offset by a corresponding income stream and net foreign assets do not change. It follows that net foreign asset dynamics are determined residually and do not influence the level of consumption in each region.

Further, current accounts need to cancel on the union level and asset markets need to clear

$$CA_t + CA_t^* = 0, \qquad n\mathcal{E}_t D_{t,t+1} + (1-n)\,\mathcal{E}_t D^*_{t,t+1} = 0 \tag{2.51}$$

(2.51) implies that there are no net savings on the union level which is equivalent to the statement that net asset holdings are zero.[68] The real net external position (real net foreign wealth) in each region needs to be sustainable from an intertemporal point of view. All claims by households from abroad need to be redeemable by households at home (and vice versa) given the contingency occurs. Therefore, by iterating the external balance of the home economy

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irrevocably fixed. Also, net foreign asset dynamics are determined residually.

[67] We used that due to the assumption of the LOP in the tradable sector, $\frac{P_F^*}{P^*_{H,t}} = \frac{P_F}{P_{H,t}}$. $C^U_{T,t} - C^*_{T,t}$ denotes net exports of Foreign, and $(1-n)\,\mathcal{E}_t\frac{D^*_{t,t+1}}{P^*_t}\frac{1}{1+i_t} - (1-n)\,\frac{D^*_{t-1,t}}{P^*_t} = (1-n)\,\frac{P_F^*}{P_t^*}T_t^{1-n}\left[C^U_{T,t} - C^*_{T,t}\right]$.

[68] We can combine current accounts such that

$$\begin{aligned} nQ_t^{1-\gamma}\left[C^U_{T,t} - C_{T,t}\right] &= -(1-n)\,Q_t^{*1-\gamma^*}\left[C^U_{T,t} - C^*_{T,t}\right] \\ C_{T,t} &= C^U_{T,t} + \frac{1-n}{n}\frac{Q_t^{*1-\gamma^*}}{Q_t^{1-\gamma}}\left[C^U_{T,t} - C^*_{T,t}\right] \end{aligned}$$

forward, one obtains

$$\begin{aligned} D^r_{t-1,t} &= \mathcal{E}_t D^r_{t,t+1} \frac{1+\pi_t}{1+i_t} - (1+\pi_t)\, Q_t^{1-\gamma} \left[C^U_{T,t} - C_{T,t} \right] \\ &= \left(\prod_{s=0}^{T} \frac{1+\pi_{t+s}}{1+i_{t+s}} \right) D^r_T - \mathcal{E}_t \sum_{s=0}^{T} \left\{ \left(\prod_{k=0}^{s-1} \frac{1+\pi_{t+k}}{1+i_{t+k}} \right) (1+\pi_{t+s})\, Q_{t+s}^{1-\gamma} \left[C^U_{T,t+s} - C_{T,t+s} \right] \right\} \end{aligned}$$

The no Ponzi-game condition requires that the present value of the portfolio $\frac{1+\pi_{t+s}}{1+i_{t+s}} D^r_T$ far in the future is zero, such that as a transversality condition we need that $\lim_{T\to\infty} \left(\prod_{s=0}^{T} \frac{1+\pi_{t+s}}{1+i_{t+s}} \right) D^r_T = 0$. Hence all claims need to be redeemed, once the state realises and no consumption can be left unpaid.[69] Hence the initial asset position is sustainable, if

$$D^r_{t-1} = -\mathcal{E}_t \sum_{s=0}^{T} \left\{ \left(\prod_{k=0}^{s-1} \frac{1+\pi_{t+k}}{1+i_{t+k}} \right) (1+\pi_{t+s})\, Q_{t+s}^{1-\gamma} \left[C^U_{T,t+s} - C_{T,t+s} \right] \right\} \tag{2.52}$$

In the intertemporal setting, consumption is a function of total wealth which equals the expected present discounted value of all future current account balances. If the initial net foreign wealth is negative $D^r_{t-1} < 0$, the present discounted value of home per capita consumption of tradables needs to be lower than the present discounted value of tradable consumption per capita in the union for some periods in the future. In other words, home needs to have net positive exports in some periods in the future that make the initial asset position sustainable and vice versa in case of positive initial foreign assets.

2.2.5 Aggregate Demand and Market-Clearing Conditions

As there is no investment demand in the model, aggregate demand for a generic tradable good h produced at Home is given by union private sector demand (domestic private absorption and net trade) and domestic public sector demand

$$y^d_{H,t}(h) = \left(\frac{p_{H,s}(h)}{P_{H,t}} \right)^{-\rho_H} \left(T_t^{1-n} C^U_{T,t} + G_{H,t} \right) \tag{2.53}$$

where union aggregate tradable consumption is obtained by summing tradable consumption over all agents in the union $C^U_{T,t} = \int_0^1 C^j_{T,t} dj$. Non-tradables demand is restricted to demand from home agents, hence

$$y^d_{N,t}(h) = \left(\frac{p_{N,t}(h)}{P_{N,t}} \right)^{-\rho_N} \left(C_{N,t} + G_{N,t} \right) \tag{2.54}$$

[69] We allowed for contracts with claims on sectoral consumption in states for some period far in the future.

and the respective demand functions at foreign are

$$y_{F,t}^{d*}(f) = \left(\frac{p_{F,t}^{*}(f)}{P_{F,t}^{*}}\right)^{-\rho_F^*} \left(T_t^{-n} C_{T,t}^U + G_{F,t}^*\right), \qquad y_{N^*,t}^{d*}(f) = \left(\frac{p_{N^*,t}^{*}(f)}{P_{N^*,t}^{*}}\right)^{-\rho_{N^*}^*} \left(C_{N^*,t}^* + G_{N^*,t}^*\right)$$

For the derivation of $y_{H,t}^{d}(h)$ and $y_{F,t}^{d*}(f)$ we have used that the LOP holds in the tradable sector, i.e. a tradables basket has the same union currency price whether bought at home or foreign as argued above.

The aggregate demand functions (describing the expenditure side of GDP) are obtained by summing over all differentiated goods in the respective basket. In equilibrium, the production and expenditure side of GDP are equal. Therefore union real aggregate demands for the produce in each sector from the home and foreign country given by Y_t^H and Y_t^F for home tradables H and foreign tradables F production, as well as service production demand given by Y_t^N and $Y_t^{N^*}$ are equal to sectoral real gross value added given by $Y_{H,t}$,$Y_{N,t}$, $Y_{F,t}$, and $Y_{N^*,t}$, such that $Y_t^J = Y_{J,t}$

$$Y_t^H \equiv \left[\frac{1}{n}\int_0^n \left(y_{H,t}(h)\right)^{\frac{\rho_H - 1}{\rho_H}} dh\right]^{\frac{\rho_H}{\rho_H - 1}} = T_t^{1-n} C_{T,t}^U + G_{H,t} = Y_{H,t} \tag{2.55}$$

$$Y_t^N \equiv \left[\frac{1}{n}\int_0^n \left(y_{N,t}(h)\right)^{\frac{\rho_N - 1}{\rho_N}} dh\right]^{\frac{\rho_N}{\rho_N - 1}} = C_{N,t} + G_{N,t} = Y_{N,t} \tag{2.56}$$

$$Y_t^F \equiv \left[\frac{1}{1-n}\int_n^1 \left(y_{F,t}^*(f)\right)^{\frac{\rho_F - 1}{\rho_F}} df\right]^{\frac{\rho_F}{\rho_F - 1}} = T_t^{-n} C_{T,t}^U + G_{F,t}^* = Y_{F,t} \tag{2.57}$$

$$Y_t^{N^*} \equiv \left[\frac{1}{1-n}\int_n^1 \left(y_{N^*,t}^*(f)\right)^{\frac{\rho_{N^*}^* - 1}{\rho_{N^*}^*}} df\right]^{\frac{\rho_{N^*}^*}{\rho_{N^*}^* - 1}} = C_{N^*,t}^* + G_{N^*,t}^* = Y_{N^*,t} \tag{2.58}$$

We have assumed that the price elasticity for foreign produced tradables is same in both countries whether consumed in the country of origin or exported, i.e. $\rho_F = \rho_F^*$ and analogously for home produced tradables $\rho_H = \rho_H^*$. We deem this a reasonable assumption given that the law of one price holds. Sectoral real output Y_J is already aggregated, as it is summed over all heterogenous goods produced in each region. Gross value added (in volume, i.e. measured in constant prices) PY_t can then be aggregated without further weighting by country size

$$PY_t \equiv P_H Y_{H,t} + P_N Y_{N,t}, \qquad P^* Y_t^* \equiv P_F^* Y_{F,t} + P_{N^*}^* Y_{N^*,t} \tag{2.59}$$

where P and P^* denote the regional GDP deflators (the steady state price levels of the base period). P_H and P_N are deflators of the tradable (industry) and non-tradables (services) sectors, analogously for foreign production with P_F^* and $P_{N^*}^*$ as derived under (2.7) to (2.9).[70] Note that by construction of the price indices and the assumptions about market structure, the weights of firms are time-invariant. Hence the price indices can also be interpreted as sectoral GDP deflators (Paasche Indices).

Market clearing requires that private and public sector consumption expenditures and expenditures on savings equal union gross value added, evaluated at current value union currency

$$P_t^U C_t^U + P_t^U \mathcal{E}_t D_{t,t+1}^U + nP_t G_t + (1-n)\, P_t^* G_t^* = P_t Y_t + P_t^* Y_t^* = P_t^U Y_t^U$$

A dynamic stochastic general equilibrium is defined as a vector of relative prices $\{E_t, T_t, Q_t, Q_t^*\}$ and sequence of allocations for which at each date t

- Goods markets clear at the union level, labour markets clear at the domestic level
- Monetary policy commits to following (2.46)
- Governments set fiscal policies according to (2.47)
- Interregional financial markets and the money market clear at all dates, as there cannot be net state-contingent claims on the union level nor holding of money outside member states. Also, the regional current accounts sum to nil,

$$n\mathcal{E}_t \frac{D_{t,t+1}}{P_t} + (1-n)\,\mathcal{E}_t \frac{D_{t,t+1}^*}{P_t^*} \equiv \mathcal{E}_t D_{t,t+1}^U = 0 \tag{2.60}$$

$$nM_t + (1-n)M_t^* = M_t^U \tag{2.61}$$

$$CA_t + CA_t^* = 0 \tag{2.62}$$

It seems worth to emphasize that our model nests well-established models as special cases[71]:

- For the country size n of the home economy towards 1, the two region model shrinks to one economy which can be regarded as a standard small-open economy with two production sectors with sticky prices and hybrid inflation dynamics (the case considered in chapters 3 and 4, augmented for a role of physical capital)
- Given that households only consume tradables, i.e. $1-\gamma = 1-\gamma^* = 0$, in absence of service production in each region $\frac{Y_H}{Y} = \frac{Y_F}{Y} = 1$, and no backward-looking price-setting

[70] Nominal GDP on the union level then is

$$GDP_t^U \equiv GDP_t + GDP_t^* \equiv P_{H,t}Y_{H,t} + P_{N,t}Y_{N,t} + P_{F,t}^*Y_{F,t} + P_{N^*,t}^*Y_{N^*,t}$$

[71] We therefore follow the 'correspondence principle', namely that conclusions derived from a model extension are especially useful when the model nests accepted models as special cases.

$\varpi_J = 0$ the Benigno (2004) framework is obtained. In the one-sector case and with backward-lookingness in inflation dynamics only in the home economy, the model of Benigno and López-Salido (2006) is obtained.

- In the absence of nominal rigidities, the model shrinks to a two-country flex-price real business cycle model where all fluctuations in economic activity are driven by developments in potential output and relative prices only.

2.3 Log-Linear Approximation of the Model

As there exists no closed form solution of the system of non-linear difference equations for variables in levels described in the preceding sections, the solution is approximated by the solution to its local log-linear approximation about the deterministic steady state. The derivation of the steady state can be found in appendix A.3. Equations in the model will be classified as to whether they describe business cycle dynamics under sticky or flexible prices. The discrimination between these two types of log-linearisations is non-trivial. We can disentangle fluctuations in the business cycle that are driven by relative prices only and reflect re-allocations of the factors of production under full competition and flexible prices ('efficient fluctuations') from those that are obtained under inertial response of prices caused by nominal and real rigidities ('inefficient fluctuations'). The gap between these two measures will be mainly influenced by the degree of structural heterogeneity between members. Sticky-price movements can be considered to represent the observable evolvement of macroeconomic series, whereas efficient flex-price fluctuations cannot be observed. In case, gaps are closed at all dates, there would be no welfare losses present that should be addressed by policy.

2.3.1 Log-Linear Flexible-Price Fluctuations

For an arbitrary variable X_t we can define

$$\tilde{X}_t \equiv \ln\left(X_t/X_t^{eff}\right) = \ln\left(X_t/X^{eff}\right) \tag{2.63}$$

where $\tilde{X}_t \times 100\%$ denotes the deviation of X_t about its period t first best steady state value X_t^{eff} in percent. As flexible price fluctuations are by assumption determined about the efficient steady state, all flexible price movements will be Pareto efficient.[72] This assumption about X_t^{eff} requires that besides nominal also real rigidities are absent such that prices are fully flexible

[72] This result requires that there are only efficient shocks in the model, i.e. shocks that do not yield additional distortions like mark-up shocks.

and the economy is fully competitive. Firms then have no pricing power over their product due to the homogeneity in goods supply. Later on, we will assume that the inefficient steady state X can replicate the outcome of the efficient steady state in each period (such that long-run allocations will coincide). We will require that there is a fiscal redistribution mechanism in place (a labour income subsidy) to cure the market imperfection from real rigidites. Note that as the focus of this chapter is on fluctuations in the short to medium run, the steady state is not prone to shocks (there are no permanent shocks) and there is no deterministic steady state growth present. Consequently, we can omit time subscripts from the steady state value $X_t^{eff} \equiv X^{eff}$. The reasoning holds, whether the variable under consideration is of a real or nominal nature.

Under the first best steady state, markups vanish

$$\frac{\rho_J}{\rho_J - 1} = 1 \tag{2.64}$$

as $\rho_J \rightarrow \infty$ in all sectors $J = H, N, F, N^*$. Fluctuations will be efficient in the sense that they are affected by real disturbances (structural shocks) only which cannot be influenced by policy.[73] Fluctuations in internal real exchange rates reflect optimal consumption decisions between tradables and non-tradables consumption

$$\tilde{Q}_t = \tilde{C}_{N,t} - \tilde{C}_{T,t}, \qquad \tilde{Q}_t^* = \tilde{C}_{N^*,t}^* - \tilde{C}_{T,t}^* \tag{2.65}$$

An increase in $\tilde{Q}_t$ and $\tilde{Q}_t^*$ indicates that non-tradables have become relatively cheaper such that N and N^* consumption increases. To express the potential rate fluctuations in the nominal interest rate $\tilde{\imath}_t$, we invert the union Euler equation under flexible prices, obtained by weighting the country-specific Euler equations by the respective country size

$$\tilde{\imath}_t = \left(n\frac{1}{\rho} + (1-n)\frac{1}{\rho^*}\right)^{-1} \left(\mathcal{E}_t \tilde{C}_{t+1}^U - \tilde{C}_t^U\right)$$

where fluctuations in overall private sector consumption in the union are given by $\tilde{C}_t^U$.[74] Under flexible prices there are no movements in the aggregate price index. As all firms reset prices optimally taking as given the aggregate index, the price index will remain unchanged. As a result, nominal and real interest rates coincide, such that $\tilde{\imath}_t = \tilde{r}_t$.

[73] This result is a consequence of additive separability in the utility function.

[74] Using $n\tilde{C}_t + (1-n)\tilde{C}_t^* = n\mathcal{E}_t\tilde{C}_{t+1} - n\frac{1}{\rho}\tilde{\imath}_t + (1-n)\mathcal{E}_t\tilde{C}_{t+1}^* - (1-n)\frac{1}{\rho^*}\tilde{\imath}_t$ one obtains

$$\tilde{C}_t^U = \mathcal{E}_t\tilde{C}_{t+1}^U - \left(n\frac{1}{\rho} + (1-n)\frac{1}{\rho^*}\right)\tilde{\imath}_t \tag{2.66}$$

When prices are fully flexible, each firm z in sector J sets its price $p_{J,t}(z)$ as a constant markup over current nominal marginal cost $P_{J,t}MC_{J,t}(z)$

$$p_{J,t}(z) = \frac{\rho_J}{\rho_J - 1} P_{J,t} MC_{J,t}(z)$$

As equilibrium real marginal cost $MC_{J,t}(z)$ is constant across firms, each firm's relative price $\frac{p_{J,t}(z)}{P_{J,t}}$ will be constant as well. This implies that all firms in each sector set $p_{J,t}(z) = P_{J,t}$. Real marginal cost turns out to be the inverse of the markup and cannot be influenced by optimal price resetting of a single firm as all firms behave accordingly due to their symmetry. Hence

$$MC_{J,t} = \left(\frac{\rho_J}{\rho_J - 1} \right)^{-1}$$

As real marginal costs are constant under flexible prices, fluctuations in real marginal cost are zero at all dates, such that $\widetilde{MC}_{J,t} = 0$ for all t. Therefore also $\tilde{P}_{J,t} = 0$ at all dates such that there are no fluctuations in the sectoral price index and in consequence also $\tilde{P}_t = 0$.

From the sectoral labour market equilibria (2.42) to (2.44) and using that $\tilde{C}_t = \tilde{C}_{T,t} + (1-\gamma)\tilde{Q}_t$, $\tilde{C}^*_t = \tilde{C}^*_{T,t} + (1-\gamma^*)\tilde{Q}^*_t$ and that $\tilde{Y}_{H,t} = (1-n)\tilde{T}_t + \tilde{C}^U_{T,t} + g_{H,t}$ as well as $\tilde{Y}_{F,t} = -n\tilde{T}_t + \tilde{C}^U_{T,t} + g_{F,t}$ one can derive efficient fluctuations in tradable consumption in both regions

$$\tilde{C}_{T,t} = \frac{1-\rho}{\rho}(1-\gamma)\tilde{Q}_t - \frac{1+\eta_H}{\rho}(1-n)\tilde{T}_t - \frac{\eta_H}{\rho}\tilde{C}^U_{T,t} + \frac{\eta_H}{\rho}(S_{H,t} - g_{H,t}) \qquad (2.67)$$

$$\tilde{C}^*_{T,t} = \frac{1-\rho^*}{\rho^*}(1-\gamma^*)\tilde{Q}^*_t - \frac{1+\eta_F}{\rho^*}(-n)\tilde{T}_t - \frac{\eta_F}{\rho^*}\tilde{C}^U_{T,t} + \frac{\eta_F}{\rho^*}(S_{F,t} - g_{F,t}) \qquad (2.68)$$

where we also used that $\widetilde{MC}_{J,t} = 0$. $S_{J,t}$ denotes a supply shock which is related to the preference/productivity shock $A_{J,t}$ by

$$S_{J,t} \equiv -\frac{V_{y_J A_J}}{V_{y_J y_J}} \frac{1}{Y_J} A_{J,t} \qquad (2.69)$$

as derived in the appendix under (A.57). Consumption of tradables by the private sector increases with positive supply shocks and decreases in government fiscal spending. Consequently, one observes a direct crowing out effect of private for public consumption, as $g_{H,t} > 0$, $g_{F,t} > 0$ will lead to $\tilde{C}_{T,t} < 0$, $\tilde{C}^*_{T,t} < 0$. Consumption of tradable goods will also depend on developments of internal relative prices (which is one main argument for a two-sector setup). As the parameter of risk aversion $\rho > 1$, an appreciation in the internal real exchange rate $\tilde{Q}_t < 0$, indicating that T goods have become cheaper compared to N goods increases T consumption. At the same time, a worsening in the terms of trade $\tilde{T}_t > 0$ (a fall in the exported goods price of the home region) might counteract the increase in consumption (given the decrease in $\tilde{Q}_t$ was triggered by a price decrease in home tradables relative to foreign tradables and not by a price rise in non-tradables). The net effect depends on the responsiveness of consumption and work effort. For the share of non-tradables in the consumption basket approaching zero, $\gamma \to 1$, the effect of the internal real exchange rate vanishes. For non-tradable consumption one obtains accordingly

$$\tilde{C}_{N,t} = \gamma \frac{\rho - 1}{\rho + \eta_N} \tilde{Q}_t + \frac{\eta_N}{\rho + \eta_N} \left(S_{N,t} - g_{N,t} \right), \qquad \tilde{C}^*_{N^*,t} = \gamma^* \frac{\rho^* - 1}{\rho^* + \eta_{N^*}} \tilde{Q}^*_t + \frac{\eta_{N^*}}{\rho^* + \eta_{N^*}} \left(S_{N^*,t} - g_{N^*,t} \right) \tag{2.70}$$

A depreciation of the internal real exchange rate, $\tilde{Q}_t > 0$, implying that N goods have become relatively cheaper compared to T goods, always leads to an increase in non-tradable consumption. Again, positive supply shocks (a positive preference shock to labour supply) increases consumption, whereas fiscal spending crowds out private sector consumption. Analogous arguments hold for the foreign region.

The terms of trade in log-linear form can be written as

$$\tilde{T}_t = \frac{\eta}{1 + \eta} \left(G^R_t - S^R_t \right) \tag{2.71}$$

where complete nominal risk sharing and the law of one price in tradables allows us to write the terms of trade independent of movements in the internal real exchange rates. Also, we imposed that the inverse of the labour supply elasticity in tradables sectors $\eta_J \equiv \frac{\mathcal{V}_{y_J y_J}(Y_J, 0) Y_J}{\mathcal{V}_{y_J}(Y_J, 0)}$, $J = H, N$ is same across regions such that $\eta_H = \eta_F = \eta$. This assumption is also made by Benigno (2004) and Beetsma and Jensen (2005b) where in both contributions this measure applies to overall work effort in a region as there is no sectoral decomposition of gross value added included. Under these assumptions, $\tilde{T}_t$ only depends on relative supply shocks in tradables production $S^R_t = S_{F,t} - S_{H,t}$ and demand shocks $G^R_t = G_{F,t} - G_{H,t}$ to tradables consumption as in the cited works. If fiscal spending increases at Home relatively to Foreign, the terms of trade will improve, $\tilde{T}_t < 0$, whereas a productivity shock in home industry will deteriorate the terms of trade (home production becomes relatively cheaper such that prices of Home produced tradables decrease).

2.3.2 Log-Linear Sticky-Price Fluctuations

For an arbitrary variable X_t we denote the percentage deviation about its inefficient steady state X_t^{ss} (the steady state where real rigidities prevail, i.e. the natural rate) as

$$\hat{X}_t \equiv \ln\left(X_t/X_t^{ss}\right) = \ln\left(X_t/X\right) \tag{2.72}$$

In case X_t denotes the gross inflation rate, $1 + \pi_{J,t}$ or gross real and nominal interest rates, $1 + r_t$, $1 + i_t$, $\hat{X}_t$ is the deviation in the (net) rate $\pi_{J,t}$, r_t, and i_t from its respective steady state value in percentage points.[75] By the definition of a steady state, prices are flexible in X as well, however there is still pricing power of firms and real marginal costs are different from output prices. As long-run developments are of no role in this work, there is no movement in the steady state and we write $X_t^{ss} \equiv X^{ss} = X$. In the inefficient steady state, output is inefficiently low. In order to prevent a stabilisation bias in monetary policy, a fiscal transfer scheme needs to be in place, which will yield that

$$X = X^{eff} \tag{2.73}$$

at all dates, given the scheme is effective. Thus in long-run equilibrium, potential and natural rate values will coincide. Details on the construction of the redistribution scheme can be found in appendix A.3 on page 249.

National Accounting and Euler equations

The aggregate demand functions can be written as[76]

$$\begin{aligned} \hat{Y}_t^H &= (1-n)\hat{T}_t + \hat{C}_{T,t}^U + g_{H,t}, & \hat{Y}_t^N &= \hat{C}_{N,t} + g_{N,t} \\ \hat{Y}_t^{F*} &= -n\hat{T}_t + \hat{C}_{T,t}^U + g_{F,t}, & \hat{Y}_t^{N*} &= \hat{C}_{N^*,t}^* + g_{N^*,t} \end{aligned}$$

Consumption Euler equations under sticky prices are obtained by log-linearising the Euler equations

$$\hat{C}_t = \mathcal{E}_t\hat{C}_{t+1} - \frac{1}{\rho}(\hat{\imath}_t - \mathcal{E}_t\pi_{t+1}), \qquad \hat{C}_t^* = \mathcal{E}_t\hat{C}_{t+1}^* - \frac{1}{\rho^*}(\hat{\imath}_t - \mathcal{E}_t\pi_{t+1}^*)$$

where $\mathcal{E}_t\pi_{t+1}$ denotes expected consumer price inflation at home and $\mathcal{E}_t\pi_{t+1}^*$ is expected consumer price inflation at foreign where expectation is conditional on information available at

[75] For example, in case of the inflation rate, we obtain that $\hat{\pi}_{J,t} \equiv \ln \frac{1+\pi_{J,t}}{1+\pi_{J,t}^{ss}} = \pi_{J,t} - \pi_{J,t}^{ss}$.

[76] The government spending shock $G_{J,t}$ is already linear in the model in levels and does not need to be log-linearised. After log-linearising $Y_{J,t}$ and dividing by Y_J one therefore obtains that $g_{J,t} = \frac{G_{J,t}}{Y_J}$ for $J = H, N, F, N^*$.

the beginning of period t (before stochastic shocks realise). We used that as the steady state price levels exhibit no deterministic growth over time (there is no steady state money supply growth), the log-deviation of the inflation rate about its trend coincides with the inflation rate $\hat{\pi}_t \equiv \ln(P_t/P_{t-1}) - \ln\left(P_t^{ss} - P_{t-1}^{ss}\right) = \pi_t$, as $P_t^{ss} = P_{t-1}^{ss} = P^{ss}$.

Union overall consumption expenditures can be written as

$$\hat{C}_t^U = \frac{nPC}{P^U C^U}\hat{C}_t + \frac{(1-n)\,P^* C^*}{P^U C^U}\hat{C}_t^* = n\hat{C}_t + (1-n)\,\hat{C}_t^* \tag{2.74}$$

where from complete risk sharing $C = C^*$, such that $P = P^* = P^U$. Union tradable consumption fluctuations are given by

$$\hat{C}_{T,t}^U = \frac{\gamma n}{n\gamma + (1-n)\,\gamma^*}\hat{C}_{T,t} + \frac{(1-n)\,\gamma^*}{n\gamma + (1-n)\,\gamma^*}\hat{C}_{T,t}^*$$

using that the LOP holds for each good in the tradables basket and hence for the overall index. When the share of tradables in overall consumption is same across regions $\gamma^* = \gamma$, we would obtain that consumption fluctuations for tradables are a weighted average of the regional ones, $\hat{C}_{T,t}^U = n\hat{C}_{T,t} + (1-n)\hat{C}_{T,t}^*$.

Real Marginal Cost and Phillips Curves

Turning to the supply side of GDP, by log-linearising marginal cost in each of the sectors about steady state and substituting out factors that appear both under sticky and flexible prices (namely supply shocks $S_{J,t}$ and demand shocks $G_{J,t}$) we obtain that real marginal cost $\widehat{MC}_{H,t}(h)$ for firm h in sector H is given by

$$\widehat{MC}_{H,t}(h) = (1-n)\,(1+\eta_H)\,(\hat{T}_t - \tilde{T}) - (1-\gamma)\,(\hat{Q}_t - \tilde{Q}_t) + \eta_H(\hat{C}_{T,t}^U - \tilde{C}_{T,t}^U) + \rho(\hat{C}_t - \tilde{C}_t) \tag{2.75}$$

Real marginal cost for firm h in the Home tradable sector H depends both on the gap of inefficient and efficient fluctuations in the terms of trade $\hat{T}_t - \tilde{T}$ and the gap in the internal real exchange rate $\hat{Q}_t - \tilde{Q}_t$. Analogously for a firm f producing in the Foreign tradable sector F

$$\widehat{MC}_{F,t}(f) = -n\,(1+\eta_F)\,(\hat{T}_t - \tilde{T}) - (1-\gamma^*)\,(\hat{Q}_t^* - \tilde{Q}_t^*) + \eta_F\left(\hat{C}_{T,t}^U - \tilde{C}_{T,t}^U\right) + \rho^*(\hat{C}_t^* - \tilde{C}_t^*) \tag{2.76}$$

A positive gap in the terms of trade increases real marginal cost at Home and lowers costs abroad. The effect is counteracted by increasing weight of non-tradables in the overall consumer basket in each region (i.e. with decreasing γ and γ^* respectively). For non-tradables in both countries we obtain

$$\widehat{MC}_{N,t}(h) = \eta_N(\hat{C}_{N,t} - \tilde{C}_{N,t}) + \rho\left(\hat{C}_t - \tilde{C}_t\right) + \gamma(\hat{Q}_t - \tilde{Q}_t) \quad (2.77)$$

$$\widehat{MC}_{N^*,t}(f) = \eta_{N^*}(\hat{C}^*_{N^*,t} - \tilde{C}^*_{N^*,t}) + \rho^*\left(\hat{C}^*_t - \tilde{C}^*_t\right) + \gamma^*(\hat{Q}^*_t - \tilde{Q}^*_t) \quad (2.78)$$

With $\widehat{MC}_{J,t}(j)$ and the assumptions about forward and backward-looking price setting given by (2.33) and (2.35), one can derive sectoral inflation dynamics. The hybrid Phillips curve describes the relationship between the change in the sectoral price index $\pi_{J,t}$ and average real marginal cost $\widehat{MC}_{J,t}$ in each sector

$$\pi_{J,t} = \lambda^b_J \pi_{J,t-1} + \frac{\lambda^{mc}_J}{\kappa^{mc}}\widehat{MC}_{J,t} + \lambda^f_J \mathcal{E}_t \pi_{J,t+1} \quad (2.79)$$

where the deep parameters are collected in

$$\lambda^b_{J,t} = \frac{\omega_J}{\theta_J + \omega_J(1 - \theta_J(1-\beta))}, \qquad \lambda^{mc}_{J,t} = \frac{(1-\omega_J)(1-\theta_J)(1-\theta_J\beta)}{\omega_H(1-\theta_J+\theta_J\beta)+\theta_J}$$

$$\lambda^f_{J,t} = \frac{\beta\theta_J}{\omega_J}\lambda^b_{J,t}, \qquad \kappa^{mc} = 1 + \eta_J\rho_J$$

for $J = H, N, F, N^*$.[77] $\lambda^b_{J,t}$ captures intrinsic inflation persistence and increases in the share of backward-looking firms ω_J, as $\lambda^b_{J,t}(\omega_J) > 0$ and $\lambda^b_{J,t}(\omega_J)' > 0$. Consequently, the higher is the share of non-optimising firms, the more inertial actual inflation $\pi_{J,t}$ will be and the less expected inflation determines current price movements.

Price rigidity θ_J, endogenous persistence ω_J , the elasticity of substitution between goods within a sector ρ_J, and the inverse of the elasticity of labour supply η_J affect the slope of the Phillips curve $\frac{\lambda^{mc}_J}{\kappa^{mc}}$. $\frac{\lambda^{mc}_J}{\kappa^{mc}}$ can thus be interpreted as the elasticity of current inflation with respect to average real marginal cost $\widehat{MC}_{J,t}$. The sectoral price elasticity of demand in sector J enters the Phillips curve via the term κ^{mc}. For any given value of the inverse of the labour supply elasticity η_J, increasing competition makes current inflation less responsive to current real marginal cost and the Phillips curve becomes flatter. For the share of backward-looking firms ω_J approaching zero, the forward-looking New Keynesian Phillips curve is obtained.

[77] Note that (2.79) can be linked with $\pi_{J,t} = \lambda^b_J\pi_{J,t-1} + \lambda^{mc}_J mc^J_t + \lambda^f_J\mathcal{E}_t\pi_{J,t+1}$ where $mc^J_t \equiv \frac{\widehat{MC}_{J,t}}{1+\eta_J\rho_J}$. This case describes hybrid inflation dynamics under constant returns to scale, the case that will be considered in detail in chapters 3 and 4 of this dissertation. Under that assumption, real marginal cost and average real marginal cost coincide at all dates such that ρ_J does not matter in determining the dynamics. In this chapter, labour is the only variable factor of production, such that one obtains (2.79).

GDP and the Current Account

For aggregate output (real union GDP at constant prices), we obtain that fluctuations are given by

$$\hat{Y}_t^U + \hat{P}^U = n\hat{Y}_t + (1-n)\,\hat{Y}_t^* \tag{2.80}$$

where we used that economic weight and country size coincide, such that $\frac{PY}{P^U Y^U} = n$. We also used that $\hat{X}^U \equiv \ln X^U - \ln X^U = 0$. Within each region, fluctuations in real GDP are given by

$$\begin{aligned} \hat{Y}_t + \hat{P} &= \hat{Y}_t = \frac{P_H Y_H}{PY}\hat{Y}_{H,t} + \left(1 - \frac{P_H Y_H}{PY}\right)\hat{Y}_{N,t} \\ \hat{Y}_t^* + \hat{P}^* &= \hat{Y}_t^* = \frac{P_F Y_F}{PY}\hat{Y}_{F,t} + \left(1 - \frac{P_F Y_F}{PY}\right)\hat{Y}_{N^*,t} \end{aligned}$$

where $\frac{P_H Y_H}{PY}$ is the share of industry gross value added to overall gross value added in the H region (as presented in table 2.1 for each country), and $\frac{P_F Y_F}{PY}$ accordingly for the F region. Further we used that as gross value added is defined in steady state constant price levels, $\hat{P} = \hat{P}^* = 0$. The aggregate equilibrium can be written as[78]

$$\hat{Y}_t^U = n\hat{C}_t + (1-n)\,\hat{C}_t^* + \frac{n}{Y^U}g_t + \frac{1-n}{Y^U}g_t^*$$

where we used that $C = C^*$ and that government shocks are zero in steady state, such that $Y^U = C^U$.[79] Eventually, overall public demand in each region is

$$g_t = \frac{P_H Y_H}{PY}g_{H,t} + \left(1 - \frac{P_H Y_H}{PY}\right)g_{N,t}, \qquad g_t^* = \frac{P_F Y_F}{PY}g_{F,t} + \left(1 - \frac{P_F Y_F}{PY}\right)g_{N^*,t} \tag{2.81}$$

where $g_{J,t} = \frac{G_{J,t}}{Y_{J,t}}$.

[78] Supply in levels equals private sector and public consumption expenditures and aggregate savings at the union level. In real terms

$$\begin{aligned} PY_t + P^*Y_t^* &= nPC_t + (1-n)P^*C_t^* + nPG_t + (1-n)\,P^*G_t^* + nPD_t + (1-n)P^*D_t^* \\ &= P^U C_t^U + P^U G_t^U \end{aligned}$$

We used that $E = 1$ implies that $P = P^*$ and that portfolios are in zero net supply, $nPD_t + (1-n)P^*D_t^* = 0$. As in steady state $P^U Y^U = nPC + (1-n)P^*C^*$, foreign real private consumption expenditures as share of union real GDP can be written as $\frac{P^*C^*}{P^U Y^U} = \left(1 - n\frac{PC}{P^U Y^U}\right)\frac{1}{1-n}$.

[79] Note that government expenditures are shocks and are therefore already linear and hence only need to be re-scaled.

Current Account Dynamics

We can use these results to determine real external asset dynamics for the home region

$$\mathcal{E}_t\hat{D}^r_{t,t+1} = \hat{\imath}_t + \frac{1}{\beta}\hat{D}^r_{t-1,t} - \frac{1}{\beta}\pi_t + \frac{1+\beta}{\beta}\left(1-\frac{C_T}{C_T^U}\right)^{-1}\hat{C}^U_{T,t} - \frac{1+\beta}{\beta}\left(\left(\frac{C_T}{C_T^U}\right)^{-1}-1\right)^{-1}\hat{C}_{T,t} \tag{2.82}$$

As the asset market needs to clear on the union level, we can determine foreign's real external assets from the union asset market equilibrium. We arrive at[80]

$$\mathcal{E}_t\hat{D}^{r*}_{t,t+1} + \hat{E}_t = \mathcal{E}_t\hat{D}^r_{t,t+1} \tag{2.84}$$

Log-deviations in the real exchange rate under sticky prices are obtained from log-linearising (2.19)

$$\hat{E}_t = (1-\gamma)\,\hat{Q}_t - (1-\gamma^*)\,\hat{Q}^*_t \tag{2.85}$$

and the risk sharing condition (2.26) is then

$$-\rho\hat{C}_t + \hat{E}_t = -\rho^*\hat{C}^*_t \tag{2.86}$$

where the degree of overall risk aversion within each region ρ and ρ^* and the real exchange rate $\hat{E}_t$ prevent fluctuations in overall real consumption $\hat{C}_t$, $\hat{C}^*_t$ to equalise across regions. Therefore, the correlation between consumption cycles will not be perfect, albeit nominal income can be perfectly insured. This result indicates that movements in the real exchange rate might explain the large discrepancy between consumption correlations across members, as illustrated in table 2.1.

[80] Deflating by P_t

$$0 = n\frac{\mathcal{E}_t D_{t,t+1}}{P_t} + (1-n)\,\frac{\mathcal{E}_t D^*_{t,t+1}}{P^*_t}\frac{P^*_t}{P_t}$$

one obtains that

$$0 = n\frac{D}{P}\left(1+\hat{D}^r_{t,t+1}\right) + (1-n)\frac{D^*}{P^*}E\left(1+\hat{D}^{r*}_{t,t+1}+\hat{E}_t\right) \tag{2.83}$$

and subtracting the steady state $0 = n\frac{D}{P} + (1-n)\frac{D^*}{P^*}E$

$$0 = n\frac{D}{P}\hat{D}^r_{t,t+1} + (1-n)\frac{D^*}{P^*}E\left(\hat{D}^{r*}_{t,t+1}+\hat{E}_t\right) = n\frac{D}{P}\mathcal{E}_t\hat{D}^r_{t,t+1} - n\frac{D}{P}\left(\mathcal{E}_t\hat{D}^{r*}_{t,t+1}+\hat{E}_t\right)$$

from which follows (2.84).

Monetary Policy Rule

The policy rule given by (2.46) log-linearises to

$$\hat{\imath}_t = \begin{array}{c} r_i \hat{\imath}_{t-1} + (1-r_i)\left\{\bar{\pi}_t^U + r_\pi\left(\hat{\pi}_{t-1}^U - \bar{\pi}_t^U\right) + r_Y\left(\hat{Y}_t^U - \tilde{Y}_t^U\right)\right\} \\ +r_{\triangle\pi}\left(\hat{\pi}_t^U - \hat{\pi}_{t-1}^U\right) + r_{\triangle Y}\left(\hat{Y}_t^U - \tilde{Y}_t^U - \left(\hat{Y}_{t-1}^U - \tilde{Y}_{t-1}^U\right)\right) + \eta_t^i \end{array} \tag{2.87}$$

where $\hat{\imath}_t \equiv i_t - r = i_t - i$ is the deviation of the nominal interest rate (the policy rate) from its steady state value r in levels as explained under 2.3.1. The natural rate is determined by households' rate of time preference ζ. r coincides with i as there is no steady state growth in the price level. Consequently, as we log-linearise around a zero inflation steady state, the time varying inflation target is set to zero at all dates $\bar{\pi}_t^U = 0$.

$\hat{Y}_t^U \equiv \ln \frac{Y_t^U}{Y_t^{U,ss}} 100\%$ and $\tilde{Y}_t^U = \ln \frac{Y_t^{U,flex}}{Y_t^{flex}} 100\%$ denote respectively, the percentage deviation of GDP Y_t^U about its natural rate value $Y_t^{U,ss}$ and the percentage deviation of GDP under flexible prices $Y_t^{U,flex}$ about potential GDP Y_t^{flex}. Besides current union CPI inflation $\hat{\pi}_t^U \equiv \pi_t^U$ and the output gap $\hat{Y}_t^U - \tilde{Y}_t^U$, the rule also stabilises the change in the output gap as well as the change in inflation, in case $r_{\triangle\pi} > 0$ and $r_{\triangle Y} > 0$. Therefore the rule exhibits 'speed limit' terms such that acceleration in output growth and inflation is of concern, too. Observe that the output gap $\hat{Y}_t^U - \tilde{Y}_t^U$ is defined as deviation of actual output from its evolvement under flexible prices, corrected for steady state movements. Hence the rule only responds to deviations in output fluctuations $\hat{Y}_t^U$ from efficient fluctuations $\tilde{Y}_t^U$, where fluctuations in $\tilde{Y}_t^U$ are only driven by exogenous shocks and hence cannot be influenced by policy. The underlying assumption is that efficient fluctuations (responses under flexible prices caused by 'real shocks' $S_{J,t}$ and $G_{J,t}$) should not be stabilised at all as they reflect movements in relative scarcities and relative prices that allow for optimal consumption and labour supply decisions by households.

η_t^i denotes an interest rate shock that might capture policy mistakes and $r_i > 0$ indicates an interest rate smoothing objective. Interest rate smoothing is an element of inertial monetary policy responses which allows to affect the formation of private sectors' expectations, given that commitment to a rule like (2.87) is possible. The rule is inspired by the fact that the monetary authority should use elements inherent in a backward-looking rule to ensure determinacy of the outcome.

2.3.3 Solution to the Approximated Model

As stressed above, the non-linear system is not solvable analytically but the dynamics can be described by the solution to its log-linearised approximation about a well-defined deterministic steady state for small enough perturbations. Throughout the analysis we assume that the inefficient steady state can replicate the efficient outcome, such that sector specific tax rates τ^J are fully effective in eliminating the equilibrium distortion

$$X^{eff} = X \tag{2.88}$$

We therefore determine fluctuations under flexible and sticky prices about a unique steady state. As the model is linearised up to first order, impulse response functions are simply the algebraic forward iteration of the model's policy or decision rule.[81] The solution of the log-linear system can be written in state space form as

$$\boldsymbol{z}_t = \boldsymbol{A}\boldsymbol{z}_{t-1} + \boldsymbol{B}\boldsymbol{s}_t \tag{2.89}$$

where $\boldsymbol{z}_t$ denotes the vector of model variables. Exogenous innovations ε_J in structural errors $S_{J,t}$ and $G_{J,t}$ are collected in $\boldsymbol{s}_t$.[82] Together with the measurement equations, a state space system is formed. As we rely on calibration in this chapter, all variables are perfectly observable and the measurement equations represent identities and can therefore be disregarded. The model is solved by means of the generalised Schur-decomposition, see Klein (2000) implemented in the toolkit `dynare`, see Juillard (2001).

In modelling $\boldsymbol{s}_t$, the analysis is influenced by the research by Eickmeier (2006) who employs a non-stationary dynamic factor model to analyse comovements and heterogeneity in the euro area. The author finds five factors or shocks that drive the euro area economy and identifies four common domestic (euro-area) shocks (productivity, labour supply, aggregate real demand, monetary policy) as well as a foreign (US) shock, see Eickmeier (2006, p. 22). Further, the results suggest that movements of output and inflation in individual countries in the euro area are heterogenous due more to idiosyncratic shocks rather than to the asymmetric transmission of common shocks. Accordingly, we assume that the stochastic behaviour of the system described by (2.89) derives from ten structural shocks: country-specific and sector-specific real 'supply' shocks $\{S_{H,t}, S_{N,t}, S_{F,t}, S_{N^*,t}\}$, a productivity shock common to industries across the union $S_{T,t}$ as well as four country-specific and sector-specific real 'demand' shocks $\{G_{H,t}, G_{N,t}, G_{F,t}, G_{N^*,t}\}$. Further there is a monetary policy shock η_t^i, which can also be classified as a common shock as it affects union monetary policy.[83] Due to the setup of the model,

[81] For an overview to the solution methods see DeJong (2007) and the `dynare` manual published on the `dynare` website, see `http://www.cepremap.cnrs.fr/dynare/`.

[82] As structural shocks S_J, G_J are persistent, they will be included in $\boldsymbol{z}_t$.

[83] As the terms of trade shock is a function of other real supply shocks, it cannot be counted an independent structural shock.

there is also a terms of trade shock, which is endogenously defined. This shock is directly connected to relative supply and demand shocks in the industry sectors, as $\tilde{T}_t = \frac{\eta}{1+\eta}\left(G_t^R - S_t^R\right)$. Note that the common shock therefore does not affect the terms of trade $\tilde{T}_t$. All shocks besides the interest rate shock are persistent through time.[84] The evolvement of structural shocks can therefore be described as

$$S_{J,t} = \rho_{S_J} S_{J,t-1} + \epsilon_{S_{J,t}} \qquad J = \{N, N^*\} \tag{2.90}$$

$$S_{J,t} = \rho_{S_J} S_{J,t-1} + \epsilon_{S_{J,t}} + S_{T,t} \qquad J = \{H, F\} \tag{2.91}$$

$$S_{T,t} = \rho_{S_T} S_{T,t-1} + \epsilon_{S_{J,t}} + \epsilon_{S_{T,t}} \tag{2.92}$$

$$g_{J,t} = \rho_{G_J} g_{J,t-1} + \epsilon_{G_{J,t}} \tag{2.93}$$

$$\tilde{T}_t = \frac{\eta}{1+\eta}\left(G_t^R - S_t^R\right) \tag{2.94}$$

$\epsilon_{S_{J,t}}$ represents the innovation to a supply shock in sector J which is iid with mean zero and of finite variance. Therefore, the rational expectation of innovations conditional on period t information are zero (their occurrence in the course of period t cannot be foreseen when rational expectations $\mathcal{E}_t$ are formed at the beginning of t). Occurrence of shocks will lead households to respond by deviating from steady state choices where deviations are efficient in case prices are flexible.[85] When investigating impulse responses, we typically assume that any shock deviates by one standard deviation from equilibrium, such that $\epsilon_{S_{J,t}} = \sigma_{S_{J,t}}$ in $k = 0$ and $\epsilon_{S_{J,t+k}} = 0$, $k > 0$. Hence overall, the fluctuations in the shock derive from the fluctuation in its innovations.[86] For the monetary shock η_t^i there is no serial correlation assumed.[87] Note that in steady state both $S_H = \epsilon_{S_H} = 0$, provided that initially $S_{H,-1} = 0$.

[84] The variance of shocks will be lower than the variance in its innovations. Note that in `dynare`, we can only calibrate the standard error in the innovation, not the standard error of the shock process itself. However, $\sigma^2_{S_{J,t}} = \frac{\sigma^2_{\varepsilon_{S_{J,t}}}}{1-\rho^2_{S_J}}$. Consequently, one could calibrate the standard error in the shock section in dynare as $\sigma_{\varepsilon_{S_{J,t}}} = \sqrt{\left(1-\rho^2_{S_J}\right)\sigma^2_{S_{J,t}}} = \sqrt{\left(1-\rho^2_{S_J}\right)}\sigma_{S_{J,t}}$. Smets and Wouters (2007) report the standard error of the innovation when actually referring to the shock itself. Hence no further calculations are necessary in that case, where we follow that approach. Persistency implies that past innovations have a prolonged impact on current developments.

[85] Under rational expectations and given that all households behave in a forward looking - hence optimising - way, expected shocks will have no impact on the decisions of households. Unexpected transitory shocks will lead to temporary deviations from steady state and permanent shocks will lead to migration towards a new steady state.

[86] In case of no persistence, $\rho_{S_J} = \rho_{g_J} = 0$, the shock coincides with its innovation at each point in time and the standard error is same.

[87] We need not specifiy the level of technology explicitly as shocks to the disutility of work effort were introduced. Technology implicitly needs to be constant in steady state and unequal to zero such that there is a positive level of production in long-run equilibrium.

2.4 Calibration and Baseline Impulse Responses

2.4.1 Structural Parameters Insensitive for Results

There is a range of structural parameters that are not critical for our results outlined in the introduction in paragraph 2.1.2. As the regions form a currency union, the long-run natural rate of interest is equal in both regions. As r equals the subjective rate of time preference in each region, discount factors equalise across regions, $\beta = \beta^*$. As the data frequency is quarterly, we set $\beta = 0.99$ such that the quarterly long-run return in the union is $r = 1/0.99 - 1 \simeq 1.01\%$. Labour supply in response to shocks is guided by the inverse of the elasticity of producing goods (the inverse of the elasticity of labour supply). We assume equality across sectors $J = H, N, F, N^*$

$$\eta_J = \eta \tag{2.95}$$

as is common in related work and where $\eta_J \geq 0$.[88] Also, the inverse of the intertemporal elasticity of substitution of consumption (the coefficient of relative risk aversion[89]) is no source of heterogeneity

$$\rho = \rho^* \tag{2.96}$$

where $\rho \equiv -\frac{\mathcal{U}_{CC}(C)}{\mathcal{U}_C(C)} C$ which is standard as well.[90] Welfare costs from fluctuations in regional gaps are influenced by these parameters, as they determine how households adapt to economic fluctuations that may affect regions differently. In case of flexible prices (under efficient fluctuations), η and ρ will be the only sources of welfare costs. Costs are rising in η and ρ as households are averse towards consumption risk as well as fluctuations in work effort (see also Beetsma and Jensen 2005, p. 12). We set $\rho = 3$ and $\eta = 3$ thereby following Beetsma and Jensen (2005b) and Rumler (2007). Values on both measures are lower in Benigno (2004) set at 1/6 and 2/3 respectively, but also same across regions, such that there arise no qualitiative differences.

The union monetary policy rule (2.87) closes the model in case policy targets union aggregates only. The rule aims at reflecting current monetary policy decisions and serves as a benchmark. Results under this rule will later on be compared to the welfare-maximising

[88] The higher η_J, the less responsive is work effort following shocks $A_{J,t}$. For $\eta_J = 0$, labour supply would be infinitely elastic, meaning that small changes in the economic environment would lead to large changes in labour supply. For $\eta_J \to \infty$, labour supply would become increasingly invariant in response to shocks.

[89] Our utility function is in the class of additively time separable utility functions. Then changing C_{t+1}/C_{t+2} affects $U_C(C_{t+2})$ only if it affects C_{t+2}.

[90] For $\rho = 0$, the household is not risk-averse and is indifferent between stochastic and non-stochastic period utility flows if they are associated with the same expected value. At the same time, the household is willing to shift large proportions of consumption through time, if interest rate changes occur. In that case the elasticity in substituting consumption tomorrow for consumption today approaches infinity.

policy ('the optimal linear regulator') and to the policy where coefficients are chosen optimally ('the optimal simple rule'). (2.87) is calibrated with values implied by the Bayesian estimation in Smets and Wouters (2003) for euro area data. The parameter vector calibrated with the estimated mode of the posterior distribution is

$$\{r_i, r_\pi, r_{\triangle\pi}, r_Y, r_{\triangle Y}\} = \{0.94, 1.67, 0.20, 0.15, 0.17\} \tag{2.97}$$

The response of the policy rate to inflation, $\frac{\partial \ln(1+i_t)/(1+r)}{\partial \ln(1+\pi_{t-1})} \equiv r_\pi = 1.67$ implies that the interest rate i_t responds more than one-for-one to inflation, such that the Taylor (1993) principle is satisfied. Further there is evidence for a substantial degree of interest rate smoothing captured by r_i which is commonly found in the literature, see Gorter et al. (2007). In addition, the rule displays significant positive short-term reaction to the current change in inflation represented by $r_{\triangle\pi} > 0$ and the current real growth rate captured by $r_{\triangle Y} > 0$. There is also a positive reaction to the current output gap, as indicated by $r_Y = 0.15$, such that policy acts in a countercyclically way.

2.4.2 Structural Parameters Sensitive for Results

Parameters determining nominal rigidity (aggregate price rigidity θ and inflation persistence ϖ), country size n, the composition of output, $\frac{P_H Y_H}{PY}$, $\frac{P_F Y_F}{P^* Y^*}$, as well as of consumption, γ, γ^* are considered main sources of heterogeneity between members in this chapter as outlined in the introduction (see the table 2.1 on page 16). They are therefore considered as crucial for the results regarding welfare effects of business cycles to be determined in the following. Also the effects of market structure captured by steady state mark-ups $\frac{\rho_J}{\rho_J - 1}$ are taken into account. Our analysis is richer than related frameworks: The analysis in Benigno (2004) is based on (only) varying the degree of price rigidity θ and the results in Benigno and López-Salido (2006) on varying the degree of nominal rigidity, i.e. both θ, ϖ. Our two-sector structure instead makes it possible to detect sectoral sources of aggregate rigdity and persistence. We will see that the expenditure share $0 \leq \gamma \leq 1$ of tradable goods in the basket C_t coupled with differing degrees of sectoral rigidity θ_J, ϖ_J will have essential influence on welfare and hence the conduct of optimal monetary policy.

Nominal rigidity as presented in table 2.1 leads to further measures of price inflexibility. We follow Benigno and López-Salido (2002, p. 23) in defining aggregate duration of price contracts D^U for the union as a geometric average

$$D^U = D^n (D^*)^{1-n} \tag{2.98}$$

Duration of price contracts in the union in quarters is thus a geometrically weighted average of duration of contracts in the home region, D, and duration at foreign D^* where weights correspond to economic weights of regions $\frac{Y}{Y^U}$ and $\frac{Y^*}{Y^U}$. We extend this aggregation method to the sectoral case. In each country, overall duration is then a composite of tradable duration D_H and non-tradable duration D_N, where weights are determined by contributions to gross value added Y_H/Y, Y_F/Y for tradables and $1-Y_H/Y$, $1-Y_F/Y^*$ for non-tradables. Therefore

$$D = D_H^{Y_H/Y} D_N^{1-Y_H/Y}, \qquad D^* = D_F^{Y_F/Y^*} D_{N^*}^{1-Y_F/Y^*} \tag{2.99}$$

In the Benigno (2004) framework, $Y_H/Y = Y_F/Y^* = 1$ and $\varpi_J = 0$. Likewise, nominal rigidity in sector J is split in a forward-looking and a backward-looking component, where $D_J^f = \frac{1}{1-\theta_J}$ and $D_J^b = \frac{1}{1-\varpi_J}$ such that $D_J = D_J^f D_J^b$. D_J is the sectoral equivalent of the expression proposed in Benigno and López-Salido (2002, p. 19). Using that in general, duration is a composite of θ and ϖ, nominal rigidity D_J and structural inflation dynamics parameters are linked by

$$D = \frac{1}{1-\theta}\frac{1}{1-\varpi}, \qquad D^* = \frac{1}{1-\theta^*}\frac{1}{1-\varpi^*}, \qquad D_J = \frac{1}{1-\theta_J}\frac{1}{1-\varpi_J} \tag{2.100}$$

where estimates on θ and ϖ are available. Then, by fixing θ_H, and θ_F, as well as ϖ_H and ϖ_F, one can determine deep parameters in the services sector such that the aggregate values remain intact. Average price duration at home D^f (as compared to duration of optimising contracts D) is a weighted average of sectoral average price duration D_H^f and D_N^f

$$D^f = \left(D_H^f\right)^{Y_H/Y} \left(D_N^f\right)^{(1-Y_H/Y)}$$

With θ given by estimates and θ_H calibrated, θ_N can be derived from the duration for service price contracts

$$D_N^f = \left(\frac{D^f}{\left(D_H^f\right)^{Y_H/Y}}\right)^{\frac{1}{1-Y_H/Y}} \tag{2.101}$$

from where the Calvo parameter in the N sector can be extracted, as $\theta_N = 1-\frac{1}{D_N}$. Accordingly one can proceed in case of θ_{N^*}, ϖ_N, and ϖ_{N^*}. Note that in case of only forward-looking price-setters in all sectors $D = D^f$, $D^* = D^{*f}$.

2.4.3 Scenarios Considered in Experiments

Experiments outlined in the introduction are based on four scenarios. In order to explore the impact of heterogeneity of member states, in any scenario, the homogenous case (the case of absence of heterogeneity in the critical parameters) is compared to the case where countries structurally differ. Under heterogeneity, always the Home region experiences treatment whereas Foreign remains structurally same. This also implies that the Foreign region size $1-n$ will not change. We label the scenarios *(1) Baseline Scenario*, *(2) Current Euro Area Scenario, (3) Large Member Scenario* and *(4) Enlarged Euro Area Scenario.* Table 2.2 on page 76 summarises the appropriate calibration for each of the scenarios in the homogenous and the heterogenous case. Scenario (1) provides the calibration for experiment I evaluated in 2.5. Scenarios (2) - (4) provide the underlying calibration for the evaluation of the second experiment performed under 2.6 as well as the third experiment under 2.7. We employ data from *Eurostat* throughout the analysis in order to determine the country weight n_i and to determine the share of gross value added in industries to overall gross value added $\frac{Y_H}{Y}$, $\frac{Y_F}{Y^*}$. These measures were already presented in table 2.1.[91]

As only aggregate nominal rigidity $\frac{1}{1-\theta}\frac{1}{1-\varpi}$ is identified from estimates, we have one degree of freedom regarding assumptions about sectoral rigidity within each country. An economic argument for lower rigidities in tradables sectors can be seen in the relative price pressures firms face that engage in the export sector (albeit keeping their pricing power). Following Razin and Binyamini (2007) it can be argued that, among others, goods mobility contributes to a flattening of the inflation-output (variability) trade-off in the Phillips-curves. We therefore assume that exporting firms experience these globalisation forces that lead to decreases in θ_H, θ_F and ϖ_H, ϖ_F where all tradables firms at Home and Foreign are affected equally. Lower rigidity will make producer price inflation rates less responsive to marginal cost movements. In order to obtain consistency with the reported estimates on aggregate rigidities in Rumler (2007), rigidity and persistency in the N and N^* sectors need to adjust properly such that estimates on ϖ and θ remain valid when aggregating sectoral values. Consistency is obtained by using the methods of aggregation described in the preceding paragraph. Setting $\theta_H = \theta_F = 0.25$ and $\varpi_H = \varpi_F = 0.40$ allowed to produce reasonable matches of empirical moments to be discussed below. For given aggregate nominal rigidity in each region, values for θ_N, θ_{N^*} as well as ϖ_N, ϖ_{N^*} are obtained using (2.100) and (2.101). In detail, the scenarios can be described

[91] For steady state values, we average annual values from 2000 to 2006 taken from annual national sectoral accounts. γ, which denotes the share of tradable goods in consumption, is proxied by the weight of industry goods in consumption, obtained from the COICOP classification, and averaging annual data. Note that as all industry goods are traded, γ also serves as an additional measure of openness of a region to the other region.

as follows:

(1) Baseline Scenario: In this scenario, we explore major effects heterogeneity has in a currency union in general. In the homogenous case, the regions resemble characteristics of the aggregate euro area. Both regions exhibit the same structure and do not differ with respect to price rigidity, inflation inertia, output movement and sectoral composition of GDP. Region sizes are equal also, such that $n = 0.5$ in order to eliminate effects that are just due to region size but neither nominal nor real rigidities. Estimates on deep parameters guiding inflation dynamics are taken from Rumler (2007) for the aggregate euro area based on a weighted average of New Keynesian Phillips curve estimates for nine major euro area countries for the time span 1980Q - 2003Q4. Both regions face the same degree of nominal and real rigidities such that both regions resemble the aggregate. Specifically, $\varpi = 0.55$, $\theta = 0.67$, $\varpi_H = 0.40$, and $\theta_H = 0.25$, where ϖ_N and θ_N are determined residually. In the heterogenous case, the Home region features forward-looking price setting in both the H and N sector whereas Foreign remains price-sticky as before. Specifically we assume that $\varpi = \theta = \varpi_H = \theta_H = 0.05$. The degree of nominal rigidity is then at a low of $\frac{1}{1-\theta}\frac{1}{1-\varpi} = 1.11$ quarters. In order to match the assumed degree of aggregate rigidity, it follows that $\varpi_N = \theta_N = 0.05$. Rumler (2007) assumes that aggregate mark-ups are 10%. We assume that there arise no sectoral differences when disaggregating this measure, therefore $\rho_J = 11$.

(2) Current Euro Area Scenario**:** In this scenario, we focus on the special role Germany has as largest member of the euro area in influencing the euro area business cycle. In the homogenous case, both regions therefore resemble the structural characteristics of Germany. In the heterogenous case, Home resembles the aggregate characteristics of major euro area countries excluding Germany whereas Foreign remains Germany. Accordingly, the economic size of the Home region is set to $n = \frac{Y}{Y^U} = 0.69$ which represents gross value added of other major euro area members excluding Germany as share of union gross value added. We obtain under heterogeneity that $\varpi = 0.4138$ and $\theta = 0.6922$. Hence average duration of prices is lower in the euro area excluding Germany compared to Germany and rigidity amounts to 5.44 quarters, much lower than the value compiled for Germany (over 12 quarters). For gross value added in industry, the weighted share is $\frac{Y_H}{Y} = 0.2157$. To be consistent with scenario (1), again the price elasticity of demand equals $\rho_J = 11$ which will be assumed throughout in case current euro area members are concerned.

***(3) Large Member Scenario*:** In this scenario, we acknowledge that economic activity in the union is driven to a large extent by activity in Germany and France that together generate more than 55% of euro area GDP. Again, in the homogenous case, both regions are calibrated with estimated parameters for Germany. In the heterogenous case, we model the home region such that it resembles France whereas the foreign region remains Germany. The size of the regions is constructed such that the relative contribution of Germany (34.3%) and France (22.3%) to euro area GDP are taken into account. Accordingly, $n = 0.39$ as the union size is normalised to one as throughout the analysis.[92] Note that in case of homogeneity, (3) (only) differs from (2) by the fact that Home region size differs.

***(4) Enlarged Euro Area Scenario*:** Eastward enlargement of the euro area is on the horizon. It is especially worthwhile to investigate the consequences of potentially increased structural heterogeneity. In the homogenous case, Home and Foreign are structurally similar and resemble characteristics of the aggregate euro area, as described in scenario (1). In the heterogenous case, we model the home region such that it matches average structural characteristics of the EU9 whereas Foreign remains the current euro area. The weight of the home region is equal to $n = 4.2\%$ such that in case of heterogeneity, Home matches the economic weight of new members in the then enlarged euro area. Enlargement therefore will not affect the size of the union in total as the size of the union is normalised to one. In chapter 3 of the dissertation, we estimate hybrid Phillips curves for each of the new members. From that source we average $\varpi^{EU9} = 0.4$ and $\theta^{EU9} = 0.55$. Average nominal rigidity is therefore around 3.7 quarters such that rigidity is less than in the current euro area. Following Lendvai (2005), it can be argued that the share of rule of thumb firms is higher in new members but prices by forward-looking firms are reset more often than in the current euro area. Following the literature, the elasticities of substitution between goods produced within a sector are assumed to be lower in new member states reflecting lower competition, such that we calibrate $\rho_H = \rho_N = 6.8$. Consequently, the markup of output prices over real marginal cost is around 18%. For the weight of tradables in overall consumption we again refer to the COICOP weight, which implies $\gamma^{EU9} = 0.41$.[93]

2.4.4 Structural Shocks

The calibration of shocks aims at matching stylised facts regarding moments and persistency obtained from contrasting simulated time series using the model as data generating process

[92] The factor required, such that the two country weights add up to 1, is given by $\frac{100}{34.3+22.3}$.
[93] Doyle et al. (2006) propose a value of 0.5.

(1) Baseline	H region size	Inflation dynamics				Mark ups industry	Mark ups services	Size industry	Size services	Open ness
	$n = \frac{Y}{Y^U}$	ω	θ	ω_H	θ_H	$\frac{\rho_H}{\rho_H - 1}$	$\frac{\rho_N}{\rho_N - 1}$	$\frac{Y_H}{Y}$	$1 - \frac{Y_H}{Y}$	γ
Homogenous										
H	0.5	0.55	0.67	0.40	0.25	1.10	1.10	0.22	0.78	0.40
F	0.5	0.55	0.67	0.40	0.25	1.10	1.10	0.22	0.78	0.40
Heterogenous										
H	0.5	0.05	0.05	0.05	0.05	1.10	1.10	0.22	0.78	0.40
F	0.5	0.55	0.67	0.40	0.25	1.10	1.10	0.22	0.78	0.40
(2) Current Area	H region size	Inflation dynamics				Mark ups industry	Mark ups services	Size industry	Size services	Open ness
	$n = \frac{Y}{Y^U}$	ω	θ	ω_H	θ_H	$\frac{\rho_H}{\rho_H - 1}$	$\frac{\rho_N}{\rho_N - 1}$	$\frac{Y_H}{Y}, \frac{Y_F}{Y^*}$	$1 - \frac{Y_H}{Y}$	γ
Homogenous										
H: DE	0.69	0.40	0.87	0.40	0.25	1.10	1.10	0.25	0.75	0.40
F: DE	0.31	0.40	0.87	0.40	0.25	1.10	1.10	0.25	0.75	0.40
Heterogenous										
H: EA ex. DE	0.69	0.41	0.69	0.40	0.25	1.10	1.10	0.22	0.78	0.40
F: DE	0.31	0.40	0.87	0.40	0.25	1.10	1.10	0.25	0.75	0.40
(3) Large Member Area	H region size	Inflation dynamics				Mark ups industry	Mark ups services	Size industry	Size services	Open ness
	$n = \frac{Y}{Y^U}$	ω	θ	ω_H	θ_H	$\frac{\rho_H}{\rho_H - 1}$	$\frac{\rho_N}{\rho_N - 1}$	$\frac{Y_H}{Y}, \frac{Y_F}{Y^*}$	$1 - \frac{Y_H}{Y}$	γ
Homogenous										
H: DE	0.392	0.40	0.87	0.40	0.25	1.10	1.10	0.25	0.75	0.40
F: DE	0.608	0.40	0.87	0.40	0.25	1.10	1.10	0.25	0.75	0.40
Heterogenous										
H: FR	0.392	0.51	0.70	0.40	0.25	1.10	1.10	0.18	0.82	0.39
F: DE	0.608	0.40	0.87	0.40	0.25	1.10	1.10	0.25	0.75	0.40
(4) Enlarged Area	H region size	Inflation dynamics				Mark ups industry	Mark ups services	Size industry	Size services	Open ness
	$n = \frac{Y}{Y^{U\prime}}$	ω	θ	ω_H	θ_H	$\frac{\rho_H}{\rho_H - 1}$	$\frac{\rho_N}{\rho_N - 1}$	$\frac{Y_H}{Y}$	$1 - \frac{Y_H}{Y}$	γ
Homogenous										
H=EA	0.042	0.55	0.67	0.40	0.25	1.10	1.10	0.22	0.78	0.40
F=EA	0.958	0.55	0.67	0.40	0.25	1.10	1.10	0.22	0.78	0.40
Heterogenous										
H=EU9	0.042	0.40	0.55	0.40	0.25	1.17	1.17	0.26	0.74	0.40
F=EA	0.958	0.55	0.67	0.40	0.25	1.11	1.11	0.22	0.78	0.40

Table 2.2: Calibration of parameters critical for results.

with empirically equivalent time series on activity, consumption and inflation rates in each of the scenarios.[94] Estimates on volatility and persistence are adjusted where necessary to reproduce empirical stylised facts regarding the movement of regional output, consumption, and inflation. We also calibrate structural shocks for which no values in the literature are available, namely idiosyncratic shocks to sectoral output supply and demand in order to match empirical volatility measures displayed in table A.2 on page 242 in the appendix as close as possible.[95] For describing the comovement between member state series X_t and union series X_t^U, model-based contemporaneous correlations $\mathrm{Corr}(X_t,\ X_t^U)$ for home and foreign consumption and output

[94] Simulated series are obtained by taking the model as data generating process where all exogenous stochastic processes face innovations that occur at random over time.

[95] We first experimented using estimates for the productivity shock in Smets and Wouters (2003) that provides a direct analogon of the value for $\sigma\left(\varepsilon_{S_T}\right)$. Their contribution takes a union wide perspective. In our setting, the common supply shock would then however overstate the variability of π_t^U in the structural shock decomposition to a large extent.

are shown that reflect empirical measures presented in table 2.1 on page 16.[96] The analysis is based on comparing sticky-price model-based fluctuations with fluctuations in empirical series.

The standard deviation for the common supply shock is taken from Smets and Wouters (2003). We therefore set $\sigma_{\epsilon_{S_T}} = 0.598\%$ which is the value for the mode of the estimated maximum posterior estimated therein. We use this value also as 'starting value' for calibrating idiosyncratic supply-shocks $\{S_H, S_F, S_N, S_{N^*}\}$. For the *Current Euro Area* and the *Large Member Scenario*, we double standard deviations of idiosyncratic shocks to industries, ϵ_{S_H} and ϵ_{S_F} which allows to better match standard deviations in the cyclical component of empirical series. The volatility in the common industry supply shock ε_{S_T} is calibrated with the value for volatility in the productivity shock given therein. Eventually, taking into account that according to Eickmeier (2006) at least 10% of variation in GDP is explained by monetary factors, we increase the standard deviation of η^i by a factor one and a half in all three scenarios.

For the monetary policy shock we set $\sigma_{\eta^i} = 0.162\%$ and fiscal shocks are determined by $\sigma_{\epsilon_{G_J}} = 0.325\%$. Values for persistence of shocks are summarised in table A.1 in the appendix. Analogously to Natalucci and Ravenna (2005) we can calibrate the excess productivity in home industry over home services as a temporary, yet persistent shock, that dies out after approximately six years. Hence we choose $\rho_{S_H} = 0.85$.

We also allow for correlation in country-specific shocks to industry supply, such that

$$\mathsf{Corr}(\epsilon_{S_H}, \epsilon_{S_F}) \equiv \rho_{\epsilon_{S_H}, \epsilon_{S_F}} = 0.5 \tag{2.102}$$

where we assume that $\epsilon_{S_{H,t}} \to \epsilon_{S_{F,t}}$ in the Cholesky ordering. We claim that purely idiosyncratic shocks to an industry will always cause spillovers to foreign, even if there is no common supply shock $S_{T,t}$ present. Note that the assumption of having correlated idiosyncratic shocks across industries might seem to render the common tradable shock $S_{T,t}$ redundant. However, the Cholesky ordering for the correlated idiosyncratic shocks will imply that fluctuations in $Y_{H,t}$ triggered by $\epsilon_{S_{H,t}}$ will not lead to much fluctuation in $Y_{F,t}$. A common shock only modeled this way would therefore induce high variability in one country but not in the other albeit there is high correlation in the occurrence of the shocks. In line with the empirical facts as presented in Eickmeier (2006) the calibration will respect that most fluctuations are triggered by $S_{T,t}$. The assumption about $\mathsf{Corr}(\varepsilon_{S_H}, \varepsilon_{S_F})$ will therefore not contribute essentially to the welfare results

[96] Time series employed refer to quarterly national accounts data, in millions of 1995 Euros and exchange rates. Data on series in levels is logged and filtered with the Hodrick and Prescott (1997) filter in order to obtain the cyclical component of each series. For the euro area of large members, we weigh the cyclical country contributions by their economic size and calculate the standard deviation and correlation thereof.

Note that as output under flexible prices is not observable, we calibrate the standard deviation of the cyclical component of actual output, for given nominal and real rigidities, in case of each scenario.

discussed below.

We further think it is useful to assume correlation in fiscal spending across sectors within each country. Therefore

$$\mathsf{Corr}(\epsilon_{G_H}, \epsilon_{G_N}) \equiv \rho_{\epsilon_{G_H}, \epsilon_{G_N}} = 0.5 = \mathsf{Corr}(\epsilon_{G_F}, \epsilon_{G_{N*}}) \equiv \rho_{\epsilon_{G_F}, \epsilon_{G_{N*}}} \tag{2.103}$$

As a second constraint besides reproducing empirical volatility and persistence in key macroeconomic series in countries considered in scenarios, we try to replicate the structural shock decomposition of GDP and CPI inflation in accordance with the forecast error variance decomposition of the structural factor model presented in Eickmeier (2006) in table 8. The author detects five common shocks, namely two euro area supply shocks, one euro area demand shock, one common monetary policy shock and a US shock that empirically drive activity in the euro area. 34% of variability in union GDP (Y_t^U) can be explained by common euro area productivity shocks and labour supply shocks (where productivity accounts for 15% and labour supply for 19%). 22% is due to demand shocks and 14% to monetary policy shocks. The US shock eventually accounts for 18% which leaves 12% for idiosyncratic shocks. As there is no possibility of a foreign shock in our framework (there is no third country representing the rest of the world), we encompass the foreign demand shock within the idiosyncratic demand shocks in the model. Hence 34% of Y_t^U (of its cyclical component $\hat{Y}_t^U$) should be explainable by S_T, our source of the common shock across industries in the model.[97] Demand shocks $\{G_J\}$ should account for 40% of variation and the monetary policy shock η^i for 14%.

Table A.3 on page 243 in the appendix illustrates the resulting decomposition of fluctuations in real output, real consumption, and inflation rates according to their structural shock contributions in each of the scenarios. We can observe which structural shocks contribute quantitatively to fluctuations in the aggregates in the model, assuming that monetary policy is characterised by (2.87) and national fiscal policies implement their fiscal rules (3.27). For the *Current Euro Area Scenario*, variables with a superscripted asterisk indicate developments in Germany whereas Home variables summarise developments in the euro area excluding Germany according to our definition of scenario 1. Likewise in case of the *Large Member Scenario* where variables indicated with an asterisk again represent Germany and Home variables represent developments in France. In the *Enlarged Euro Area Scenario*, superscripted variables present values for the current euro area and Home variables show values for new EU members. We observe that the shock decomposition of GDP implied by the model is broadly in line with the

[97] In case of GDP, 12% of variation is attributed to country-specific movements and in case of inflation the share is 10%. 18% of variability in GDP is explained by the foreign demand shock and 15% in case of inflation. We refer to median outcomes.

results for the forecast error decomposition obtained in the factor model. However the model tends to overemphasise the contribution of the common productivity shock in driving regional CPI inflation rates and the rate for the union as a whole. Regarding reduced form volatilities, the model tends to understate the volatility in empirical inflation in general.[98]

2.4.5 Member State Heterogeneity in the Baseline Scenario

As a first step towards evaluating the welfare costs of fluctuations in a currency union in the following three experiments, we investigate the performance of member states when exposed to transitory shocks along the business cycle.[99] Impulse responses of key macroeconomic variables are triggered by innovations in structural shocks (2.90) to (2.94). By assuming that innovations can have persistent effects, structural shocks will be autocorrelated. In the following, we concentrate on the effects a positive shock to industry productivity S_H (a 'supply shock') and a positive shock to fiscal spending on services g_N (a 'demand shock') have on macroeconomic activity, inflation rates on the producer and consumer stage as well as relative prices. As the model structure implies that the union size is 1 and as the size of the Home region n will not change when moving from homogenous to heterogenous members, changes in the economic setup of the H region can be compared to the homogenous case. Impulse responses are based on the calibration of the *Baseline Scenario* as described under 2.4.3.

Supply Shocks under Heterogeneity

Figures 2.1 and 2.2 on page 81 illustrate the transmission of the supply shock at Home and spillovers to Foreign in case of equal rigidities across regions.[100] The innovation in the supply shock S_H improves overall efficiency of production in the traded-goods sector. As the shock was unexpected by rational agents, output will increase thereby increasing the physical marginal product of labour such that wages in the H sector can increase. As we assumed correlation between the industry shocks, output in the foreign region will increase as well. Due to monopolistic competition, firms have pricing power over their product and firms that are able to

[98] A sort of trade-off materialises. On the one hand, this chapter tries to embody as much information as possible found in related work. On the other, this might not always contribute to the overall fit regarding moments and comovement between key variables generated by the model. Hence as a further step, a system wide estimation of the model could be worthwile.

[99] By structural shocks, we mean shocks that affect the variable under consideration directly. The effect is therefore not the result of spillovers of other variables that might possible affect the left hand side variable.

[100] The figures illustrate deviations of key macroeconomic variables from steady state in %, i.e. for any variable

$$x = \ln X_t / X^K \times 100\%$$

where X^K is the efficient/inefficient but subsidised steady state of the variable. The impulse response is triggered by an innovation amounting to one standard deviation.

lowering prices will do so in order to sell more which leads to an increase in profits. Labour mobility across sectors within each country will equalise (after-tax) wages across sectors. As the N sector did not receive a gain in productivity, output prices in the N sector will tend to increase. In both regions, the internal real exchange rates Q_t and Q_t^* will thus appreciate as service-goods have become relatively more expensive. Whereas this basic transmission mechanism is same in the homogenous and heterogenous union, price adjustment will be different under heterogeneity, which will be elucidated in the following.

In the homogenous union, when prices are sticky, only a portion of firms in the H and N sector will be able to reset prices after the common, unexpected productivity shock. Only a portion $1-\theta_J$ of firms can reset prices and $\varpi_J \times 100\%$ of newly set prices will be set in a non-optimising way. If product prices cannot be adjusted, prices no longer reflect current factor costs whereas for firms that can reset prices this is the case. Price dispersion across firms due to price rigidity causes the supply shock to affect real marginal cost of production in both sectors and therefore sectoral producer price inflation rates and accordingly CPI inflation. Hence there arises a distortion in production as output of firms that cannot reset prices will be inefficiently low. Also relative prices between goods in the same sector and across sectors are distorted such that households no longer make efficient choices. Real average marginal cost in the tradables sector $MC_{H,t}$ will drop and an increase in average marginal cost in the non-tradable sector $MC_{N,t}$ will result. The effect on sectoral inflation is stretched over time and becomes more inertial the higher the time-span optimising firms face until they can reset prices again and the higher the share of firms that do not optimise at all. As developments are equally rigid at home and abroad, the appreciation in internal real exchange rates (non-tradables have become relatively more expensive) has no effects on the external real exchange rate: As producer price indices in both industry sectors respond inertial, the terms of trade movements are minor. Accordingly, competitiveness of regions remains unaffected.

Under structural heterogeneity and fully flexible prices at Home (impulse responses labeled 'het.' in figures 2.1 and 2.2), output and factor income from labour (wages) at Home increase instantaneously. As all firms will behave accordingly, the drop in $p_H^o(h)$ causes an equal drop in $P_{H,t}$ and real marginal costs remain unaffected. Consequently, they are set as a constant markup over real marginal cost. Hence, given the shock would be one on-off, no effect of the shock on the sectoral inflation rate $\pi_{H,t}$ would materialise, which becomes clear from (A.94) in the appendix. Then also the consumer price inflation would remain unaffected. As the shock is autocorrelated, the decisions of agents will however have consequences for tomorrow's response to the shock, such that adjustments are stretched over time. Then, also under flexible prices

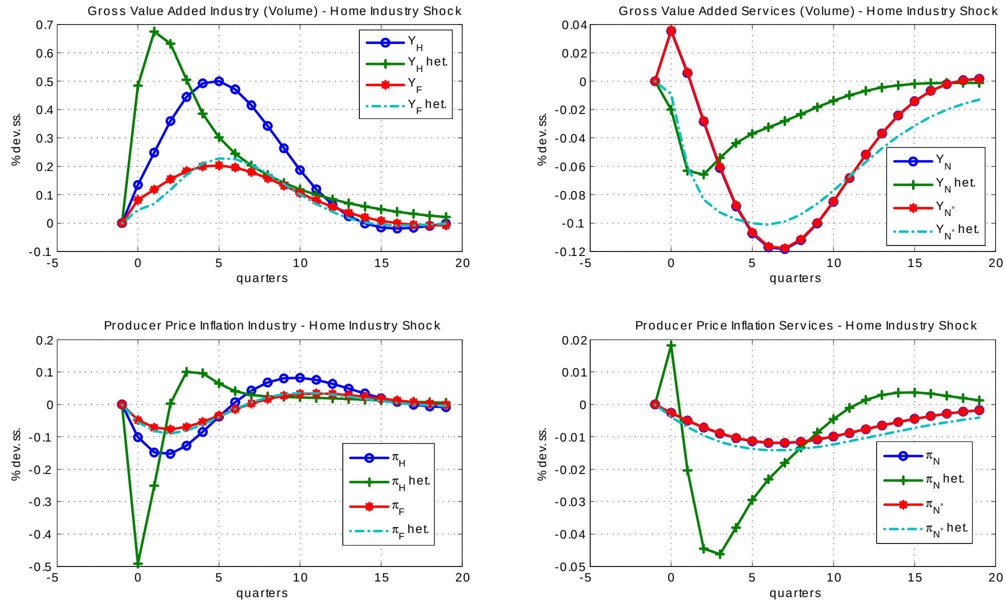

Figure 2.1: Innovation in industry productivity S_H - response of gross value added and producer price inflation rates.

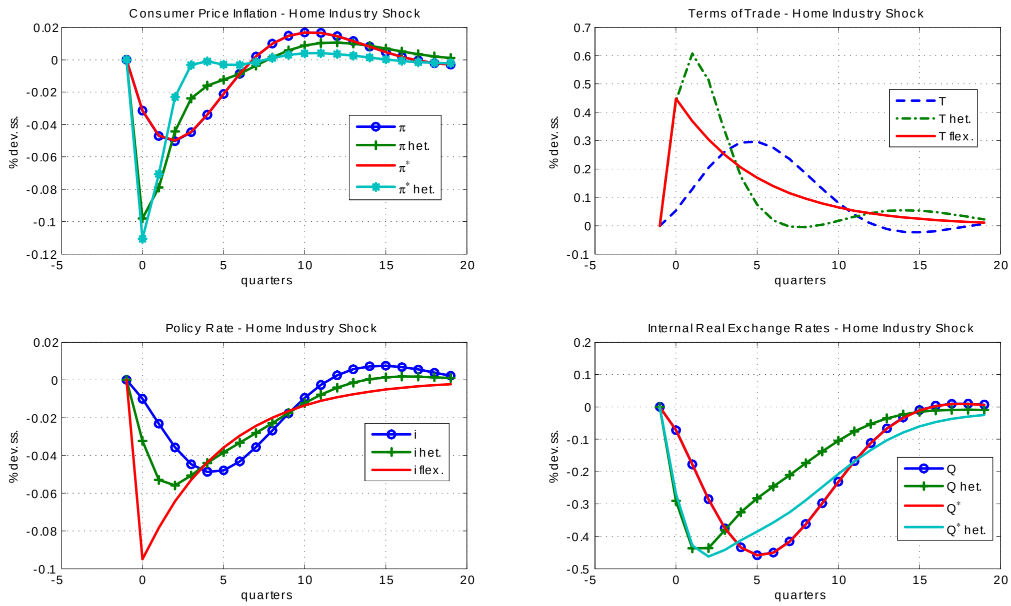

Figure 2.2: Innovation in industry productivity S_H - response of the CPI inflation rate, the policy rate, the internal real exchange rates, and the terms of trade.

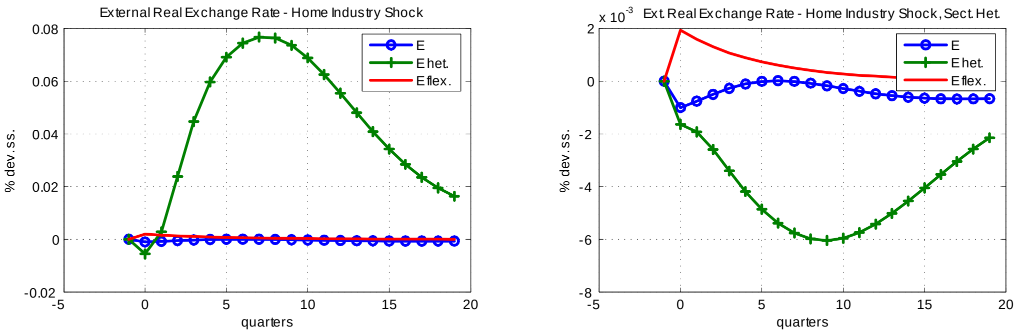

Figure 2.3: Innovation in industry productivity S_H - response of the external real exchange rate.

the deviation from steady state can last for some quarters. This explains why there can be also inflation differentials under flexible prices. To maintain profit margins, firms in the N sector will charge higher final goods prices to decrease real wages again. Under flexible prices, all firms behave in such a way, such that P_N remains fixed and N firms have to lower output until real wages drop again, which becomes clear from the upper right plot in figure 2.1. Hence output in the N sector responds efficiently. As there is now a different speed of adjustment across members present, home producer price inflation $\pi_{H,t}$ initially drops stronger compared to producer price inflation in foreign $\pi_{F,t}$. A large deterioration (an increase) in the terms of trade T_t shows up. Accordingly, the home economy can improve its external competitiveness and a large external real depreciation in the external real exchange rate shows up, as the right panel of figure 2.3 makes clear.

The response of monetary policy to heterogeneity across regions becomes clear from inspecting the setting of the policy rate under the Smets-Wouters rule (our proxy for actual policy of the ECB), shown in the lower left panel of figure 2.2. Under homogeneity (equal rigidities across countries), the interest rate response is rather muted. Intrinsic inflation persistence due to the presence of backward-looking firms causes adjustments in prices to be stretched over time. This allows monetary policy to stabilise activity with a lower decrease of its policy rate as the expectations channel is active where agents believe that monetary policy will continue using its instrument rule. However, this policy is not fully efficient when compared with the response of the natural rate of interest $\tilde{\imath}_t$ (the straight line) which would prevail in case prices would be fully flexible. A stronger response in policy would be desirable as it brings output and prices back to equilibrium faster thereby resulting in lower overall fluctuations over time. Accordingly, under heterogeneity when home is flexible, the interest rate response is more aggressive which spurs a faster adjustment back to the steady state.

We also investigate the case when Home exhibits higher flexibility only in the industry sector in case of structurally different members. Figures A.1 and A.2 on page 244 in the appendix illustrate the results. When services differ substantially in price and output dynamics from industry goods, determinants in the services sector can have a large impact on overall macroeconomic activity and CPI inflation. We obtain that price rigidity in the home service sector in case of heterogeneity now even leads to a loss of external competitiveness. The external real exchange rate appreciates instead of depreciating following the supply shock which is shown in figure 2.3: As only some firms can reset prices in the services sector, the decrease in prices following the positive supply shock is lower than before, which becomes clear from comparing the lower right panels of figures 2.1 and A.1. Accordingly, as service prices decrease less at home than abroad and as the real exchange rate is driven by the large weight in non-tradables, a real appreciation builds up. The result emphasises the impact non-tradables can have in shaping the overall performance of a region.

Demand Shocks under Heterogeneity

We further investigate the outcome of prolonged demand shocks, triggered by higher fiscal spending at Home.[101] Again where compare the case where both regions exhibit the same degree of nominal rigidities to the case where Home is flexible. Fiscal spending is correlated across sectors within a region and only falls on the produce of the respective region. We assume that a shock to service expenditures (most government expenditures can be expected to fall on services) also triggers a surge in demand for industry goods, such that the Cholesky ordering is $\varepsilon_{G_{N,t}} \rightarrow \varepsilon_{G_{H,t}}$. Relative price movements against the backdrop of joint short-run interest rate determination will lead to spill-over effects of the demand shock to the other region via the external real exchange rate E_t within periods and via the expected real interest rate $i_t - \mathcal{E}\pi_{t+1}$ between periods. Figure 2.4 illustrates the transmission of the transitory demand shock to sectoral output and inflation rates. Figure 2.5 presents results for the CPI inflation rate, internal and external relative prices and the policy response of the central bank.

A prolonged higher spending on home non-tradables by the home government increases domestic production in the service sector and as government purchases also fall on H goods, industry output will increase as well. The resulting labour market tightening can only be resolved by offering higher wages in both sectors as the labour market remains in equilibrium throughout such that households provide more hours of work. As the increase in demand is not accompanied by an increase in sectoral productivity[102], output prices have to rise in order

[101] Due to the specification employed, the spending shock can be thought of as a general shock to the expenditure side of GDP and can therefore be classified as a general ‘demand’ shock.

[102] Diminishing returns in labour input will cause the marginal revenue product of labour to be lower for any

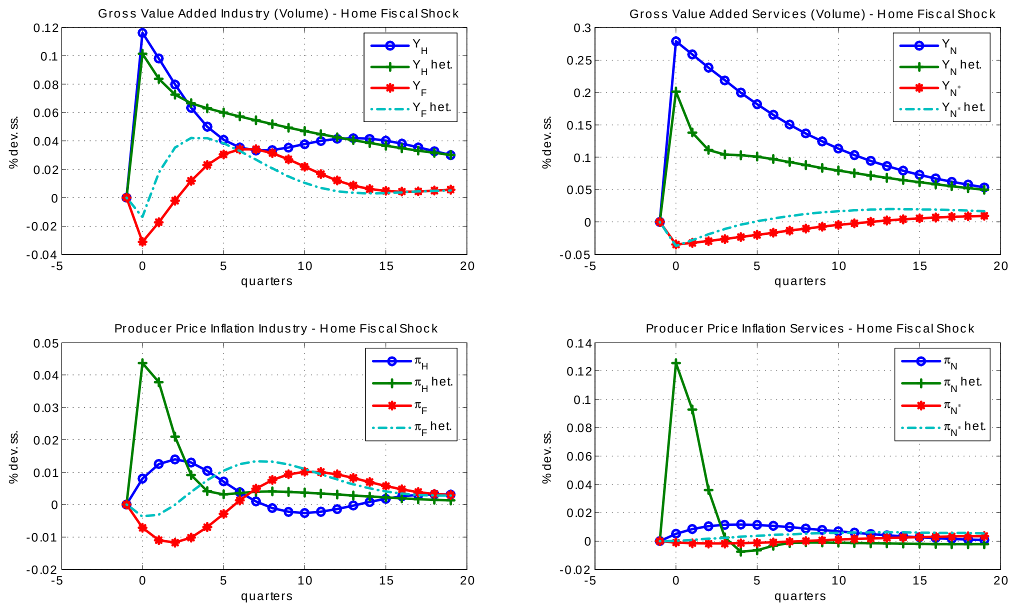

Figure 2.4: Innovation in fiscal expenditures G_N - response of gross value added and producer price inflation rates.

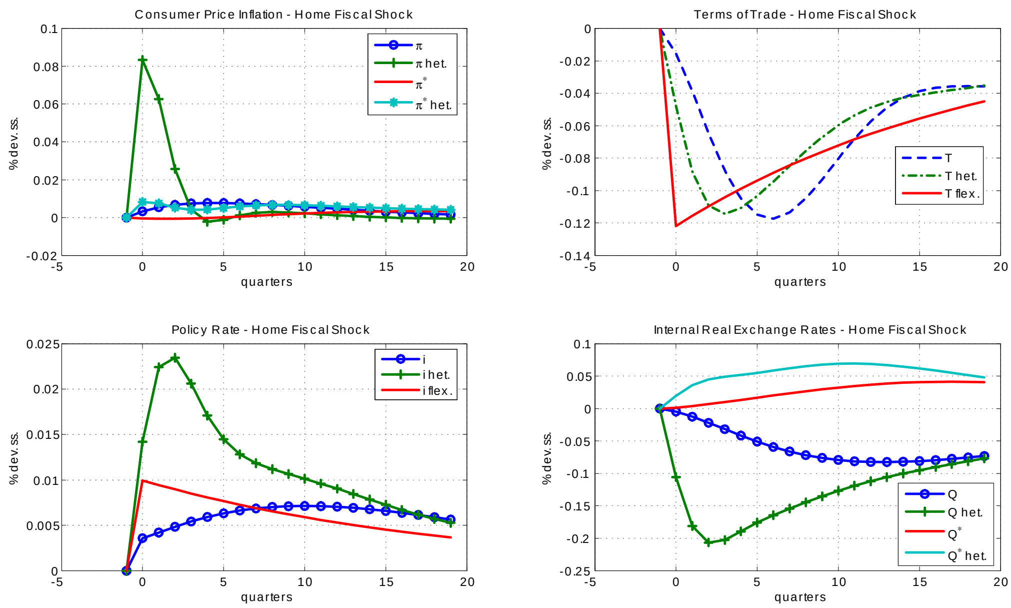

Figure 2.5: Innovation in fiscal expenditures G_N - response of the CPI inflation rate, the policy rate, the internal real exchange rates, and the terms of trade.

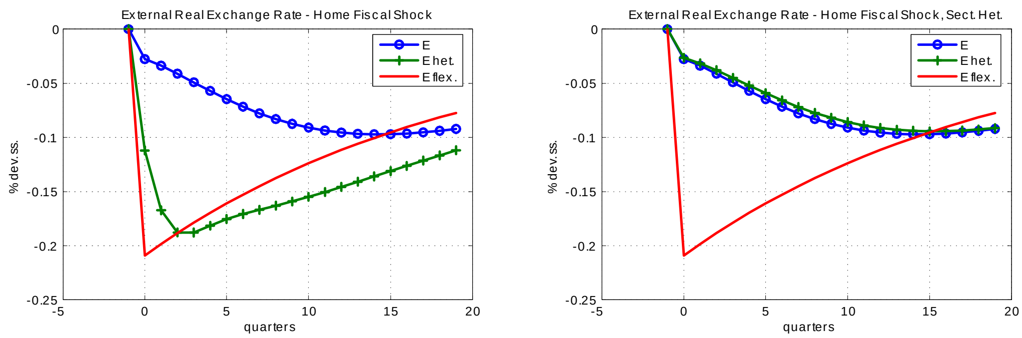

Figure 2.6: Innovation in fiscal spending G_N - response of the external real exchange rate.

to meet the higher demand. Accordingly, real marginal cost will increase, thereby increasing producer price inflation in both sectors. Price increases can then passed on consumers by firms that are able to reset prices such that cost increases eventually feed into home consumer price inflation. A loss in competitiveness of the home region builds up, indicated by a strong appreciation of the real exchange rate E_t that derives from the appreciation in Q_t as figures 2.5 and 2.6 make clear. At the same time, there is a complete crowding out observable of private for public consumption (not shown). The loss in competitiveness of the home region depreciates the foreign internal real exchange rate Q_t^*. As the terms of trade of the home region improve, foreign production of tradable goods increases due to Home consumers' revaluation of wealth that increases spending on foreign goods.

In case the home region is flexible in all sectors, the inflation adjustments happen faster on the sectoral level, and higher flexibility is transmitted to foreign production sectors via the competitiveness channel. Lower inertia in the transmission of the shock originating at Home moves the peak in impulse responses closer to the initial period. Heterogeneity aggravates the internal real appreciation and the improvement in the terms of trade is initially stronger. Both effects fuel higher CPI inflation in the periods following the shock as the upper left panel of figure 2.5 reveals. Accordingly, the appreciation of the external real exchange rate is stronger under heterogeneity which derives from the stark appreciation of the internal real exchange rate as the left panel of figure 2.6 displays. As there is no home bias in consumption of industry goods, the improvement in the terms of trade causes expenditure switching from home to foreign produced tradable goods. As non-tradable goods consumption constitutes a large share in overall consumption, the increase in the consumer price index is to a large extend driven by the higher producer price inflation rate in that sector. Monetary policy dampens the inflationary pressure by an interest rate increase. As before, the response under heterogeneity

additional hour of work.

is closer to the 'first-best' (i.e. when monetary policy would be neutral due to price flexibility). In case it is assumed that higher heterogeneity is obtained only in the Home industry sector, the real exchange rate appreciation is more muted. Overall nominal rigidity at Home has decreased less, such that the external real exchange rate adjusts more inertially. The difference, when moving from the homogenous to the heterogenous union is then minor, as the right panel of figure 2.6 shows.

2.5 Experiment I: Main Welfare Implications of Member State Heterogeneity

2.5.1 Welfare Costs from Business Cycles in General

In order to evaluate the costs from member state heterogeneity in the four scenarios and to derive policy recommendations regarding structural reform priorities, we set up a welfare framework that results from the structural equations of the model. We extend the standard public finance approach used in Benigno (2004) to assess the efficiency losses that can arise in case monetary policy targets aggregates only. The advantage of an optimisation-based approach is that the specification of individuals' decision problems that is used to explain the effects of monetary policy can also be used for purposes of welfare analysis, as argued in Rotemberg and Woodford (1997, p. 2). Lucas (1987) represents a seminal contribution regarding the assessment of costs that arise from business cycles along the trend growth path. Risk-averse households dislike macroeconomic volatility as fluctuations affect the amount of goods and services they get to consume. They are forced to contend with volatile and unpredictable consumption rather than stable and predictable consumption growth. The losses in utility $\mathcal{U}(\cdot)$ in (2.1) basically derive from the concavity of utility from consumption streams: For any shock of same absolute size, positive shocks to consumption that provide additional utility from increased consumption of goods and services add less additional period utility than a negative shock withdraws, as utility increases at a diminishing rate. Hence, when positive and negative shocks occur at random under a certain symmetrical distribution, volatile consumption streams will yield less utility than sure consumption streams, $\mathcal{U}(\mathcal{E}_t[C_t]) > \mathcal{E}_t[\mathcal{U}(C_t)]$. Thus, a household would be better off in a world without consumption volatility.

Risk-averse household therefore need to receive a premium ψ in order to accept volatile consumption such that

$$\mathcal{U}(\mathcal{E}_t[C_t]) = \mathcal{E}_t[\mathcal{U}((1+\psi)C_t)]$$

Concavity is linked to the Arrow-Pratt measure of relative risk aversion. For risk aversion being zero ($\rho = \rho^* = 0$), utility from consumption is linear. Hence if households are not risk-averse at all, welfare costs from business cycles would be nil in the original Lucas framework.[103] Taking a second-order Taylor approximation of $\mathcal{U}(\mathcal{E}_t[C_t])$ about steady state consumption $\mathcal{E}_t[C_t] = C$ and assuming that C_t follows a conditional log-normal distribution one obtains that

$$\psi \simeq -\frac{1}{2}\frac{\mathcal{U}''}{\mathcal{U}'}(\mathcal{E}_t[C_t])\,\sigma_C^2 = \frac{1}{2}\rho\sigma_C^2 \tag{2.104}$$

where σ_C^2 is the (reduced form) cyclical variance of realised (i.e. ex-post) consumption expenditures and $\rho \equiv -\frac{\mathcal{U}''}{\mathcal{U}'}C$. Using annual data on real per capita consumption for the U.S., Lucas found ψ to be at a low of 0.008 percent of yearly steady state consumption.[104] Individuals would therefore be willing to sacrifice no more than one-hundredth of 1 percent of their consumption to achieve macroeconomic stability (Barlevy, 2005, p. 33). Given that yearly private consumption expenditures per household in the U.S. are about \$25,500 (in constant 2004 terms), the yearly costs from business cycles would be little more than \$2 (ψC). These costs seem negligible such that the gains of eliminating cycles by stabilisation policy completely would be low. A range of papers prompted by this exercise largely corroborated the findings considering the magnitude of costs, reviewed in Lucas (2003).[105] Therefore, economic policies should rather be of structural nature, i.e. targeted at promoting economic growth rather than stabilising real consumption and output about trend. However, it has to be taken into account that the analysis of Lucas is *ex post* as the ex post developments of real consumption can be observed only, i.e. after stabilisation policies *have had* an impact. In other words, the *ex ante* volatility in consumption before stabilisation policies took place cannot be observed. Low gains therefore might illustrate that stabilisation policies over the postwar period have actually been very successful in mitigating volatility in C_t and left not much room for improvement. This point is also acknowledged in Lucas (2003) that provides the update to the original 1987 calculation. Our dynamic general equilibrium approach allows also for an *ex ante* assessment of costs based on intertemporally valid descriptions of optimal policy.

When the analysis of welfare costs is extended to include nominal and real rigidities such

[103] As the period utility function $\mathcal{U}(\cdot)$ belongs to the class of additively time separable homothetic utility functions, risk aversion is inversely related to the intertemporal elasticity of substitution. Accordingly, increasing risk aversion (increases in ρ and ρ^*) induce the households to be less willing to substitute consumption tomorrow for consumption today following interest rate changes.

[104] Lucas assumed that $\rho = 1$ and $\sigma_C^2 = 0.013^2$ where $\sigma_C^2 \equiv Var(\ln \frac{C_t}{C})$.

[105] Tallarini (2000) provides the exception. He argues that far greater values of risk aversion are needed to accord with the premium on risky equity. Consequently, estimates on costs of business cycles are found to be much larger, between 2 percent and 12 percent of lifetime consumption. See Barlevy (2005) for the discussion of that result and related findings.

that price inertia matters, as has become standard since the seminal work by Rotemberg and Woodford (1997), goods and (potentially labour market frictions) summarised by the disutility of labour effort term $\mathcal{V}(y_{J,t}, A_{J,t})$ in (2.1) enter the overall assessment of welfare consequences of business cycles. Welfare costs from fluctuations in consumption and other macroeconomic objectives might be amplified by the effect movements in the overall price level and inflation persistence can have on optimal production plans by firms and consumption plans by households. Prices should reflect relative scarcities and prices of products that are valued more should be higher than those which are valued less. Price increases in one good should lead to a decrease of demand and an increase of demand for other related goods. For firms, changes in prices across firms should reflect changes in the consumption preferences for goods and the price system should transmit these real adjustments in a transparent way. Consequently, the price level should be a numeraire and monetary policy be aimed at assuring price stability.[106] Under fully flexible prices, just the composition of the price index would change but not the price index as such, which ultimately implies a constant price level.

However, in case of rigidities some firms will be unable to reset their product price when economic conditions change (changes in real marginal costs, e.g. triggered by labour preference shocks $A_{J,t}$). Relative prices between firms within and across sectors will be distorted. As there is only a portion of firms that resets prices, the aggregate (producer) price index P_J in sector J will change, as the increase in P_J is a weighted average of increases in some specific goods prices $p_J(i)$ and other unchanged prices $p_J(j)$. Accordingly, inflation builds up. Firms that are able to readjust their price will set the price $p'_J(i)$ relative to the aggregate price index as symmetry between firms implies that the aggregate index is the average price set by all firms. Therefore $\frac{p'_J(i)}{P'_J}$ is set as a constant markup over real marginal cost. For firm j, whose price is sticky, $\frac{p_J(i)}{P'_J}$ still applies, albeit the price level has gone up from P_J to P'_J for both firms.

Movements in P_J might therefore not reflect changes in the structure of prices of goods within a sector but just reflect the effect P'_J has on $p'_J(i)$. Hence, price rigidity leads to inefficient relative price changes between any two firms, as $p'_J(i)/p_J(j) > p_J(i)/p_J(j)$. The increase in the aggregate price level will tend to decrease profits of firms that could not reset prices and output will be reduced by firms in order to decrease real marginal cost. Rule of thumb firms do not optimise at all, and they will increase the stickiness in the price index and therefore increase the costs from inflation: A firm that receives a price signal - but finds itself unable to reset prices optimally - is forced to set an inefficient relative price relative to past period's aggregate index. As the time span until the same firms is allowed to reset prices again

[106] Most of the following arguments are similar to those presented by the ECB in the discussion of the benefits of price stability, see `http://www.ecb.int/mopo/intro/html/benefits.en.html`.

can be considerable, the relative price remains inefficient for a long time. Backward-looking behaviour of some firms causes even fewer prices to be adjusted and higher inflation will lead to larger inefficient relative price changes.[107] The more rigid the sector is, i.e. the higher the degree of friction in the adjustment of relative prices following shocks, the longer it will take until the relative price ratio again reflects the 'true' relative prices and the price dispersion across firms has vanished.

Households on the other hand only observe $p'_J(i)/p_J(j)$. This measure no longer allows them to make well-informed (intratemporal) consumption decisions based on their demand functions and to allocate their income efficiently, i.e. with no other ways left to raise utility when there would be price stability. At the same time, the (inefficient) substitution effect is accompanied by a negative income effect as the nominal income of the households buys less goods and assets than before. Further, they will provide less labour, as the real wage $\frac{W_J}{P'_J}$ has decreased (firms that reset prices contribute to the increase in P'_J and firms that are not able to reset prices need to decrease real marginal cost by laying off workers). Thus, households are less willing to give up leisure in order to obtain higher consumption.[108] At the same time, the ex-ante real interest rate $i_t - \mathcal{E}_t \triangle P_{J,t+1}$ that guides the decision between consumption today and consumption tomorrow is lowered, causing current consumption to increase and portfolio holdings written in nominal terms to be devalued which provides lower consumption in the future and decreases income of lenders. Forward-looking households consume out of their lifetime wealth and an unexpected rise in $P_{J,t}$ causes an arbitrary redistribution of wealth and decreases long-run consumption possibilities. At the same time, inflation risk premia in interest rates increase and creditors will demand for higher compensation in nominal income terms for holding nominal assets. This further reduces incentives to invest in assets.[109]

As real income of all sorts (profit income in form of dividend payments, labour income, redeemed claims) is lowered, household's consumption will decrease leading to lower period utility and accordingly to a decrease in lifetime utility. Hence, the costs of business cycles considered to be 'negligible' by Lucas (1987) might become considerable, already for low degrees of nominal and real rigidities, as the analysis will show. Also, our analysis takes the role of conditional expectations about future prices explicitly into account and determines welfare costs from price distortions *ex ante* (and not *ex post*, i.e. when adjustments have taken place, as implicitly assumed in the Lucas exercise).

[107] A further aspect is due to inefficiencies that derive from firms being forced to hedge against the negative impact of inflation or deflation.

[108] There is no 'surprise' inflation motive that affects labour supply under rational expectations.

[109] Households might further be affected by the distortionary impact that increases in the aggregate price level have on income taxation ('cold progression') and social transfers (devaluation of fixed pension schemes).

Average life-time welfare $\mathcal{W}_t$ for an household at home and $\mathcal{W}_t^*$ for an household at foreign can be derived from a second-order approximation of life-time utility U_t^j (2.1) of the representative agent. A second order approximation is needed as optimal policies will not have first order effects. Average per period welfare at home w_t is obtained by aggregating over all instantaneous utility flows Home citizens receive and by averaging such that

$$w_t \equiv \frac{1}{n}\int_0^n U_t^j dj \tag{2.105}$$

w_t increases in utility from consuming the goods basket C_t and decreases in disutility from effort spent on providing labour to the home monopolistic-competitive firms. By aggregation we obtain average utility from consumption at home according to $\frac{1}{n}\int_0^n \mathcal{U}(C_t^j)dj = \mathcal{U}(C_t)$.[110] As all households can pool idiosyncratic risk to income from sectoral employment, the consumption decision is identical across all households, $C_t^j = C_t$. We hence write

$$w_t = \mathcal{U}(C_t) - \frac{1}{n}\int_0^n \mathcal{V}(y_{H,t}(h), A_{H,t})dh - \frac{1}{n}\int_0^n \mathcal{V}(y_{N,t}(h), A_{N,t})dh \tag{2.106}$$

Analogously for foreign

$$w_t^* = \mathcal{U}(C_t^*) - \frac{1}{1-n}\int_n^1 \mathcal{V}(y_{F,t}(f), A_{F,t})df - \frac{1}{1-n}\int_n^1 \mathcal{V}(y_{N^*,t}(f), A_{N^*,t})df$$

The welfare objective for each region thus corresponds to the formulation in Aoki (2001). Instead of investigating the welfare consequences of business cycle dynamics in a two-sector two-country currency union, Aoki (2001) highlights the role a sticky and flexible price production sector have for welfare of a country.[111]

Given these sources for losses in lifetime-utility for the household, one could then ask in the sense of Lucas (1987) how much would households be willing to pay to live within a world of no volatility nor uncertainty about the evolvement of the economy and where no distortions in relative prices arise from movements in the aggregate price level. Hence, welfare consequences of inefficient business cycle movements need to be determined. Also, one could ask how successful monetary policy can be in eliminating inefficient movements in the aggregate price level P_t^U by using its control over the policy instrument i_t under different assumptions about the conduct of monetary policy and under inclusion of additional policy targets. Both questions will be answered in the following.

[110] Note that $\frac{1}{n}\int_0^n \mathcal{U}(C_t^j)dj = \frac{1}{n}\int_0^n \mathcal{U}(C_t)dj = \mathcal{U}(C_t)\frac{1}{n}\int_0^n 1dj = \mathcal{U}(C_t)$.

[111] Note that as money demand does not enter w_t nor w_t^*, we assume the cashless-limit such that utility from liquidity services provided by holding money is negligible for welfare, see Woodford (2003).

2.5.2 Welfare Objective and Loss Function

Home welfare measured in steady state consumption units $\mathcal{U}_C(C)C$ is the expected present discounted value of all future utility streams w_{t+s} evaluated in t

$$\frac{\mathcal{W}_t}{\mathcal{U}_C(C)C} = \mathcal{E}_t \sum_{s=0}^{\infty} \beta^s w_{t+s}$$

Accordingly for foreign

$$\frac{\mathcal{W}_t^*}{\mathcal{U}_{C^*}(C^*)C^*} = \mathcal{E}_t \sum_{s=0}^{\infty} \beta^s w_{t+s}^*$$

Union lifetime welfare is then obtained by aggregating over all citizens in the union

$$\mathcal{W}_t^U = n\mathcal{W}_t + (1-n)\,\mathcal{W}_t^* \tag{2.107}$$

We therefore employ a Utilitarian perspective: utility can be summed across regions and the larger a region, the higher its contribution to union welfare $\mathcal{W}_t^U$. Note that in our framework, equal levels of steady state consumption and production at home and foreign will not turn out to be a long-run equilibrium result per se, such that the steady state equilibrium is not unique. This is a direct consequence of allowing for different consumption and production structures which affects steady state levels of sectoral and overall consumption. We assume that $Q = Q^* = 1$ which implies that $E = 1$ and therefore $C = C^*$ as argued in section A.3 in the appendix. We concentrate on this steady state allocation and hence restrict attention to an equilibrium where consumption levels equalise across members, such that union average consumption coincides with domestic consumption levels, as by definition $nC+(1-n)C^* \equiv C^U$, see (A.13). It is then possible to rewrite union welfare in terms of the average consumption level C^U

$$\frac{\mathcal{W}_t^U}{\mathcal{U}_{C^U}(C^U)C^U} = n\frac{\mathcal{W}_t}{\mathcal{U}_{C^U}(C^U)C^U} + (1-n)\frac{\mathcal{W}_t^*}{\mathcal{U}_{C^U}(C^U)C^U} = \mathcal{E}_t \sum_{s=0}^{\infty} \beta^s w_{t+s}^U$$

It is common in the literature to state the policy problem of maximising welfare as one of minimising loss, where per period union loss is related to w_{t+s}^U by

$$L_{t+s}^U = -w_{t+s}^U \tag{2.108}$$

Lifetime loss $\mathcal{L}_t^U$ is correspondingly given by $\mathcal{L}_t^U = -\mathcal{W}_t^U$ such that further

$$\frac{\mathcal{W}_t^U}{\mathcal{U}_{C^U}(C^U)C^U} = -\frac{\mathcal{L}_t^U}{\mathcal{U}_{C^U}(C^U)C^U} = -\mathcal{E}_t \sum_{s=0}^{\infty} \beta^s L_{t+s} \tag{2.109}$$

As the union size is equal to one and by the assumption of complete markets, total per period union loss L^U_{t+s} approximates the negative of period utility of the average citizen of the currency union. As all elements in L^U_{t+s} are scaled by marginal utility $U_C(C)C^U$, one can assess the contributions of each element in the loss function in terms of steady state consumption loss C^U. A second-order approximation of per period union welfare w^U_{t+s} is provided in appendix A.7.[112] Per period union loss L^U_{t+s} can then be written as

$$L^U_{t+s} = \begin{array}{c} -\lambda_C\left(\hat{C}^U_{t+s}\right)^2 - \lambda_C n(1-n)\left(\hat{C}_{t+s} - \hat{C}^*_{t+s}\right)^2 \\ + \sum\limits_{J\in\{H,N,F,N^*\}} \left\{\lambda_{Y_J}\left(\hat{Y}_{J,t+s} - \tilde{\Psi}_{J,t+s} - \frac{\Phi^J}{\eta_J+1}\right)^2 + \gamma_{\pi_J}\left(\lambda_{\pi_J}\pi^2_{J,t+s} + \lambda_{\triangle\pi_J}\left(\triangle\pi_{J,t+s}\right)^2\right)\right\} \\ -t.i.p. + o||3|| \end{array} \tag{2.110}$$

where deep parameters are collected in

$$\begin{aligned}
\lambda_C &= \frac{1-\rho}{2}, \quad \lambda_{Y_H} = n\frac{\eta_H+1}{2}Q^{1-\gamma}\left(n\gamma + (1-n)\gamma^*\right), \\
\lambda_{Y_F} &= (1-n)\frac{\eta_F+1}{2}Q^{*1-\gamma^*}\left(n\gamma + (1-n)\gamma^*\right), \quad \lambda_{Y_N} = n\frac{\eta_N+1}{2}\left(1-\gamma\right), \\
\lambda_{Y_{N^*}} &= (1-n)\frac{\eta_{N^*}+1}{2}\left(1-\gamma^*\right), \quad \gamma_{\pi_J} = \lambda_{Y_J}\left(\frac{\eta_J+1}{2}\right)^{-1}, \\
\lambda_{\pi_J} &= \frac{1}{2}[\rho_J^{-1} + \eta_J]\frac{\rho_J^2}{1-\beta\theta_J}\frac{\theta_J}{1-\theta_J}, \quad \lambda_{\triangle\pi_J} = \lambda_{\pi_J}\frac{1}{\theta_J}\frac{\varpi_J}{1-\varpi_J}
\end{aligned}$$

for $J = H, N, F, N^*$. The t.i.p. part contains terms independent of policy (e.g. the negative of the steady state level of welfare and combinations of structural shocks). Products of flex-price deviations are not policy relevant either. As these measures are by assumption only disturbed by real shocks that cannot be influenced by policy, they are put in the t.i.p. part as well. $\tilde{\Psi}_{J,t}$ comprise real disturbances to supply $S_{J,t}$ and demand $g_{J,t}$ that affect sectoral output J under flexible prices and therefore cannot be influenced by monetary policy

$$\tilde{\Psi}_{J,t} = \frac{\eta_J S_{J,t} + g_{J,t}}{\eta_J + 1} \tag{2.111}$$

where $J = H, N, F, N^*$. The expressions $\frac{\Phi^J}{\eta_J+1}$ in (2.110) describe the steady state distortion that arises from monopolistic competition. If $\Phi^J > 0$, fiscal policy cannot fully offset the distortion and a gap between inefficient and efficient steady state allocations builds up, such

[112] Loss functions discussed in the literature can be obtained as special cases of (2.110). Setting the share of backward-looking firms and service producing firms to zero, $\varpi_H = \varpi_F = 0$ and $\gamma = \gamma^* = 1$, and equalising deep parameters guiding risk aversion and disutility from labour supply across regions, we obtain the Benigno (2004) specification. Setting n to 1 we obtain the closed economy two sector model. In case there are no movements in the aggregate price index, the inflation terms disappear in the loss function.

that $X < X^{eff}$. Then an inflationary bias would arise in equilibrium: The central bank would be tempted to trigger a surprise inflation to increase steady state output towards the efficient level. Rational agents know the structure of the economy and the loss function and will foresee this motive such that there arises no effect on output but prices have risen. We abstract from this incentive, assuming that fiscal policy commits to setting

$$\Phi_J = 0 \tag{2.112}$$

which implies that the gaps between the first-best and second-best steady state allocations in output and consumption can be fully closed. Therefore

$$y_J = c = c^* = t = q = q^* = 0 \tag{2.113}$$

where $J = H, F, N, N^*$. $y_J \equiv -\ln\left(Y_J/Y_J^{eff}\right)$, $c \equiv -\ln\left(C/C^{eff}\right)$, and $c^* \equiv -\ln\left(C^*/C^{*eff}\right)$ denote deviations in steady state allocations from the efficient level. Relative price gaps in steady state are accordingly, $t \equiv -\ln\left(T/T^{eff}\right)$, $q \equiv -\ln\left(Q/Q^{eff}\right)$, and $q^* \equiv -\ln\left(Q^*/Q^{*eff}\right)$.[113] The economic meaning of (2.113) is that fiscal policy in each region is fully efficient in setting sector-specific labour subsidies τ^J in order to eliminate real rigidities. The long-run equilibrium distortion caused by monopolistic competition between firms can be fully offset and steady state labour supply decisions are as in the first best case.[114]

The consumption terms on the right hand side of (2.110) resemble terms entering the risk premium in the original Lucas exercise, see (2.104). However, due to nominal and real rigidities, structural inflation dynamic parameters ϖ_J, θ_J and price elasticities of demand ρ_J and labour supply elasticities $1/\eta_J$ contribute to potential costs also. In the absence of price rigidities and intrinsic inflation persistence (ϖ_J, $\theta_J \to 0$) and in the absence of monopolistic competition ($\rho_J \to \infty$), risk aversion ρ and volatility in real consumption will be decisive for the overall welfare losses as in the original calculation.

As risk aversion is greater than one, $\rho > 1$, period loss L^U_{t+s} is positively affected by variability in all elements shown. Deviations in sticky price developments from developments under flexible prices lead to utility losses and a non-trivial role for monetary policy in the stabilisation process arises. The first term on the right hand side of (2.110) implies that the target variable for consumption is equal to zero $\tilde{C}_t = 0$ which implies that it would be optimal

[113] See appendix A.4.4 on page 258 for details.

[114] As explained in Benigno and Woodford (2006), one could alternatively derive a second order approximation to the equilibrium conditions that allows to eliminate the linear terms in the second order approximation of the welfare criterion above.

if consumption would not fluctuate at all, in line with the Lucas (1987) result. L^U_{t+s} increases in fluctuations in sectoral output gaps $\left(\hat{Y}_{J,t} - \tilde{\Psi}_{J,t} - \frac{\Phi^J}{\eta_J+1}\right)^2$ due to the disutility experienced when substituting labour for leisure for output production. Also variability in inflation rates $\pi_{J,t}$ reduces welfare as sticky relative prices do not allow for optimal choices of households and firms as explained above. As the 'desired' inflation rates are zero due to the flexibility of prices in steady state, the policy maker wants to attain the lowest inflation rates possible in all sectors such that allocations of variables can be implemented that correspond to the flex-price equilibrium where all producer inflation rates are zero. It turns out that persistency in inflation matters as well, as $\triangle\pi_{J,t}$ terms have positive weight also.[115]

2.5.3 The Role of Price Rigidity, Country Size, and Market Structure

In the following, we investigate quantitatively the effects different sources of heterogeneity have on the various elements in the loss function (2.110). We are especially interested in the way penalty coefficients $\left\{\lambda_C, \lambda_{Y_J},\ \gamma_{\pi_J}, \lambda_{\pi_J}, \lambda_{\triangle\pi_J}\right\}$ change when moving from structural homogeneity across members to heterogeneity which might shift monetary policy priorities. Table 2.3 on page 95 lists parameter combinations of key structural parameters that illustrate the weight on regional sectoral characteristics. We focus attention on the effect heterogeneity has on the Home region, by means of numerical evaluations. We continue in employing the calibration underlying the baseline scenario as described under 2.4.[116] Accordingly, under heterogeneity, deep parameters guiding inflation dynamics and markups are assumed to differ across regions. In case of heterogenous regions (the right part of panels (1) to (3)), the degree of nominal rigidity is again reduced by assuming that price rigidity in the Home region becomes irrelevant, $\theta_H = \theta_N \to 0$, as well as that inflation persistence vanishes, $\varpi_H = \varpi_N \to 0$. As stressed above, in order to allow for welfare comparisons, the size of the Home region n remains unchanged when moving from homogeneity to heterogeneity.

We first consider the case where both countries are of equal size illustrated in panel (1) of table 2.3.[117] We observed that objectives for sectoral inflation stabilisation $\pi^2_{J,t}$ and growth rates of inflation terms $(\triangle\pi_{J,t})^2$ receive a higher weight $\lambda_{Y_J}\lambda_{\pi_J}$ and $\lambda_{Y_J}\lambda_{\triangle\pi_J}$ than consumption or sectoral output targets already for low degrees of nominal rigidity.[118] Consequently, nominal

[115] We will see below that for an empirically relevant calibration the weights $\lambda_{\triangle\pi_J}$ are indeed considerable. Inclusion of 'speed limit' terms in ad-hoc rules is therefore welfare-improving, given that inflation persistence is present in the data.

[116] It needs to be emphasised that the results are robust to different assumptions about parameters in case of heterogeneity.

[117] In case of flexibility, we again use the same calibration as when producing the impulse responses earlier. Therefore, $\theta = \theta_H = 0.05$ and $\varpi = \varpi_H = 0.05$. Values for services are calculated residually from the implied duration of price contracts D^f.

[118] For the euro area calibration, with average price duration in each region of around 3 quarters (and equal

(1) Nominal rigidity	Homogenous area		Heterogenous area	
Varying flexibility in prices	$\theta = \theta^* = 0.67$		$\theta_H = \theta_N \to 0$	
Weight on targets in L^U_{t+s}	Home	Foreign	Home	Foreign
$\gamma_{\pi_H}\lambda_{\pi_H}$	16.5	16.5	$\downarrow$	$\longleftrightarrow$
$\gamma_{\pi_H}\lambda_{\triangle\pi_H}$	44.1	44.1	$\downarrow$	$\longleftrightarrow$
$\gamma_{\pi_N}\lambda_{\pi_N}$	587.7	587.7	$\downarrow$	$\longleftrightarrow$
$\gamma_{\pi_N}\lambda_{\triangle\pi_N}$	1122.6	1122.6	$\downarrow$	$\longleftrightarrow$
Varying intrinsic inflation persistence	$\varpi = \varpi^* = 0.55$		$\varpi_H = \varpi_N \to 0$	
Weight on targets in L^U_{t+s}	Home	Foreign	Home	Foreign
$\gamma_{\pi_H}\lambda_{\pi_H}$	16.5	16.5	$\longleftrightarrow$	$\longleftrightarrow$
$\gamma_{\pi_H}\lambda_{\triangle\pi_H}$	44.1	44.1	$\downarrow$	$\longleftrightarrow$
$\gamma_{\pi_N}\lambda_{\pi_N}$	587.7	587.7	$\longleftrightarrow$	$\longleftrightarrow$
$\gamma_{\pi_N}\lambda_{\triangle\pi_N}$	1122.6	1122.6	$\downarrow$	$\longleftrightarrow$
(2) Country size, $n = 0.042$	Homogenous area		Heterogenous area	
Varying flexibility in prices	$\theta = \theta^* = 0.67$		$\theta_H = \theta_N \to 0$	
Weight on targets in L^U_{t+s}	Home	Foreign	Home	Foreign
$\gamma_{\pi_H}\lambda_{\pi_H}$	1.4	31.6	$\downarrow$	$\longleftrightarrow$
$\gamma_{\pi_H}\lambda_{\triangle\pi_H}$	3.7	84.2	$\downarrow$	$\longleftrightarrow$
$\gamma_{\pi_N}\lambda_{\pi_N}$	49.4	1129.8	$\downarrow$	$\longleftrightarrow$
$\gamma_{\pi_N}\lambda_{\triangle\pi_N}$	94.3	2158.9	$\downarrow$	$\longleftrightarrow$
Varying intrinsic inflation persistence	$\varpi = \varpi^* = 0.55$		$\varpi_H = \varpi_N \to 0$	
Weight on targets in L^U_{t+s}	Home	Foreign	Home	Foreign
$\gamma_{\pi_H}\lambda_{\pi_H}$	1.4	31.6	$\longleftrightarrow$	$\longleftrightarrow$
$\gamma_{\pi_H}\lambda_{\triangle\pi_H}$	3.7	84.2	$\downarrow$	$\longleftrightarrow$
$\gamma_{\pi_N}\lambda_{\pi_N}$	49.4	1129.8	$\longleftrightarrow$	$\longleftrightarrow$
$\gamma_{\pi_N}\lambda_{\triangle\pi_N}$	94.3	2158.9	$\downarrow$	$\longleftrightarrow$
(3) Market structure, $\rho_H = \rho_N \to \infty$	Homogenous area		Heterogenous area	
Varying flexibility in prices	$\theta = \theta^* = 0.67$		$\theta_H = \theta_N \to 0,$	
Weight on targets in L^U_{t+s}	Home	Foreign	Home	Foreign
$\gamma_{\pi_H}\lambda_{\pi_H}$	16.5	16.5	$\uparrow$	$\longleftrightarrow$
$\gamma_{\pi_H}\lambda_{\triangle\pi_H}$	44.1	44.1	$\uparrow$	$\longleftrightarrow$
$\gamma_{\pi_N}\lambda_{\pi_N}$	587.7	587.7	$\uparrow$	$\longleftrightarrow$
$\gamma_{\pi_N}\lambda_{\triangle\pi_N}$	1122.6	1122.6	$\uparrow$	$\longleftrightarrow$
Varying intrinsic inflation persistence	$\varpi = \varpi^* = 0.55$		$\varpi_H = \varpi_N \to 0$	
Weight on targets in L_{t+s}	Home	Foreign	Home	Foreign
$\gamma_{\pi_H}\lambda_{\pi_H}$	16.5	16.5	$\uparrow$	$\longleftrightarrow$
$\gamma_{\pi_H}\lambda_{\triangle\pi_H}$	44.1	44.1	$\downarrow$	$\longleftrightarrow$
$\gamma_{\pi_N}\lambda_{\pi_N}$	587.7	587.7	$\uparrow$	$\longleftrightarrow$
$\gamma_{\pi_N}\lambda_{\triangle\pi_N}$	1122.6	1122.6	$\downarrow$	$\longleftrightarrow$

Table 2.3: Weight of member states' inflation objectives in the union loss function (2.110): The role of nominal rigidity, country size, and market structure. Calibration based on baseline scenario as described in section 2.4.

and real rigidities have considerable effects for aggregate welfare that are not taken into account in the Lucas (1987) exercise as volatility in consumption can only be observed ex-post, i.e. in reduced form. In the following, we therefore only report the weight on inflation objectives $\pi^2_{J,t}$ and $(\triangle\pi_{J,t})^2$. Our finding yields supports for the idea that monetary policy should mainly be concerned with securing price stability on the aggregate *and* sectoral level.

As we set aggregate price rigidity in accordance with the available estimates and as we varied industry rigidity in order to match aggregate inflation rates, price rigidity is much higher in services in the baseline scenario as described above. Accordingly the weight of service price inflation is much larger than that of industry. In case Home is price-flexible in both sectors ($\theta_H = \theta_N \to 0$) presented in the right part of panel (1) of table 2.3, the weight on inflation terms $\lambda_{Y_J}\lambda_{\pi_J}$ decreases in the objective function for both sectors at home, as expected. As the share of services is higher in gross value added, the weight on service inflation and service inflation growth $\lambda_{Y_J}\lambda_{\triangle\pi_J}$ is higher in total than for industry. Inflation persistence in a sector (rationalised by the presence of rule-of-thumb firms) increases the weight of the inflation objective for that sector which is also found in Benigno and López-Salido (2006). Accordingly when assuming that most firms are optimising ($\varpi \to 0$), the weights of speed limit terms are reduced, as the lower part of panel (1) in table 2.3 shows. It also becomes clear that rule-of-thumb firms leave the weight on current inflation unaffected but only affect the acceleration of inflation.

The effect of regional size n for optimal objectives for monetary policy is illustrated in panel (2) of table 2.3 on page 95. Asymmetry in size across regions is of special relevance when facing enlargement of the euro area to the east. The weight of the Home region is such, that it resembles the weight new EU members would have in the enlarged euro area while all other structural parameters are same across regions. Consequently, the effects of structural heterogeneity in the case that both country groups differ largely in economic size can be investigated. One observes that the smaller region receives lower attention than the larger region, as is the case when only aggregate developments are targeted. As the foreign region is now of higher importance for the policy maker, the weights of foreign objectives increase. One can also investigate, whether it would be rewarded for the small region to be closer to the first best, i.e. if it is possible to receive higher weight in union monetary policy when price rigidity induced by the H region decreases, such that $\theta \to 0$. It turns out that it is optimal to underrepresent the smaller, more efficient region. Therefore, optimal policy is more concerned about sectors that exhibit a higher share of backward-looking firms, in this case

and lower duration in industry sectors), the weight on $\hat{C}$ is 1.0, the weight on regional gross value added $\hat{Y}_J - \tilde{\Psi}_J$ is 0.4 for industry goods as well as 0.6 for services.

foreign. The same conclusions carry over when turning to the effect of reducing endogenous inflation persistence at home ($\varpi \to 0$), such that all firms are forward-looking in both sectors of the home economy. As a result lower overall price duration at home will not be associated with higher weight given that regions only differ in nominal rigidity, as the Benigno (2004) proposition 4 suggests.[119]

In panel (3) of table 2.3 we present results in case Home turns competitive. Thus low price rigidity is coupled with a high degree of product market deregulation when moving from homogeneity to heterogeneity ($\rho_H,\ \rho_N\ \to\ \infty$). Again, regions are assumed to be equally sized.[120] Price elasticities ρ_J affect welfare as price dispersion terms are derived from sectoral demand functions (2.53) and (2.54). We see that for the same degree of nominal rigidities across sectors, the weight on inflation developments in the more competitive sectors (the sectors with the lower markups and higher price elasticity) is *increased* substantially. The role of competition is therefore contrary to the role of price rigidity: Lower product market regulation, that comes along with a lower degree of heterogeneity of goods supplied is always 'rewarded' by a higher weight in common monetary policy. Turning to the lower panel of the same table, one observes that increasing the number of forward-looking firms would reduce the weight of speed limit terms such that lower inflation persistence overcompensates the increased weight.

2.5.4 The Role of the Composition of the Member State HICP

Inspecting L_t, one observes that welfare maximisation of household's utility implies that the policy maker cares more about developments that are of higher importance in the member state's consumer basket, reflected by a high consumption weight γ, γ^* for tradables and $1-\gamma$, $1-\gamma^*$ for non-tradables. Due to the symmetric shape of the loss function, for the same degree of rigidity across sectors within a country, the sector with higher weight in the CPI will receive higher weight which directly becomes clear from inspecting penalty coefficients λ_{Y_J}. However at the same time, price rigidity and inflation persistency matter also, which follows from inspecting partial derivatives $\frac{\partial \lambda_{\pi_J}}{\partial \theta_J} > 0$, $\frac{\partial \lambda_{\triangle \pi_J}}{\partial \theta_J} > 0$ and $\frac{\partial \lambda_{\triangle \pi_J}}{\partial \varpi_J} > 0$. It becomes clear that there are two - potentially - opposing forces that determine the overall weight of sectoral inflation rates in (2.110): the degree of nominal rigidity and preferences about goods as expressed in the harmonised consumption basket on the member state level. As a result, it might be the case that rigidity is most crucial in sectors that are of low weight in consumption. Welfare losses arising from developments in these sectors would then be negligible and trigger

[119] As a reminder, the proposition states that "[i]f prices are sticky in both regions and if monetary authority can commit only to the class of the 'inflation targeting' policies, then it is optimal to give higher weight to the region with higher degree of nominal rigidity".

[120] We thus set $\rho_H = \rho_N = 20$ in case Home is competitive.

no concern of optimally chosen policy. Consequently, nominal rigidity and price duration that derive from weighing price developments from the supply side (where shares in gross value added, $\frac{Y_J}{Y}$ and $\frac{Y_J}{Y^*}$, serve as weights) might provide different signals of price pressures than measures based on weights in the HICP from the demand side, γ and γ^*. We aim at exploring this trade-off quantitatively in the following.

The upper panel of figure 2.7 on page 101 inspects the impact of sectoral heterogeneity in price-setting at Home. The setup is symmetric where $\gamma = \gamma^* = n = 0.5$ is assumed such that tradable consumption has same expenditure weight, $\frac{P_T C_T}{PC} = \frac{P_T^* C_T^*}{P^* C^*}$. The loss function (2.110) is plotted where (only) price rigidity in the Home services sector is varied whereas price rigidity in industry production is held constant at $\theta_H = 2/3$.[121] Consequently, the weight on developments in the industry sector in the loss function will not change. One obtains that the policymaker puts always higher weight on the more price rigid sector, i.e. on H in case $\theta_N < \theta_H$ and on N in case $\theta_N > \theta_H$. However as becomes clear from the lower panel, as the share of services in overall consumption is increased from 50% to 80%, such that $\gamma = 0.2$, the N sector is of higher concern in L_t already for $\theta_N < \theta_H$. Consequently, the policymaker cares more about developments in the N sector albeit it is less rigid but of higher importance for households' consumption at the same time. Further, when the discrepancy between θ_H and θ_N becomes large, the 'nominal rigidity effect' will dominate the 'preference effect' such that the standard Benigno (2004) result is obtained. The same qualitative conclusions apply in case we vary the degree of intrinsic inflation persistence ϖ_N but keep ϖ_H constant such that the speed limit term in the N sector will vary.[122]

As a result, it can be welfare improving to tilt monetary policy towards the *less* price-rigid sector, given that the private sector favours the consumption of that category of goods considerably in the overall basket. The result obtained here is new and stresses the importance of taking the composition of GDP seriously when analysing rigidity in prices across members of a currency union: There is no one-to-one link between price rigidity and the weight in a central bank's loss function when one accounts for the fact that gross value added is composed of different types of goods that can be consumed at different proportions across regions.

In figure 2.8 on page 101, we explore whether as a consequence the weight of a region can actually increase in L_t even in case that overall nominal rigidity $\frac{1}{1-\theta}\frac{1}{1-\varpi}$ is lower than in the other member. We will show that this is indeed the case. We therefore directly challenge the result found in the related literature that monetary policy will generally stand by the region

[121] We normalised all ex-ante conditional variances (the date 0 expectations of quadratic terms in union loss (2.110)) to 1, for the ease of exposition.
[122] We set $\varpi_H = \varpi_F = 40\%$.

with the higher aggregate rigidity be it due to price rigidity or inflation persistence.

In the left part of figure 2.8, we restate results for the optimal conduct of monetary policy in case of different degrees of price rigidity across regions obtained in Benigno (2004) which serves as a consistency check between the two-sector and one-sector frameworks before moving on. Namely, in case of price stickiness in both regions, it is optimal to give higher weight to the region with the higher degree of nominal rigidity, see also proposition 4 in Benigno (2004). In order to reproduce the one-sector setup, we assume that consumption baskets at Home and Foreign only contain tradables, such that $\gamma = \gamma^* = 1$ which also implies that service production is non-existent, $\frac{P_N Y_N}{PY} = \frac{P_{N^*} Y_{N^*}}{P^* Y^*} = 0$ (the union is closed). Accordingly, price duration in each country is only determined by rigidity in tradables. Increasing aggregate price rigidity at Home is then obtained by increasing $\theta = \theta_H$. As intrinsic inflation persistence is of no role in the cited model, we assume it is absent, $\varpi_J = 0$. Accordingly, both regions have price setters that behave in a forward-looking manner. One observes that our framework reproduces the results found in Benigno (2004). Namely, the region with higher nominal rigidity (the higher duration of price contracts $\frac{1}{1-\theta}$) - the case where $\theta > \theta^* = 2/3$ - receives higher weight in welfare-maximising monetary policy compared to the other region, $L_t\left(\pi_t, \triangle\pi_t\right) > L_t\left(\pi_t^*, \triangle\pi_t^*\right)$.

However, acknowledging the role services have in the composition of actual gross value added as we do here, this result is challenged, as becomes clear from the middle and lower panel of the left part of figure 2.8. In correspondence to above, we assume that average duration of goods prices in sectors H, F, N^* remains fixed, such that $\theta_H = \theta_F = \theta_{N^*} = 2/3$. We therefore make use of the assumptions made in the baseline scenario explained earlier and already used in the preceding illustrations. In the two sector framework, θ can increase by increases in price rigidity in the Home service sector θ_N. θ_N is allowed to vary between 0 and 1 as before. In consequence, the total weight on Home inflationary terms in (2.110) changes when rigidity in the home service sector increases. In case the share of industry goods in the consumption basket (and accordingly in the consumer price indices P_t and P_t^*) is same across countries, $\gamma = \gamma^* = 0.5$, it is found that the region with the higher overall rigidity will receive higher weight in the loss function. Hence the results obtained in the one-sector framework still hold. In the lower panel we investigate the case where services are of higher weight in the Home consumption basket, such that $\gamma < \gamma^*$. One observes that the weight on inflation objectives in L_t for the Home economy is increased beyond foreign for lower values of aggregate price rigidity at Home than Foreign, i.e. for $\theta_N < \theta_H = \theta_F = \theta_{N^*}$. As expenditures on these items play the dominant role in the basket, their increased weight ('consumption preferences effect') nets out the reduction in weight due to the higher price flexibility that derives from $\theta_N < \theta_{N^*}$

('nominal rigidity effect'). Home and Foreign tradable goods are consequently of much lower role for consumption at Home, but their potential lower price flexibility will further increase the weight on home objectives to foreign ones (but on a smaller scale). In total, an increase in weight on Home inflation terms in L_t can result such that the policy maker is more concerned about economic developments in the Home region. As the overall nominal rigidity at Foreign is unchanged, optimal policy implies that *the weight of the region with the lower price rigidity and consequently the lower aggregate duration of prices can increase under the welfare maximising monetary policy.*

In the right part of figure 2.8, we also challenge the result in Benigno and López-Salido (2006) that provides the extension to Benigno (2004) in case one allows for intrinsic inflation persistence, i.e. $\varpi > 0$. Namely, it is claimed by the authors that as the share of backward-looking firms increases (ϖ increases) higher weight should be given to the inflation rate in the Home region as nominal rigidity increases, see Benigno and López-Salido (2006, p. 602). Analogously to above, we vary ϖ while keeping price rigidity θ constant. In the upper panel, we again can validate their result within our framework in case non-tradables do not matter. In case of the same composition of baskets across regions, the results again carry over as well. However, in case the less persistent sector is of higher weight in the HICP in one region than in the other, union monetary policy will be more concerned about developments in that region, already for lower aggregate rigidity in that region compared to the other, indicated by $L_t\left(\pi_t, \triangle\pi_t\right) > L_t\left(\pi_t^*, \triangle\pi_t^*\right)$.

Overall, we find that expenditure weights in domestic harmonised indices of consumer prices on the member state level are crucial determinants of the weight that should be given to member states' inflation terms in the loss function. Nominal rigidity alone will not do. To illustrate the workings of this effect in general equilibrium (the case where all shocks are active and variability generated by the model is minimised by policy), figure 2.9 plots duration in the services sector at Home, D_N versus the optimum country size of the Home region again assuming that consumption preferences are tilted towards services. As only D_N is varied, D increases via (2.99) whereas D^* remains constant. One observes that welfare-maximising policy optimally takes into account the heterogeneity in the composition of the basket. Accordingly, a U-shaped relation between duration and n results: Monetary policy is more concerned about the Home region already for $D < D^*$ indicated by n above 50%.

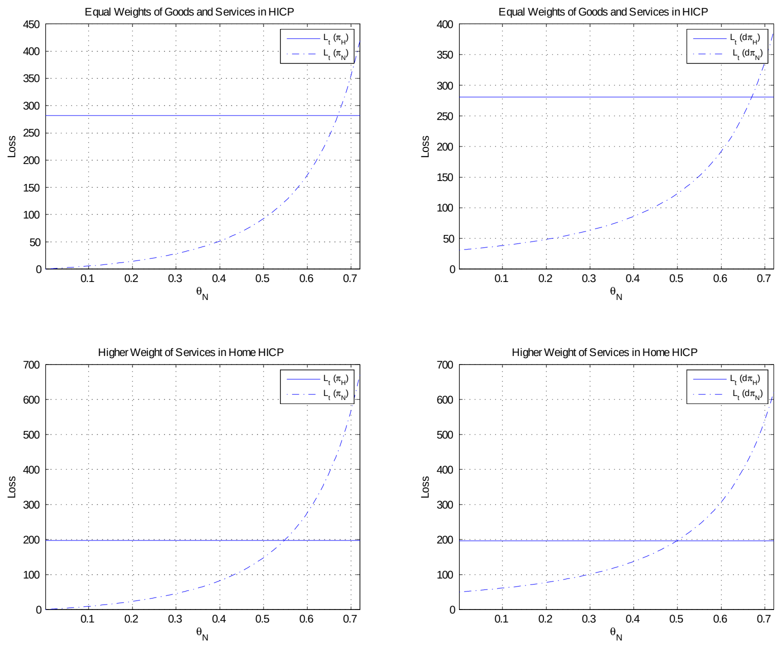

Figure 2.7: Inflation weights in the objective function (2.110) and sectoral heterogeneity at Home. Upper panel: $\gamma = 0.5$, lower panel: $\gamma = 0.2$.

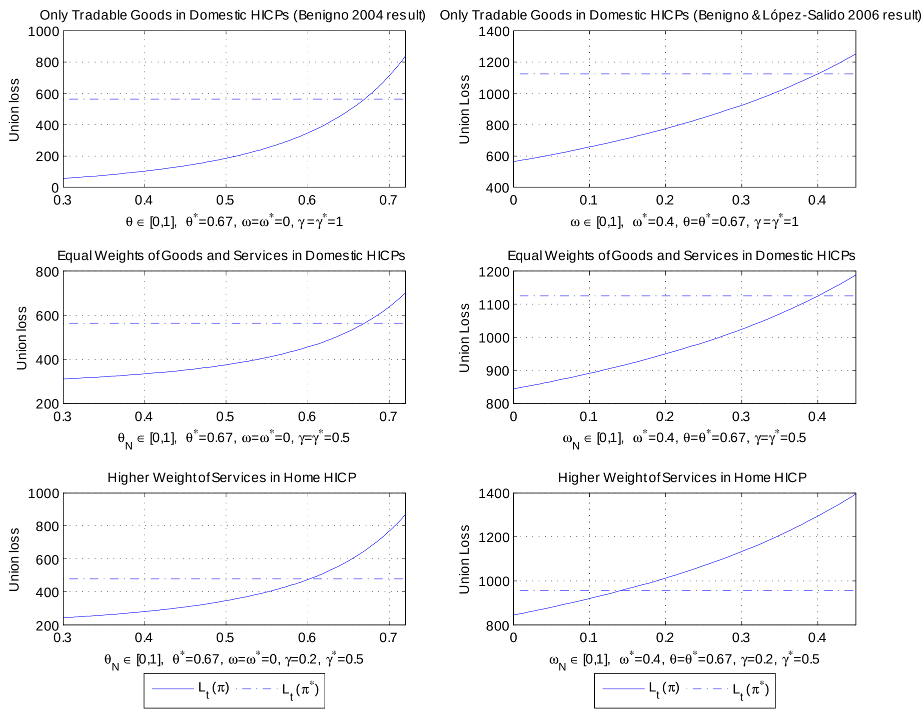

Figure 2.8: Nominal rigidity, the composition of the domestic HICP, and optimal policy. Left panel: varying price rigidity θ, right panel: varying inflation persistence ω.

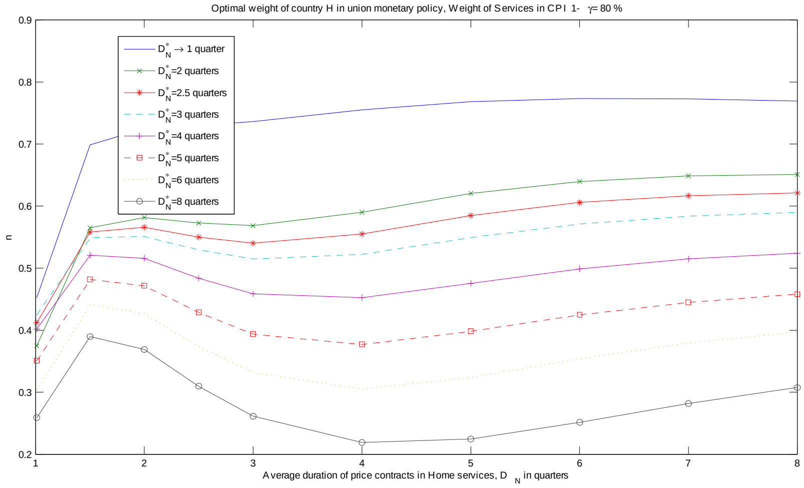

Figure 2.9: Price duration, composition of the HICP and optimal monetary policy.

2.6 Experiment II: The Optimal Course of Monetary Policy under Heterogeneity

2.6.1 Optimal Linear and Optimal Simple Policy

Equipped with the microfounded welfare objective and having investigated the basic mechanisms intrinsic in a two-region two-sector welfare function, we calculate optimal policy under various sources of member state heterogeneity and benchmark actual policy against the first-best outcome. The first best outcome refers to the case where nominal and real rigidities are absent such that all economic fluctuations are Pareto efficient and prices are flexible. The benevolent planner maximises union welfare subject to the constraints the union economy faces, i.e. the first order conditions that characterise the optimal behaviour of firms and households and respecting aggregate resource constraints. The instrument choice for the benevolent, omnipotent planner is essentially only limited by the number of endogenous variables. In the following, we will assume that the policy maker under the optimal policy can directly set relative prices $\{Q_t, Q_t^*, T_t\}$ (which also determines the external real exchange rate E_t) as well as the policy rate i_t. As there are still more objectives in (2.110) than instruments, also the optimal policy will not yield perfect stabilisation following shocks.

Optimal policy under full commitment can basically be calculated as Ramsey optimal policy or as policy from a 'timeless perspective', a term introduced by Woodford (1999). When solving for the Ramsey Policy, the policy-maker optimises once and for all at date zero and will stick to the implications of the optimal plan subsequently. One needs to take into account, that it is known since Kydland and Prescott (1977), that such a policy is generally time-inconsistent. Time-inconsistency implies that the policy maker faces the same incentives in changing policy once a period has arrived as in time zero when deciding upon the intertemporal problem. It is tempting not to honour promises made in the past but rather reoptimise by responding to the current state of the economy optimally. Two sources of distortions can therefore arise. An inflationary bias of the Barro and Gordon (1983) type as well as a stabilisation bias due to the inability to affect inflationary expectations under time-inconsistent policies that forces excessive stabilisation. The inflationary bias might result from an overly ambitious (steady state) output target of the central bank such that the target is not given by $\tilde{Y}_{J,t}$, but some value $\tilde{Y}_{J,t}+k_J$. In our setting $k_J = -\frac{\Phi^J}{\eta_J+1}$ shows up in the objective function (2.110), where Φ_J is a measure of the inefficiency, i.e. the equilibrium distortion due to monopolistic competition. Hence the policy maker would be tempted after date zero to renege on its plan and create surprise inflation in order to bring output closer to its first best outcome $\tilde{Y}_{J,t}$. However our assumptions about fiscal policy, namely that the sector-specific labour supply subsidy can fully alleviate from the distortion in steady state such that $\Phi_J = 0$ eliminates the inflationary bias and output gap targeting focusses on minimising the variability of $\hat{Y}_{J,t} - \tilde{Y}_{J,t}$. It is therefore possible to calculate the Ramsey policy without running into time-inconsistency problems from this source. We therefore need the governments to commit to implementing the fiscal scheme.

When policy cannot commit to following the optimal plan (or any other rule), it will choose policy in a discretionary way, i.e. by re-optimising (2.110) at the beginning of each period for the observed state of the economy. Therefore if commitment is not possible, the result under the fully optimal policy boils down to discretionary stabilisation. However, in such a setting, the decisions of the central bank at date t do not bind at any future dates and private sector's expectations about future inflation cannot be affected (see also Walsh 2003, p. 526). In case commitment to the optimal plan is not feasible or the central bank announces to follow discretionary stabilisation, there is room for a stabilisation bias. Given that the central bank cannot commit to follow policy through time, the private sector will rationally expect such behavior and monetary policy cannot affect future expectations about the policy stance. Therefore inflation expectations will no longer affect the path of current inflation as the inflation target is no longer nominally anchored. Hence, the central bank will need to counteract cost shocks more aggressively as it cannot smooth the stabilisation of the shock through time.

We assume in the following that these commitment problems will not materialise such that the central bank commits to minimising the intertemporal loss function (2.108) once and for all. Following the intertemporal plan requires that policy is chosen at date zero, no re-optimising takes place in subsequent periods and the rule implied by the initial optimisation will be executed whatever event occurs.[123] Hence, the constraints faced at date zero are the same as in all subsequent periods and one obtains a result as when assuming commitment under the time-less perspective.[124] Whereas in case of commitment to a Ramsey-policy one *assumes* that no re-optimising takes places and the resulting monetary policy will always be implemented, a timeless perspective policy imposes history dependence by introducing shadow costs of reneging to policy explicitly. This can either be performed as in Woodford (1999), by assuming that the date zero problem never occurred, i.e. the constraints that arise as a first order condition after the initial period were valid from the infinite past.[125] Alternatively, one can introduce date zero Lagrange multipliers that are non-zero explicitly.[126]

As our welfare function and macroeconomic set-up exhibits the linear-quadratic structure with symmetric information as preferences over goal variables are quadratic and the transmission mechanism is linear, optimal policy will satisfy the principle of certainty equivalence: The optimal response to (the best estimate of) unexpected shocks is same as if the shocks would be perfectly observed (see also Walsh 2004, p. 3). In other words, the optimal linear regulator (OLR) will only depend on endogenous variables but is independent of structural shocks. For the optimal simple rule (OSR), certainty equivalence will however not hold, as choosing policy coefficients optimally depends on the minimisation of conditional / unconditional variances of objectives in the loss function which themselves depend on structural shocks and their persistence. In the following, we illustrate how optimal policy and the optimal simple rule can be derived in general and how the methods are applied to our setting.

Optimal Policy with full commitment

The full commitment solution can be found by solving a stochastic linear regulator problem. The optimal linear regulator can be written as

$$\frac{\mathcal{L}^U}{\mathcal{U}_{C^U}(C^U)C^U} = \mathcal{E}_t \sum_{s=0}^{\infty} \beta^s L_{t+s}^U = \min_{\mathbf{u}} \mathcal{E}_0 \sum_{t=0} \beta^t \left(\boldsymbol{y}_t' \boldsymbol{W}_{11} \boldsymbol{y}_t + 2\boldsymbol{y}_t' \boldsymbol{W}_{12} \boldsymbol{u}_t + \boldsymbol{u}_t' \boldsymbol{W}_{22} \boldsymbol{u}_t \right) \quad (2.114)$$

[123] When the central bank re-optimises each period, the policy is called discretionary.

[124] Note that minimising the expected average loss is not the same as assuming that the period loss function is minimised any period as under discretionary minimisation. In the latter, the central bank cannot affect future expectations about endogenous variables, see Svensson (2007, p. 24) for details.

[125] There, the optimal policy is solved for by assuming that the first order conditions are time-invariant. History dependance is introduced by imposing date zero information as additional constraint the policy maker faces and opportunity costs from reneging of once optimal plans arise (shadow prices). This approach imposes a set of state-contingent commitments that prevent the policymaker from exploiting future private expectations along the path of the endogenous variables implied by the optimal policy plan. The resulting set of first order conditions can then be interpreted as the policy. See Ferrero (2005) for further details.

[126] For a further discussion, see the lecture notes by Svensson (2007) and the contribution by Juillard and Pelgrin (2007).

subject to the constraints posed by the equilibrium macroeconomic relationships

$$\boldsymbol{A}_1\mathcal{E}_t\boldsymbol{y}_{t+1} + \boldsymbol{A}_2\boldsymbol{y}_t + \boldsymbol{A}_3\boldsymbol{y}_{t-1} + \boldsymbol{B}\boldsymbol{u}_t + \boldsymbol{C}\boldsymbol{e}_t = \boldsymbol{0} \tag{2.115}$$

where $\boldsymbol{y}_t$ is a column vector of all endogenous variables.[127] $\boldsymbol{u}_t$ is the vector of policy instruments where in our case $\boldsymbol{u}_t = \left(\hat{\imath}, \hat{Q}, \hat{Q}^*, \hat{T}, 0\ldots, 0\right)'$. $\boldsymbol{e}_{t+1}$ is the vector of innovations to structural shocks where

$$\boldsymbol{e}_t = \left(\epsilon_{S_{H,t}}, \epsilon_{S_{N,t}}, \epsilon_{S_{F,t}}, \epsilon_{S_{F^*,t}}, \epsilon_{S_{T,t}}, \epsilon_{G_{H,t}}, \epsilon_{G_{N,t}}, \epsilon_{G_{F,t}}, \epsilon_{G_{N^*,t}}, \eta_t^i\right)$$

The optimal linear regulator implements the best possible linear policy intertemporally. However it is time inconsistent as stressed in the preceding paragraph and one needs to assume that the central bank optimises once and for all at date 0 in order to apply the full commitment solution. $\boldsymbol{W}_{11}$ denotes variances/covariances (quadratic objectives), $\boldsymbol{W}_{12}$ summarises covariance terms between other endogenous variables and instruments, and $\boldsymbol{W}_{22}$ allows for standard errors in instruments. The loss criterion (2.108) can be expressed in this form. Note that, in order for a maximum of the preference function to exist, $\boldsymbol{W}_{11}$ needs to be negative semi-definite.

Following Rudebusch and Svensson (1998, p. 30), by applying unconditional expectations, the solution to the minimisation problem (2.114) becomes

$$\mathcal{E}\frac{\mathcal{L}^{U,\min}}{\mathcal{U}_{C^U}(C^U)C^U} = -\mathcal{E}\frac{\mathcal{W}^{U,\min}}{\mathcal{U}_{C^U}(C^U)C^U} = \mathcal{E}\left[\hat{\boldsymbol{X}}_t'\boldsymbol{V}\hat{\boldsymbol{X}}_t\right] + \frac{\beta}{1-\beta}\mathsf{trace}\left(\boldsymbol{V}\sum\nolimits_{ee}\right) \tag{2.116}$$

where $\hat{\boldsymbol{X}}_t$ is the vector of variables of the state space form of the above system, $\boldsymbol{V}$ is a square matrix that solves the value function iteration of the model in its state space form, and $\sum_{ee} = \mathcal{E}\left[\boldsymbol{s}_t\boldsymbol{s}_t'\right]$ is the unconditional covariance matrix of the structural disturbances in the model . As β is close to one, once can inspect the case where the discount factor attains its limit such that the loss reported provides an upper bound of the loss actually experienced. This will have several benefits in providing interpretable results as welfare implications can then be linked to unconditional variability in target variables in the loss function. If one directly imposes $\beta = 1$ on (2.116), the solution, however, becomes unbounded. Following Svensson (2007) and Rudebusch and Svensson (1998), rescaling the lifetime loss function by the factor $(1-\beta)$, such

[127] Note that $S_{J,t}$ and $G_{J,t}$ are endogenous variables within the `dynare` nomenclatura whereas innovations to these structural shocks are referred to as exogenous shocks.

that

$$\mathcal{E}\left[\mathcal{E}_0 \sum_{s=0}^{\infty} (1-\beta)\,\beta^s L_{t+s}^U\right] \tag{2.117}$$

will not affect the optimal policy rule, as the maximand is unaffected (it will however rescale the welfare outcome).[128] In that case, we can go further, as then first order terms in (2.116) disappear and the solution condenses to

$$\mathcal{E}\left[L_t^U\right] = \mathsf{trace}\left[\boldsymbol{V}\textstyle\sum_{ee}\right] = \mathsf{trace}\left[\boldsymbol{W}_{11}\boldsymbol{y}_t^{opt}\boldsymbol{y}_t^{opt\prime}\right] = -\mathcal{E}\left[w_{t+s}^U\right] \tag{2.118}$$

$\mathcal{E}\left[L_t^U\right]$ denotes the average expected per period loss for the representative household in the union. For β approaching one, the subjective rate of time preference ς approaches zero and consumption streams far in the future are of equal importance for lifetime utility as current consumption. Accordingly, the dependence on initial conditions $\{T_0, Q_0, Q_0^*\}$ is removed from the system, where dependence on initial conditions could result in an inflationary bias.

As $\mathcal{E}\left[\boldsymbol{y}_t^{opt}\boldsymbol{y}_t^{opt\prime}\right]$ denotes the unconditional variance/covariance matrix of endogenous variables, the period average loss is in fact obtained by summing up unconditional variances of the elements in (2.110) weighted by their respective penalty coefficients derived from optimising behaviour of households. Minimising welfare under the policy therefore amounts to minimising unconditional variances of variables that appear as stabilisation targets in the welfare function. We can recover lifetime welfare by remembering that we rescaled the lifetime utility function by $(1-\beta)$ before calculating optimal policy

$$\frac{\mathcal{W}^{U,\min}}{\mathcal{U}_{C^U}(C^U)C^U} < \frac{1}{1-\beta}\mathcal{E}\left[w_{t+s}^U\right] = \mathcal{E}\frac{\mathcal{W}^U}{\mathcal{U}_{C^U}(C^U)C^U} \tag{2.119}$$

where $\beta < 1$ is the discount factor used in the model. Evaluating $\mathcal{E}\left[w_{t+s}^U\right]$ provides an upper limit for the per-period 'true' loss based on evaluating lifetime welfare.

Optimised Simple Rule

Whereas the full commitment policy provides the natural benchmark 'how good one can get' in monetary stabilisation policy, the optimal plan may be difficult to implement and communicate. Also under the OLR, aggregate demand can be affected directly by choosing optimal paths for

[128] The formula is an application of the fact that we can measure the unconditional variability in a random variable X by the statistic

$$v(X) = \mathcal{E}\left[(1-\beta)\sum_{t=0}^{\infty}\beta^t\mathcal{E}_0 X_t^2\right]$$

where except for discounting, $v(X)$ corresponds to the unconditional variance of X_t, see also Benigno (2000, p. 40). Hence $v(X)$ can in general be used for evaluating the average per period contributions of the elements in period loss L_{t+s} (2.110) that sum over time to form overall lifetime loss $\mathcal{L}^U$.

instruments $\{Q_t, Q_t^*, T_t\}$, whereas it seems clear that especially the transmission mechanism from policy rate changes to actual choices of households is important in the policy debate. The optimal rule approach can be understood as the solution of an optimal policy problem of minimising fluctuations (deviations from trend) in objectives relevant for the policy maker expressed by (2.110) subject to a given instrument rule like (2.87). The interest rate that implements the optimal plan then becomes a function of the 'deep' parameters of the model. Benigno (2004) instead defines the class of 'inflation targeting' policies as policies in which the union monetary authority aims at stabilising a weighted average of the region-specific inflation rates as

$$\delta \pi_t + (1 - \delta) \pi_t^* = 0 \tag{2.120}$$

where for $\delta = n$, the union CPI inflation rate is set to zero at all dates. In that setting, monetary policy can influence the inflation rates directly. However, our framework explicitly accounts for the transmission of monetary policy from the monetary to the real side of the economy via the (three month) money market rate i_t. Imposing (2.87) as the instrument to minimise (2.110) in fact introduces an additional structural relation as a first order condition in the optimal policy problem. (2.120) instead sets a definition within the set of optimality conditions of the optimal policy problem equal to zero.

Under both cases, structural differences between economies and the impact on fluctuations are taken into account whereas a monetary policy rule aimed at targeting aggregates only relies on values for penalty coefficient that are 'ad hoc', even when estimated. Accordingly, in the baseline case, the weight of each country in the policy rule is determined by n for the home region and $1 - n$ for the foreign region, whereas in the optimised rule the weight is determined by the region's contribution to minimising the welfare loss. As explained in experiment I, this contribution depends on both the expenditure and production side of GDP and sectoral inflation developments. Explicitly considering the transmission process here makes necessary the selection of optimal values of response coefficients in the rule $\{r_i, r_\pi, r_{\triangle\pi}, r_Y, r_{\triangle Y}\}$ and the country weight of the home region n. These are set in order to minimise the intertemporal loss of the union central bank, given by (2.110).

Therefore, the minimisation problem reads

$$\min_{\gamma} \mathcal{E}_0 \sum_{t=0} \beta^t \boldsymbol{y}_t' \boldsymbol{W}_{11} \boldsymbol{y}_t = \min_{\gamma} \sum_{t=0} \beta^t \mathsf{trace} \left(\boldsymbol{W}_{11} \mathcal{E}_0 \left[\boldsymbol{y}_t \boldsymbol{y}_t' \right] \right) \tag{2.121}$$

subject to the constraints posed by the equilibrium relationships of the structural macroeconometric model

$$\boldsymbol{A}_1 E_t \boldsymbol{y}_{t+1} + \boldsymbol{A}_2 \boldsymbol{y}_t + \boldsymbol{A}_3 \boldsymbol{y}_{t-1} + \boldsymbol{C}\boldsymbol{e}_t = \boldsymbol{0} \tag{2.122}$$

Hence the OSR is nested within the OLR when no instruments are available such that $\boldsymbol{u}_t = (0, 0, \ldots, 0)'$. Optimisation amounts to choosing optimally set coefficients on some parameters linked to endogenous variables summarised in $\boldsymbol{\gamma}$. $\boldsymbol{\gamma}$ represents the vector of parameters that are contained in the policy rule over which it is maximised. In our case $\boldsymbol{\gamma} = (r_i, r_\pi, r_{\triangle\pi}, r_Y, r_{\triangle Y}, n)'$, such that interest smoothing, inflation and output gap stabilisation, and the growth rates in the inflation rate and the output gap will be of concern for policy. Also the weighting of overall country contributions in making up aggregate contributions can vary.

It also becomes clear that certainty equivalence will not hold for the optimised rule, as the minimisation will not yield policies that only depend on the expected values of endogenous variables but rather also on the variances and covariances between endogenous variables collected in $\boldsymbol{y}_t$. As described in the preceding section, we will again evaluate the stabilisation outcome under the optimal simple rule by evaluating the weighted unconditional (minimum) covariance matrix of endogenous variables, i.e. the case where $\beta \to 1$ such that (2.117) is minimised. Again the optimal simple rule suffers from the time inconsistency problem and we still need to assume that the central bank sticks to the optimised policy rule through time when setting the interest rate.

2.6.2 Optimal Monetary Policy in the Baseline Scenario

Before determining equilibrium welfare outcomes in scenarios (2) to (4), we explore basic findings by means of impulse responses. As in section 2.4.5 on page 79, the model is calibrated according to scenario (1) such that results can be directly compared. We compare the model's outcomes when the central bank follows the Smets-Wouters rule (SWR) that proxies actual policy to the response when the policy rate is set optimally thereby choosing policy coefficients in the SWR in a loss-minimising way (OSR) and when relative prices and the policy rate can be set directly according to the optimal plan (OLR). Again, we compare the homogenous union (the union with equal rigidity and persistence across regions) with the heterogenous union (the union where Home is flexible).

Supply Shocks

Figure 2.10 illustrates the impact a shock S_H in home industry productivity has under the three monetary strategies for the homogenous union with equal nominal rigidities across members. The case where home is flexible is presented in figure 2.11. We observe that under the Smets-

Wouters rule there are large swings in regional inflation rates in the same direction and therefore also in the union CPI inflation rate. OSR and OLR can nearly perfectly stabilise inflationary developments (for the given assumptions about θ_J, ϖ_J there is a large weight on these items in the period loss function L_t). Dampening inflation allows for the highest increase in regional and overall output under the optimal rule. In the heterogenous union, the home economy can better adapt to the shock such that the drop in the home producer price inflation - which translates to a drop of the CPI inflation - is stronger and inflation returns faster to the steady state. As the shocks are correlated, the faster adjustment at Home also spills over to foreign.

Under the common supply shock S_T shown in figures A.7 and A.9 in the appendix, there arises no trade-off in stabilising regional developments in case both regions exhibit the same degree of nominal rigidity and there is comovement in CPI rates and regional sectoral and overall output. Monetary policy that targets the union CPI will accommodate the developments and both regions will benefit equally. Under the optimised rule, interest rates would be slightly increased. Optimal monetary policy would succeed in completely stabilising the CPI rates by proper adjustments in interregional relative prices and the terms of trade. Therefore, as inflation remains at its medium run target, the policy rate will not be changed at all. Again, we contrast the result to the case where Home is flexible such that adjustments at Home happen faster albeit both regions are exposed to the same shock.

The case where sectoral heterogeneity is present seems warranting a deeper exploration. As in paragraph 2.4.5, we can assume that higher flexibility is attained only in the industry sector in case members structurally differ. As before, the setup is such that aggregate price rigidity at Home θ is taken as given. Lower rigidity in the industry sector θ_H implies that rigidity in the service sector θ_N increases in order to match aggregate rigidity in a region. When comparing the outcomes under heterogeneity summarised in figures 2.11 (both sectors flexible) and 2.12 (only industry production flexible), one observes that under the Smets-Wouters rule home and foreign CPI inflation rates remain completely synchronous, as was the case when both regions exhibited the same degree of price rigidity in any sector. The intuition is that as services have much higher weight, the increased heterogeneity by the higher flexibility in the industry sector will not change the overall behaviour of CPI inflation much. Also, higher flexibility allows to better stabilise inflation rates under the OSR compared to the case where both sectors were flexible.

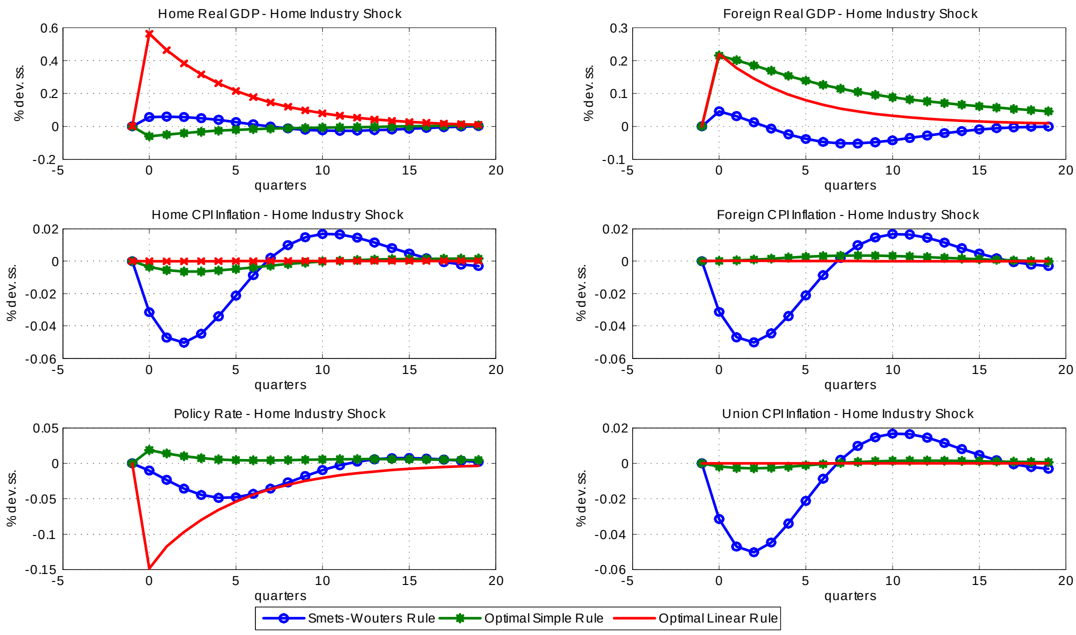

Figure 2.10: Optimal monetary stabilisation of a country-specific shock to industry productivity.

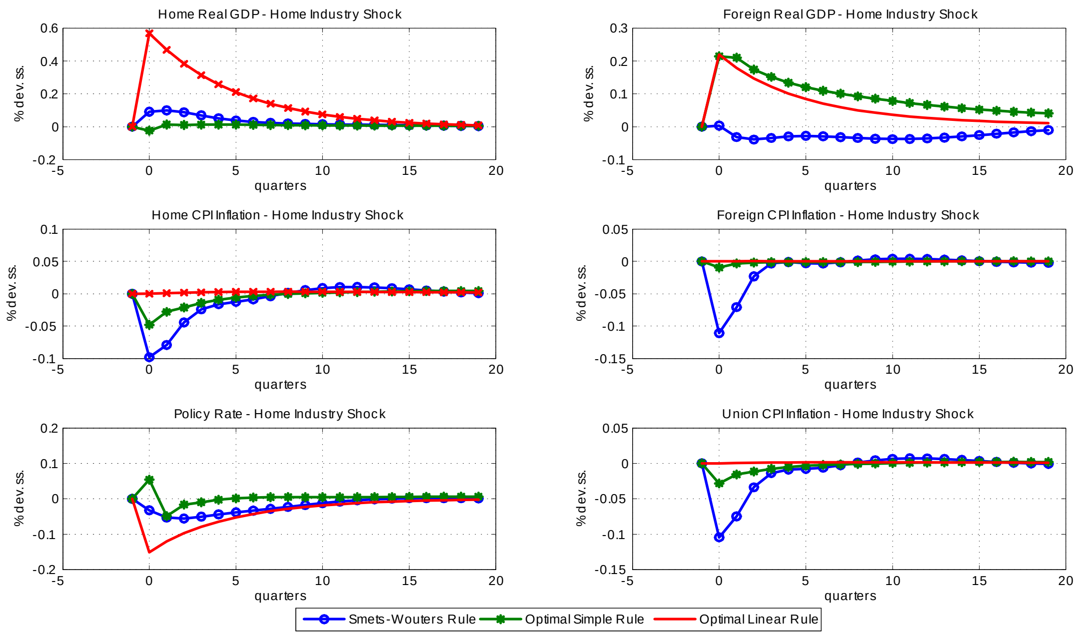

Figure 2.11: Optimal monetary stabilisation of a country-specific shock to industry productivity: Home flexible.

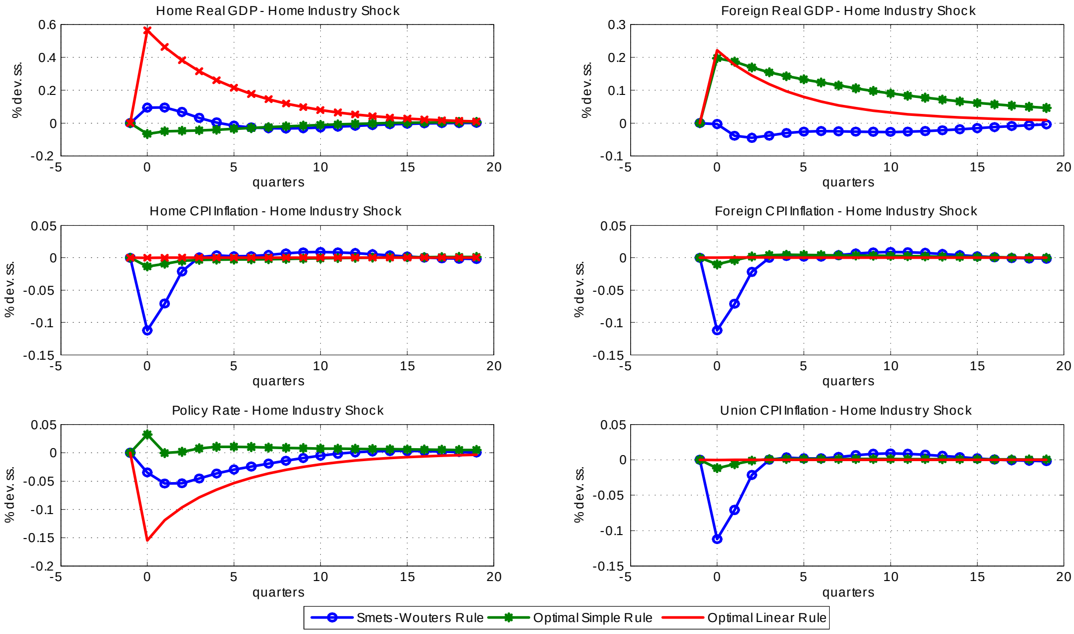

Figure 2.12: Optimal response to a country-specific supply shock under sectoral heterogeneity.

Demand Shock

Turning to fiscal spending in the Home region, monetary stabilisation based on the Smets-Wouters rule causes the CPI inflation rate at Home to remain above steady state for a long time such that the effect dies out slowly. Again, it would be optimal to eliminate the effect on regional CPI rates completely such that the union inflation rate remains unchanged, as figure 2.13 illustrates. Under the optimised rule, the effect of the shock would be distributed equally across both regions such that regional GDP increases about 0.1% equally in both regions under the OSR. As the effect of the shock on inflation rates can be reduced (OSR) or even eliminated (OLR), the policy response is lowest in case of the fully optimal policy. Turning to the effect heterogeneity has in this setup presented in figure 2.14, we observe that it remains optimal to eliminate the inflation differential between regions completely. The distribution of the shock by policy across regions is now, however, less successful in the OSR compared to the case of homogenous regions.

2.6.3 Judging the Efficiency of Monetary Stabilisation

In order to judge how well different policies perform in eliminating inefficiencies, a criterion for evaluating losses conditional on a certain policy is introduced. We follow Benigno and López-Salido (2002, p. 21) and define a consumption based welfare index based on average lifetime

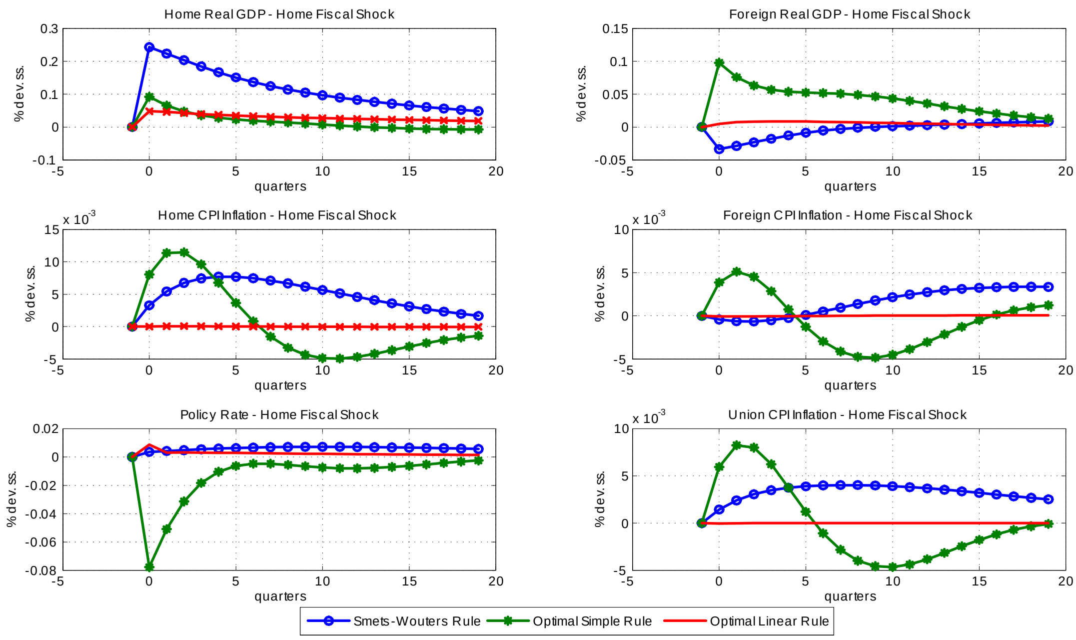

Figure 2.13: Optimal stabilisation of a fiscal shock.

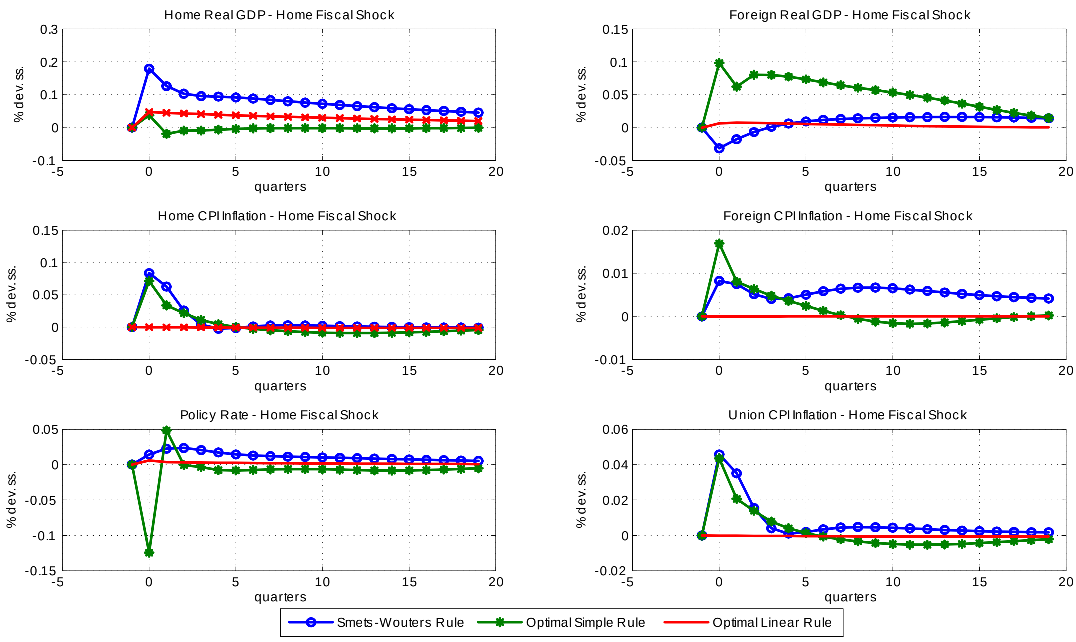

Figure 2.14: Optimal stabilisation of a fiscal shock: Home flexible.

union welfare $\mathcal{W}^U$ as

$$\begin{aligned}\chi(j) &\equiv -\mathcal{E}\left[(1-\beta)\,\mathcal{E}_0\sum_{s=0}^{\infty}\beta^s w_{t+s}^U(j)-(1-\beta)\,\mathcal{E}_0\sum_{s=0}^{\infty}\beta^s w_{t+s}^{U,C}\right]\times 100\% \\ &= -(1-\beta)\,\mathcal{E}\left[\frac{\mathcal{W}^U(j)-\mathcal{W}^{U,C}}{\mathcal{U}_{C^U}(C^U)C^U}\right]\times 100\% \end{aligned} \tag{2.123}$$

where $\chi(j)$ measures the permanent percentage shift in steady state domestic consumption units C that is lost under the policy j with respect to the efficient level for the average consumer in the union under policies $j = SWR, OSR, OLR$. We can rewrite $\chi(j)$ as the per period average welfare differential by using (2.119)

$$\chi(j) = -\mathcal{E}\left[w_t^U(j) - w_t^{U,C}\right]\times 100\% \tag{2.124}$$

As the time period in the model is one quarter, C represents the quarterly flow of consumption that goes on forever, such that $4C$ is yearly steady state consumption. The index j includes the Smets-Wouters rule (SWR), the optimised SWR rule (the OSR), and the fully optimal policy (OLR).[129] The closer policy j is to the first best, the lower will be the permanent consumption loss. By construction, $\chi(j) \geq 0$. $\mathcal{W}_t^{U,C}$ denotes the first best welfare result where economic choices by firms and households are guided by relative prices only under full foresight, i.e. the outcome in a RBC model where nominal rigidities and real rigidities are absent and all fluctuations are Pareto efficient. The efficient outcome requires that nominal and real rigidities are absent, such that prices become fully flexible and markups vanish

$$\theta_J \to 0, \qquad \omega_J \to 0, \qquad \rho_J \to \infty$$

Under real business cycles there is no cyclical activity in the aggregate price indices and all costs related to business cycles stem from fluctuations in real consumption. Optimal policy under the first best will differ from optimal policy under the second best, $\mathcal{W}^U(OLR) \neq \mathcal{W}^{U,C}$, albeit a fiscal transfer scheme is in place that removes long-run inefficiencies. The result derives from the impact, price elasticities of demand ρ_J have on inflationary terms in the loss function (2.110) in a second best world.

With $\chi(j)$ - which will be our key measure in the following to determine losses from inefficient business cycles - at hand, we can calculate the compensating variation under policy j, $cv(j)$. $cv(j)$ describes the constant euro equivalent of consumption baskets to be given to an

[129] Alternatively, we can interpret $\chi(j)$ as the increase in consumption, expressed as a percentage of period consumption in the steady state, necessary to make agents as well off in a world with nominal rigidities as in a world with perfectly flexible wages and prices, as defined in Paustian (2005).

average consumer in each scenario to be no worse off as in the absence of heterogeneity. The intuition is that households in one region of the currency union will accept heterogeneity that derives from different structural characteristics of the other region only, when additional costs in terms of steady state losses of consumption brought about increases in inefficient dynamics and spillover effects are offset. The household is therefore indifferent between homogeneity and heterogeneity in the union as long as she is compensated for the cost experienced, by receiving $cv(j)$. Analogously, if the move would yield lower losses (heterogeneity that increases price flexibility in the union, say), the household would be willing to give some of the then received consumption away to live in that union. As a result, the compensating variation would in fact be negative (χ is lower under the new regime than under the old one). From $\chi(j)$, we obtain the compensating variation as

$$cv(j) = \chi_{HET}(j) - \chi_{HOM}(j) \tag{2.125}$$

It is also useful to define a measure that allows comparing the loss that arises in case the central bank follows the interest rate rule that targets union aggregates only (the baseline Smets-Wouters rule) instead of following the loss-minimising interest rate rule (which provides the analogue to the optimal 'inflation' targeting policy in the sense of Benigno (2004, p. 315) in our context). As in Benigno (2004), we define the percentage reduction in the deadweight loss that society can obtain by using the fully optimal instrument rule OSR. The deadweight loss $DR(S)$ that arises from targeting aggregates only can be written as

$$DR(S) = \mathcal{E}\left[\frac{\mathcal{W}^{U,S}(SWR) - \mathcal{W}^{U,S}(OSR)}{\mathcal{W}^{U,S}(SWR) - \mathcal{W}^{U,S}(OLR)}\right] \tag{2.126}$$

$\mathcal{W}^{U,S}(SWR)$, $\mathcal{W}^{U,S}(OSR)$, and $\mathcal{W}^{U,S}(OLR)$ are the welfare outcomes associated respectively with the Smets-Wouters rule (SWR), the optimised Smets Wouters rule (OSR), and the optimal plan (OLR) and $S = HOM, HET$.[130]

We are now in the position to assess the overall costs from macroeconomic fluctuations that arise from disperse economic activity and inflation differentials over time in a currency union by evaluating the welfare consequences of inefficient cyclical activity. Our criteria to assess

[130] Note that by the logic of optimal policy - and as a plausibility check - the loss under the full optimal policy (OLR) needs to be smaller (in absolute terms) than the loss under the constrained optimal policy, which is here the optimal simple rule (OSR). Further, the optimal inflation targeting policy is always at least as good as the SWR rule. Hence, it always holds that

$$0 \geq -\mathcal{W}^{U}_{OLR,first\ best} \geq -\mathcal{W}^{U}_{OLR} \geq -\mathcal{W}^{U}_{OSR} \underset{<}{\geq} -\mathcal{W}^{U}_{SWR} \tag{2.127}$$

the welfare consequences from business cycle movements are both cyclical (relating to nominal rigidities and shocks) as well as structural (markups and the composition of consumption and production). Instead of focussing on impulse responses following single shocks, we perform a stochastic simulation of the model for given policies and evaluate volatility in time series that are generated by the model. Accordingly, the model is taken as data generating process where all randomness derives from innovations to structural shocks that occur over time. Weighting the moments of variables that appear in the microfounded loss function by their respective penalty coefficients implied by the microfoundations allows to obtain costs of volatility in terms of household consumption units. Also, correlation between idiosyncratic industry supply shocks as well as correlation between sectoral fiscal spending on the country level are taken into account. As each simulation can be understood as one realisation of the joint intertemporal distribution of all structural shocks, the simulation is re-run several times to wash out the effects a specific draw would have and consequently results are averaged.[131]

In the following, we judge how member state heterogeneity will be coped with under each monetary policy stance (SWR, OSR, OLR) and we assess the costs that arise from focussing attention towards aggregate developments only. We evaluate the empirically relevant scenarios outlined in section 2.4 on page 70, namely scenarios (2) to (4). We assess welfare in the *Current Euro Area* scenario where regions are grouped as Germany and the euro area excluding Germany, we then turn to the *Large Member Area*, and eventually welfare is addressed for the *Enlarged Euro Area*. In order to determine the potential role of union monetary policy to cope with structural differences of members, monetary policy either follows the benchmark rule (SWR) without taking into account welfare consequences or implements the fully optimal commitment policy (OLR). We investigate dissimilarity due to differences in price flexibility and inflation inertia as well as country size and the role of the composition of national consumption baskets. We then ask ourselves, how much needs an average euro area citizen to receive / give away in order to compensate for the losses / gains in consumption due to the dissimilarity of the other region compared to his own region under a certain monetary policy strategy. Under 2.6.4, we then address the optimal representation of member state heterogeneity in union monetary policy, thereby implementing the OSR. Accordingly, we determine the optimal weight a member should receive in the policy rule (2.87) that aims at minimising period union loss (2.110), given its overall and sectoral degree of price duration, composition of the domestic HICP as well as country size.

Losses are evaluated for the limiting case, i.e. $\beta \rightarrow 1$, as explained above. Outcomes can

[131] Alternatively one could rely on asymptotically calculated moments, obtained as solution to the Sylvester equation.

then be judged by inspecting the unconditional covariance matrix between endogenous variables of the model obtained from taking the model as data generating process under all three policies. Therefore, the reported losses provide an upper bound of the 'true' social losses experienced.[132] It has to be noted , that this procedure leaves the discount factor in the model invariant at $\beta = 0.99$. Welfare costs are assessed in terms of the permanent percentage shift in steady state domestic consumption units C, i.e. relative to the average euro area household steady state consumption flow per period. We will also report the equivalent of losses in C in money terms, expressed in euros in prices of 1995 for the EA15, by averaging yearly data for 2000-2006 of euro area gross value added at constant prices. We therefore obtain that $4P^U Y^U = €_{1995} 25934$.[133] As the time period in the model is one quarter, C and C^*, the relevant consumption flow is $P^U C^U = 25934/4 = 6484\ €_{1995}$.[134]

In the upper panel of table 2.4 we present results for the *Current Euro Area* scenario. In case there would be no heterogeneity between countries (both regions exhibit structural characteristics of Germany regarding gross value added composition, structural inflation dynamics, and consumption preferences) the results presented in the upper left panel apply. Also in the absence of heterogeneity between regions there arise welfare costs from the presence of (equal) nominal and real rigidities in the regions when policy is described by following (2.87). The costs amount to $\chi(SWR) = 0.33$ percentage points of long-run equilibrium consumption per quarter and area citizen. Commitment to the fully loss-minimising policy would make these losses from rigidities in the homogenous union negligible with $\chi(OLR) = 0.002$pp. When one takes heterogeneity between regions into account, thereby bundling other euro area countries as the Home region - presented in the right part of the upper panel - one observes a rise in welfare costs of about 0.14 percentage points (0.47 - 0.33). Therefore, the compensating variation given to German citizens to be no worse off as in case other members would have the same macroeconomic structure amounts to 0.14 percent of long run consumption. That value represents a constant money equivalent of $0.0014 \times P^U C^U = 9.1$ Euros a quarter.

The middle panel illustrates welfare effects of heterogeneity in the *Large Member Area* where the Home region accounts for 39% of the union size which represents the relative economic size

[132] We therefore set `olr_beta=1` as dynare option in the `olr()` command. Then `dynare` evaluates the minimised unconditional covariance matrix, stored in `oo_.var`. Member state economies are exposed to structural shocks to supply and demand at random which produces dynamics over the simulation horizon. Costs arise from macroeconomic volatility as it would be best from a household's point of view if the economy is in steady state (on trend) throughout.

[133] We used overall income instead of private consumption as the model implies that in steady state $P^U Y^U = P^U C^U$.

[134] We simulate the model for 20000 periods in each run over five runs, store the welfare results $\mathcal{W}^U$ in each run and average to obtain the results for the compensating variation. *cv* represents the key measure to evaluate the costs of heterogeneity in the tables.

of France compared to Germany. Under the homogenous case, only the country size is different from the scenario before. Accordingly, welfare losses for the average citizen are roughly equal as the average outcome is evaluated and members are otherwise same in their setup. Accordingly, again about 0.33 pp. of long-run consumption streams are lost in case monetary policy follows the SWR instead of implementing the optimal plan (OLR). When actual structural macroeconomic differences between Germany and France are taken into account, these losses increase to 0.40 percentage points. The resulting figure is broadly similar to the value obtained for the whole area (0.40 versus 0.47) such that one can argue that welfare results in the euro area are driven by fluctuations in its largest members mainly. The compensating variation needed to make any household in the union no worse off than in the absence of heterogeneity, i.e. the cost 'of sharing the union with France for German households' amounts to 0.07 percentage points of per period steady state consumption C, or about 4.5 constant Euros a quarter permanently in money terms when deflated by P^U. In other words, German households require about 4.5 Euros a quarter in order to be as well off as in case France would mirror the structure of Germany regarding nominal and real rigidities, the composition of production and spending, and the exposure to shocks.

For assessing welfare outcomes in the enlarged euro area, we assumed in the calibration that the home country of the union represents the group of new EU members whereas Foreign is the current euro area. Accordingly, the home region size is $n = 0.042$, where n represents the weight new members would have in the then enlarged area, as outlined in the introduction. Therefore, enlargement of the euro area does not change the size of the union, but will just affect the economic structure of the Home region, which will obey potentially different dynamics. Hence, welfare results can still be expressed in terms of welfare for the average union member albeit the absolute size of the union has increased but not its mass in the model (which is still 1). According to our calibration, euro area enlargement would leave average welfare losses experienced largely unaffected. Permanent consumption losses could even decrease from 0.47 to 0.45 when moving from homogeneity (the current area) to heterogeneity (the enlarged area). This result is obtained albeit average lifetime welfare is lower in the enlarged than in the current area but losses in the first best case actually increased (which cannot be influenced by policy). In the introduction, we stressed that estimates suggest that new EU members might exhibit more forward-looking inflation dynamics than current euro area members (see table 2.1 on page 16). At the same time, volatility in structural shock exposure is actually higher (a pattern investigated in greater detail in the following chapters of the dissertation). It turns out that higher flexibility overcompensates for the higher volatility stemming from structural

Welfare costs of heterogeneity	Homogenous area		Heterogenous area	
Current Euro Area	Foreign=DE=Home		Foreign=DE, Home=EA ex. DE	
$n = 0.69$	SWR	OLR	SWR	OLR
Lifetime welfare $\mathcal{W}^U - t.i.p$	-0.346	-0.020	-0.486	-0.019
$\chi(j)$ in % of C^U	0.328	0.002	0.470	0.001
$cv(j)$ in pp	NA	NA	0.14	-0.001
$\chi(j)$ in € per quarter	21.3	0.1	30.5	0.1
$cv(j)$ per citizen in € per quarter	NA	NA	9.1	-0.1
Large Member Area	Foreign=DE=Home		Foreign=DE, Home=FR	
$n = 0.39$	SWR	OLR	SWR	OLR
Lifetime welfare $\mathcal{W}^U - t.i.p$	-0.351	-0.019	-0.421	-0.019
$\chi(j)$ in % of C^U	0.333	0.001	0.403	0.001
$cv(j)$ per citizen in pp	NA	NA	0.07	-0.001
$\chi(j)$ in € per quarter	21.6	0.1	26.1	0.1
$cv(j)$ per citizen in € per quarter	NA	NA	4.5	-0.1
Enlarged Euro Area	Foreign=$EA15$=Home		Foreign=$EA15$, Home=$EU9$	
$n = 0.04$	SWR	OLR	SWR	OLR
Lifetime welfare $\mathcal{W}^U - t.i.p$	-0.489	-0.018	-0.555	-0.106
$\chi(j)$ in % of C^U	0.471	-0.000	0.450	0.001
$cv(j)$ in pp	NA	NA	-0.02	0.00
$\chi(j)$ in € per quarter	30.5	0.0	29.2	0.1
$cv(j)$ in € per quarter	NA	NA	-1.3	0.0

Table 2.4: Welfare results for the current euro area, the area composed of Germany and France and the enlarged euro area. $\chi(j)$ denotes the average per period permanent consumption loss for each citizen. $cv(j)$ is the compensating variation per citizen. NA=Not applicable.

shocks experienced by new members. We observe that the compensating variation needed to be given to an average citizen of the current euro area to 'accept' the heterogeneity brought about by enlargement is negative (-0.02 percentage points), implying that enlargement represents a net gain within this setting.

2.6.4 Optimal Representation of Members when Heterogeneity Lasts

In the preceding section, we contrasted welfare outcomes obtained under an interest rate rule calibrated with estimates for the euro area as a whole with the optimal plan, where policy maximises welfare of the average consumer in the union. Whereas the SWR focuses on union-wide developments only and disregards regional heterogeneity, the OLR minimises the loss criterion (2.110) thereby minimising the costs from inefficient business cycle dynamics and spillovers between regions intertemporally. A related question concerns how dissimilarities between members should be represented in joint monetary policy making, given that heterogeneity might last. Especially as in Berger and Mueller (2007) one could ask, how large and small countries should be represented in a currency union. We pointed out that, due to sectoral heterogeneity and potentially differing preferences about the composition of the overall goods basket across regions, there is no longer a one-to-one link between regional nominal rigidity and the weight received under optimal policy. In this section we also take into account that monetary strategies should be operationally plausible whereas the optimal plan derived from the full commitment policy might be difficult to implement and to communicate to the private sector in member countries.

As a consequence, we assume in this paragraph that the Smets-Wouters rule specified in (2.87) is given (the central bank commits to using it). Instead of using ad-hoc weights implied by country size and empirical results regarding response coefficients taken from Smets and Wouters (2003) only, policy parameters (elasticities) are chosen by the central bank also in accordance with households' loss-minimisation problem and respecting the model structure.[135] We allow for optimisation over all coefficients in the monetary policy rule (2.87) such that the optimal simple rule (OSR) is calculated as outlined in the optimisation program (2.121). The more instruments are available, the lower overall losses will become and the closer the OSR will resemble the outcome under the full commitment policy (OLR). We especially ask, how countries would be represented in union monetary policy once we allow the country size n to vary such that the member state contribution to the aggregate union price index and aggregate GDP can vary. Accordingly, relative economic size $\frac{Y_i}{Y^U}$ and voting share are allowed to differ. We explore how optimal size depends on country-specific real and nominal rigidities and the composition of consumption and production.[136]

Results are presented in table 2.5 on page 121. We observe that interest rate smoothing is not recommendable at all in both the homogenous and the heterogenous case in all scenarios. The weight on the response coefficient in the policy rule is nil throughout, once instruments are chosen optimally, i.e. with respect to minimising (2.110). This result might look surprising at a first glance given the large body of literature that emphasises desirability of inertia in policy that is a property of full commitment policy under inflation targeting in the New Keynesian model, see Woodford (2003). However it should be noted that persistence in structural shocks, which is high throughout scenarios, makes shock stabilisation of the central bank predictable

[135] The case with 'ad-hoc' chosen weights could be rationalised by argueing that the rule has been delegated to the central bank, as will be the case in chapter 3. As a consequence, maximising social welfare is rather a task of member state governments and the central bank is silent on utility of the household by focussing mainly on inflation developments. We use the coefficients of the Smets-Wouters rule as initial values for the numerical optimisation where $-\mathcal{W}^U$ is our loss objective. `dynare` formulates the policy problem as one of minimising loss, which is the dual problem to minising negative welfare.

[136] When one allows all parameters to vary in the loss function, it might happen in the numerical optimisation that there are values obtained for, say country size n and interest rate smoothing that are not within reasonable regions. In that case we resort to minimising the loss function over auxiliary variables $a_{c(i)} = -\log\left[\frac{lb_{c(i)} - ub_{c(i)}}{c(i) - ub_{c(i)}} - 1\right]$ where lb and ub are value bounds on $c(i)$. $a_{c(i)}$ is tranformed back to $c(i)$ after the minimisation. As an advantage, $a_{c(i)}$ will have an interior solution for sure, as for $c(i)$ approaching the lower bound, $a_{c(i)}$ will approach $-\infty$ and $+\infty$ for $c(i)$ approaching the upper bound. For all policy parameters in the rule (2.87), we will assume that $lb_{c(i)} = 0$, and $ub_{c(i)} = 5$, where the upper bounds for the policy rate smoothing parameter r_i and country size n are equal to 1.

through time such that a similar effect can be obtained. Further, monetary policy becomes very 'hawkish' about inflation (the inflation coefficient hits its upper bound, 5.00) where all stabilisation weight is put on current period's inflation rate coefficient r_π. Speed limit terms $r_{\triangle\pi}$ play no role. It is therefore optimal to disregard rule-of-thumb firms such that optimal policy strengthens the role of inflation expectations in affecting current policy. When both regions are homogenous, the weight on the output gap and the speed limit on output would be increased substantially.

The scenario for the euro area aggregate, where both regions feature structural characteristics of Germany, is illustrated in the upper left panel of table 2.5. It seems clear that the representation of homogenous members according to the country weight in joint monetary policy making ($n = 0.69$) remains unaffected when coefficients are chosen optimally. This observation is a direct consequence of Benigno's proposition, namely that in case both regions exhibit the same rigidities, an optimal inflation targeting policy will place equal weight on regions like a rule that targets aggregates only. A similar result was illustrated in figure 2.8 on page 101 in our two-sector setup. Albeit the results in Berger and Mueller (2007) are obtained in a different analytical framework, it is nevertheless interesting to compare the different implications. The authors find that given "[...] preference shocks were sufficiently similar, it would always be optimal [under both correlated and uncorrelated shocks across regions] to overrepresent small countries and to under-represent large countries". Similarity in preference shocks could be interpreted as structural similarity across regions such that there is symmetry in shock exposure for the given difference in relative economic size. In contrast to that result, the policy maker aimed at maximising welfare will not reweigh relative voting power here. We also observe, that the policymaker would increase the weight on current output and put even more weight on its speed limit compared to the empirically estimated values, where that coefficient increases from 0.0625 to the upper limit.

In case the Home region represents the characteristics of other euro area members, the weight on these countries is lowered under optimal policy as overall nominal rigidity is lower than in Germany and preferences about consumption (the CPI weights γ, γ^*) are quite similar. The highly similar composition of consumption baskets across regions prevents the 'consumption preference effect' highlighted in experiment I under 2.5.4 on page 97 to overcompensate the 'nominal rigidity effect'. As known from the Benigno (2004) model, the central bank is more concerned about the region with the overall higher price duration in such cases, where $D^{EA\setminus DE} < D^{DE}$ where D was defined in (2.100). As a result, the weight of the - less rigid - Home region is decreased to 65%. When heterogeneity is optimally taken into account, losses

Optimal representation	Homogenous area		Heterogenous area	
Current Euro Area	Foreign=Home=*DE*		Foreign=*DE*, Home=*EA ex. DE*	
$n = 0.69$	SWR	OSR	SWR	OSR
Interest rate smoothing r_i	0.80	0.00	0.80	0.00
Inflation r_π	1.70	5.00	1.70	5.00
Speed limit inflation $r_{\triangle\pi}$	0.30	0.00	0.30	0.00
Output gap r_Y	0.125	0.78	0.125	1.75
Speed limit output gap $r_{\triangle Y}$	0.0625	5.00	0.0625	0.00
Optimal Home region size n	**0.69**	**0.69**	**0.69**	**0.65**
$\chi(j)$ in % of C^U	0.33	0.14	0.47	0.17
$cv(j)$ per citizen in pp	NA	NA	0.14	0.04
$\chi(j)$ in constant € per quarter	21.4	9.1	30.5	11.0
$cv(j)$ per citizen in € per quarter	NA	NA	9.1	2.6
Deadweight loss DR in %	58.3		63.1	
Large Member Area	Foreign=Home=*DE*		Foreign=*DE*, Home=*FR*	
$n = 0.39$	SWR	OSR	SWR	OSR
Interest rate smoothing r_i	0.80	0.00	0.80	0.00
Inflation r_π	1.70	5.00	1.70	5.00
Speed limit inflation $r_{\triangle\pi}$	0.30	0.00	0.30	0.00
Output gap r_Y	0.125	0.78	0.125	1.49
Speed limit output gap $r_{\triangle Y}$	0.0625	5.00	0.0625	0.00
Optimal Home region size n	**0.39**	**0.39**	**0.39**	**0.40**
$\chi(j)$ in % of C^U	0.333	0.141	0.403	0.184
$cv(j)$ per citizen in pp	NA	NA	0.077	0.042
$\chi(j)$ in constant € per quarter	21.6	9.1	26.1	11.9
$cv(j)$ per citizen in € per quarter	NA	NA	4.5	2.7
Deadweight loss DR in %	58.0		55.3	
Enlarged Euro Area	Foreign=Home=*EA15*		Foreign=*EA15*, Home=*EU9*	
$n = 0.04$	SWR	OSR	SWR	OSR
Interest rate smoothing r_i	0.80	0.00	0.80	0.00
Inflation r_π	1.70	5.00	1.70	5.00
Speed limit inflation $r_{\triangle\pi}$	0.30	0.00	0.30	0.00
Output gap r_Y	0.125	0.78	0.125	0.88
Speed limit output gap $r_{\triangle Y}$	0.0625	5.00	0.0625	5.00
Optimal Home region size n	**0.042**	**0.042**	**0.042**	**0.00**
$\chi(j)$ in % of C^U	0.471	0.181	0.450	0.122
$cv(j)$ per citizen in pp	NA	NA	-0.021	-0.059
$\chi(j)$ in constant € per quarter	30.5	11.7	29.2	7.9
$cv(j)$ per citizen in € per quarter	NA	NA	-1.4	-3.8
Deadweight loss DR in %	61.6		73.0	

Table 2.5: Representation of heterogenous members under current policy (SWR) and with optimally chosen coefficients (OSR) in (2.87).

in the current area could be more than halved, from $\chi(SWR) = 0.47$ to $\chi(OSR) = 0.17$ (from 30.5. to 11.0 Euros in constant money terms). Accordingly, the deadweight loss (the loss from choosing (2.87) without taking into account heterogeneity) increases from 58% in case of homogenous regions to about 63% under heterogeneity.

The case where Germany and France fully determine macroeconomic activity and inflation developments in the currency union, labeled the *Large Member Area*, is shown in the middle panel of figure 2.5. Albeit average duration of inefficient contracts in Germany lasts more than 12 quarters compared to about 7 quarters in France (as illustrated in table 2.1 on page 16) and with equal weights on tradables in consumption, the optimised Smets-Wouters rule would be slightly more concerned with developments in France. Consequently, the economic weight of Germany in the loss function decreases from 61% to 60%. This result can be explained by the overall lower inflation persistency in the union brought about the lower nominal rigidities in France which can nevertheless lead to an increase in weight of the more flexible region as became clear from experiment I in paragraph 2.5.4. The deadweight loss from ignoring heterogeneity under both the homogenous union (both regions resemble Germany) compared to the heterogenous union decreases by about 3 percentage points (from 58.0% to 55.3%).

Finally, the lower panel of figure 2.5 compares optimal representation in the current versus the enlarged euro area.[137] Under homogeneity, Home and Foreign resemble regions with structural parameters guiding price and output dynamics that reflect values for the aggregate euro area and where Home has size $n = 0.042$. Given the very unequal size between the regions, it is optimal to put a large weight on the speed-limit term of the output gap. In case of homogeneity, where both regions just differ in size, union monetary policy also is concerned about developments in the home region similar as under the SWR rule such that the economic weight of the home region remains unchanged even when choosing policy optimally. This objective vanishes in the enlarged area, resulting in a weight of nil of the home region. As duration D^U in the current euro area is given by about 6.7 quarters against 3.7 quarters in new EU members from the East and as preferences between industry and service goods are similar to the current euro area, the 'Benigno-effect' dominates.

Underrepresenting the less rigid country therefore remains the dominant element in case the Home region resembles the EU9 economic characteristics. Setting the weight to zero implies that policy responds only to developments in the current euro area. However, this strategy is beneficial for the average consumer of the enlarged area (and hence also for member states from the east) which becomes visible from inspection of the compensating variation cv offered to

[137] As the enlarged area is normalised to one and as an average consumer perspective is taken such that welfare can be expressed in per capita, the welfare outcomes in both cases are comparable.

average households. The euro equivalent given to the average citizen in the euro area to accept enlargement under the OSR is -0.059 percentage points of long-run consumption. Thus the average household faces lower consumption risk in case of the enlarged area. In case only the SWR can be implemented, the compensating variation nevertheless increases and stands at -0.021. However, the rule aimed at targeting aggregates only could not internalise all benefits by leaving the country size of the home region unaffected. Accordingly, the costs from disregarding the economic structure of members increase by 11.4 percentage points (from 61.6 to 73.0) when moving from the current to the enlarged euro area.

2.7 Experiment III: Structural Reform Priorities for Reducing Heterogeneity

In the last section, we assumed that current heterogeneity between member states will remain unchanged in the periods to come. In this section, we continue the analysis by asking which sources of heterogeneity should receive highest attention for the policymaker. Eliminating heterogeneities can even be beneficial in case the compensating variation is negative after reform, or lowest compared to the alternatives. When monetary policy continues targeting aggregate union wide developments in prices and output only, the observed member state heterogeneity between current members and between current and future members needs to be coped with by other means than monetary policy: Means that can limit costs from differing regional characteristics are usually seen in measures that lead to higher similarity in economic structure and flexibility in adapting to shocks. Labour mobility across sectors and regions can - in principle - shift factor resources from regions that experienced a negative shock or structural change to regions of the currency union that experienced a surge in demand.[138] Union wide fiscal transfers from surplus regions to regions that are adversely affected could be enacted to smooth out regional demand asymmetries. Fostering of wage flexibility and increased competition in product markets will in general help that price movements signal relative price changes. Efficient reallocations and faster adaptation to changing economic conditions can then become possible without further need for monetary policy actions.

We explore whether losses mainly derive from inefficiencies in the regional organisation of markets, structural shocks, or too passive fiscal stabilisation policies. We focus on four reform options: The impact of product market deregulation[139] (a decrease in markups in

[138] For a discussion of the relationship between labour market structure and voting rights in the ECB, see Berger and Hefeker (2004).

[139] Also the empirical literature acknowledges that structural reforms to enhance competitiveness in labour and product markets might reduce the importance of price stickiness, (Altissimo et al., 2006, p. 9).

goods prices over marginal costs across firms and sectors ρ_J), fostering forward-looking price setting (a decrease in θ_J and ϖ_J), the moderation of structural shocks (lower volatility in innovations to structural shocks), and an active (countercyclical) output gap stabilisation by national governments to dampen price pressures directly where they originate from.[140] It is then possible to obtain a clear ordering of the effects different measures would have for reducing heterogeneity between members.[141]

Reform options are again discussed for the scenarios (2) - (4) introduced under 2.4 in line with the exposition in the previous paragraphs. Consequently, we first assess options for the current euro area grouped as Germany versus other major euro area countries, then move on to analysing the large member area composed of Germany and France and then turn to the enlarged euro area. Again, as in the preceding section, we take the viewpoint that the fully optimal policy is difficult to implement and communicate and that easy understandable rules like (2.87) might be easier to communicate to the private sector. Therefore, in table 2.6 outcomes under the policy rule that targets union aggregates only, the SWR (left panel), are contrasted to results under the policy rule that minimises loss, the OSR (right panel). Under the optimised simple rule, we again maximise over all available policy parameters such that inefficiencies from regional heterogeneity are minimised from the union average household's perspective. The measure of comparison is again $\chi(j)$ defined under (2.123). A higher ranking in table 2.6 is then associated with a lower $\chi(j)$. We also report the compensating variation in units of long-run consumption, *cv* as defined under (2.125).

As a general result under both the SWR and the OSR emerges that flexibilising price setting (a low duration of sectoral price contracts and a low share of non-optimising price setters) should receive the highest attention throughout the scenarios. Flexibilising goods prices in all production sectors in the union will allow to move closest to the first best in all three scenarios. This result is straightforward if one reconsiders the large weight inflation terms and inflation growth rates receive in the microfounded union loss objective (2.110) already for low levels of nominal rigidities, as argued under 2.5.3. Moderation in the volatility of structural shocks follows next. When it is assumed that volatility in innovations is 80% of the values

[140] In case of the latter, the elasticity $a_{g_J} < 0$ in sectoral fiscal rules described by (2.47) $a_{g_J} \equiv \frac{\partial g_{J,t}}{\partial\left(\hat{Y}_{J,t}-\tilde{Y}_{J,t}\right)}$ denotes the percentage change in fiscal spending following a 1% change in the output gap $\left(\hat{Y}_{J,t} - \tilde{Y}_{J,t}\right)$. An elasticity below zero indicates that stabilisation is countercyclical such that the governments stand by the monetary authority in dampening activity if the output gap is positive and vice versa. It is assumed that governments are committed to following these rules.

[141] Under product market deregulation, $\rho_J = 40$. In case of the current euro area scenario, $\rho_H = \rho_N = 41$, due to numerical considerations attached with the OLR. No rigidity in price setting implies that $\theta = \theta^* \simeq 0$ and the absence of intrinsic inflation persistence implies $\varpi = \varpi^* \simeq 0$. Lower volatility in innovations in structural shocks is obtained by assuming that standard errors are 80% of their original value. Counter-cyclical fiscal policies are obtained by assuming that in (2.47) $a_{g_J} < 0$, where $a_{g_J} = -10^{-7}$.

Reform priorities for reducing heterogeneity							
Current Euro Area	SWR		Reform	OSR			Reform
$n = 0.69$	$\chi(j)$	cv	priority	$\chi(j)$	cv	n	priority
Product market deregulation	0.93	0.22	5	0.39	0.14	0.63	5
Flexibilising price-setting	0.01	-0.00	1	0.00	-0.00	0.56	1
Moderation of shocks	0.30	0.08	2	0.11	0.02	0.64	2
Counter-cyclical fiscal policies	0.46	0.13	4	0.17	0.04	0.65	3
Status quo	0.46	0.12	3	0.18	0.04	0.65	4
Large Member Area	SWR		Reform	OSR			Reform
$n = 0.39$	$\chi(j)$	cv	priority	$\chi(j)$	cv	n	priority
Product market deregulation	0.85	0.13	5	0.31	-0.02	0.30	5
Flexibilising price-setting	0.01	0.00	1	0.00	0.00	0.52	1
Moderation of shocks	0.27	0.05	2	0.11	0.03	0.39	2
Counter-cyclical fiscal policies	0.42	0.09	4	0.18	0.04	0.39	3
Status quo	0.41	0.08	3	0.18	0.05	0.39	4
Enlarged Euro Area	SWR		Reform	OSR			Reform
$n = 0.04$	$\chi(j)$	cv	priority	$\chi(j)$	cv	n	priority
Product market deregulation	1.14	0.09	5	0.36	-0.00	0.00	5
Flexibilising price-setting	-0.08	-0.08	1	-0.08	-0.08	0.00	1
Moderation of shocks	0.29	-0.01	2	0.08	-0.04	0.00	2
Counter-cyclical fiscal policies	0.44	-0.04	3	0.11	-0.06	0.00	3
Status quo	0.46	-0.02	4	0.12	-0.07	0.00	4

Table 2.6: Priorities in reducing regional heterogeneity. χ denotes the permanent consumption loss in percentage points. cv is the compensating variation, in percentage points. n denotes optimal region size.

under the status quo, long-run losses are in the range of 0.3 percent of long-run consumption throughout scenarios for the given non-optimal policy rule (left panel) and about 0.1 in case policy would take heterogeneity into account (right panel). We also observe that the ranking of policy priorities based on χ is largely invariant across scenarios under both the SWR and the OSR. This implies that the impact of reform measures is quite effective over the range of nominal and real rigidities considered.

A more prominent role for counter-cyclical national fiscal stabilisation on the domestic level seems not considerably welfare enhancing when followed under the SWR. However, one could slightly improve upon the status quo under the OSR. Accordingly, the compensating variation cv decreases slightly under all scenarios. Product market deregulation - and therefore higher competition among firms in any sector - performs rather poorly in all scenarios and generally worse than active fiscal measures throughout. In the enlarged area, one obtains that it remains optimal to decrease the weight of new members to zero under all reform settings.[142] By obtaining a weight of zero, the union is completely homogenous and the move to reform will be attached with compensating variations lower than zero. As a result, all households receive a net again including new member states' households, albeit their region would be underrepresented. As the OSR optimally takes into account heterogeneity, the gains are larger

[142] In case of flexibilising price setting, it is assumed that $\varpi_J = \theta_J = 2 \times 10^{-1}$.

compared to the case where the rule targets aggregates only.

2.8 Conclusions

This chapter investigated the welfare consequences of dissimilarities in economic characteristics of countries that share a common currency within a calibrated two-region two-sector DSGE by means of three experiments. A two-sector setup respects the fact that there are broadly two categories of goods in gross value added across countries, namely tradables (industry goods) and non-tradables (services) that might differ substantially regarding inflation dynamics and shock exposure. We extended the two-country one-sector currency union model of Benigno (2004) to the two-sector case and included a broad range of structural dissimilarities across countries such that inefficient dynamics can have more potential sources, rather than just being derived from price inertia in the tradable goods sector. We showed that nominal rigidity alone is not sufficient to provide recommendations for the optimal course of monetary policy. Also the composition of the domestic HICP in each member state is a necessary component in order to determine welfare implications from targeting aggregates only, which resembles the official policy stance followed by the ECB. We found that considering heterogeneity more formally in policy can provide non-negligible welfare gains, given that the described empirical heterogeneity remains in place.

Based on empirically relevant calibrations of the model for three scenarios, the current euro area, the area composed of Germany and France, as well as the enlarged area, we determined the welfare costs member states experience when the joint monetary policy targets aggregates only, thereby disregarding regional heterogeneity. For the current euro area, about 0.47% of long-run consumption is lost compared to the optimal plan. Taking into account heterogeneity but otherwise following an instrument rule like the one that mimics ECB's interest rate setting could reduce the costs experienced up to 63% for the chosen calibration. In the union of Germany and France, average per period losses are in the same range as for the aggregate union which indicates that aggregate euro area losses might be driven by inefficient fluctuations and spillovers between its two largest members. Taking into account heterogeneity between the largest members in monetary policy could reduce the losses from heterogeneity up to 55%.

Following euro area enlargement, monetary policy faces the task of optimally considering macroeconomic characteristics of a much smaller group of countries with higher volatility in key variables and differences in price setting. We obtain that average losses mainly derive from the combination of two groups of countries of very unequal size. As the attainable first best outcome worsens, one eventually finds that structural heterogeneity of new EU members would actually provide a net gain as the compensating variation is negative. Thus, the average citizen would favour euro area enlargement. Regarding structural reform options, it emerges that flexibilising price setting (a lower duration of sectoral price contracts coupled with lower endogenous inflation persistence) should receive the highest attention. Moderation of volatility in structural shocks follows next whereas a more prominent role for fiscal policies does not turn out as a recommendable reform option.

Chapter 3

Can Convergence Really Be Blamed? Assessing Sustainable Compliance with Monetary Maastricht Criteria in EU9

3.1 Introduction

Countries from Eastern and Central Europe that joined the EU in 2004 and 2007 also became members of the Economic and Monetary Union (EMU), with a derogation on implementing the euro as common currency.[1] All countries announced intentions to join the euro area within the next five to ten years. Hungary has not announced a target euro area entry date yet, although conditions for announcing a date can be met soon, see Geeroms (2007). Slovenia became full member already in 2007 and the Slovak Republic will obtain the euro as legal tender in 2009. As there is no opt-out clause included in the accession treaty, the countries (labelled EU9 in the following) will sooner or later become full members of the EMU as soon as the joint fulfilment of the Maastricht criteria can be secured in each country.

The criteria are intended to assess the readiness to join a common currency area for a certain country and to minimise costs from giving up monetary and exchange rate policy as stabilisation tools within the respective country: The inflation criterion requires sustainably low year-on-year HICP inflation rates. The rate must not be more than one and a half percentage point higher than the average inflation performance in the three EU countries with the lowest rates over the past two years. The interest rate criterion requires that the spread of yields on long-term (10 year) government bonds over EU bonds with same maturity is not more than two percentage points. Sound public finances demand that the country is not in an excessive deficit situation. The planned or actual government deficit as share of GDP must not exceed three percent in normal circumstances and the ratio of government debt to GDP has to be

[1] See the volume by Berger and Moutos (2004) for a general overview on the management of EU enlargement.

not more than 60%. Eventually, the exchange rate criterion requires stability of the nominal exchange rate. A future euro area member state should have respected the normal fluctuation margins in the rate around the central parity (±15%) for at least two years in the Exchange Rate Mechanism II (ERM II). Fulfilment of the criteria needs to be sustainable as the case of Lithuania shows. Albeit the inflation rate was very close to the target of 2.6 percent, standing at 2.7 (calculated using the change in the latest available 12-month average of the HICP over the previous 12-month average), the ECB and the European Commission (EC) concluded in their convergence reports that the inflation rate was considered to be temporarily low and not sustainable in the near future, see European Central Bank (2006).

The current state of play of nominal convergence at the time of this writing for all countries is summarised in table 3.1 on page 130. Current account developments are displayed also, as the ECB and the EC can take any further useful variable into account when assessing whether a country can adopt the euro. The table reveals that each of the countries breaches at least one Maastricht criterion. The Baltic States (Estonia, Latvia, and Lithuania) mainly face a problem with high inflation and excessive current account deficits, whereas the fiscal criteria are sound. Stable exchange rates and healthy public finances prevented that high inflation rates translated to high long term rates by means of a high inflation risk premium (Geeroms, 2007, p. 292). All three countries participate in ERM II without any devaluation risks.

In contrast, most of the Visegrad States (the Czech Republic, Hungary, Poland, the Slovak Republic) have problems in complying with the fiscal deficit criterion, the only exception being Poland. Adjustment concerning public finances is worst in Hungary. High inflation rates, that where fuelled further by the sharp increase in administered prices in the beginning of 2007, together with unsound public finances translated in too high long term rates. The Slovak Republic is the only Visegrad state to participate in ERM II (since 2005). The Czech Republic, Hungary and Poland are not members of ERM II yet, and their current exchange rate regimes are deemed not to be compatible with ERM II, as table 3.2 illustrates. 2007 entrants (Bulgaria and Romania) have still a long way to go. Bulgaria breaches the HICP criterion. The currency board installed in Bulgaria in the wake of the 1998 balance of payments crisis can even so be considered beneficial as inflation and inflation expectations were brought down from two digit levels.[2] Romania misses all criteria apart from the government debt criterion.

[2] The situation of Bulgaria is treated in detail in chapter 4.

Country	*HICP* Average	*Long term rates* Apr 06 - Apr 07	*Fiscal deficit* Estimates for 2007	*Debt*	*Current account* Memo
Visegrad States					
Czech Republic	2.0	3.9	**-3.9**	30.6	-3.0
Hungary	**5.9**	**6.7**	**-6.8**	**67.1**	-3.5
Poland	1.6	5.3	**-3.4**	48.4	-3.1
Slovak Republic	**3.6**	4.3	-2.9	29.7	-4.2
Baltic States					
Estonia	**4.7**	5.1	3.7	2.7	-15.1
Latvia	**6.9**	5.5	0.2	8.0	-22.4
Lithuania	**4.1**	4.2	-0.4	18.6	-12.4
2007 Entrants					
Bulgaria	**6.4**	4.2	2.0	20.9	-16.6
Romania	**5.3**	**7.2**	**-3.2**	12.8	-12.1
Maastricht benchmark	3.0	6.4	-3.0	60	

Table 3.1: Current state of compatibility with Maastricht criteria: yearly data. Table modified from Geeroms (2007, p. 291). Bold: criterion is not passed.

3.1.1 Main Objectives and Motivation

We argue in this chapter that challenges in fulfilling the monetary Maastricht criteria derive from three sources that need to be assessed for each country in turn: the interaction of domestic monetary and fiscal policy conditional on the prevailing exchange rate regime, the market structure of goods and factor markets, and the role of transitory as well as permanent shocks in driving macroeconomic aggregates. Maastricht compliance is therefore affected by both short and long run factors which should be taken into account when assessing the prospects of euro area entry.

First, the course of domestic monetary and fiscal policies prior to euro area entry is constrained by the upper bounds provided by the criteria regarding the evolvement of the inflation rate and fiscal deficits as well as government debt. Compliance with criteria therefore imposes additional targets as policy objectives that have to be controlled by policymakers. Additional trade-offs might be created as the policy maker cannot simply focus on stabilisation of the inflation rate and minimising the deviation of GDP about trend in the sense of policy rules in the spirit of Taylor (1993).[3] Exchange rates have to be kept within a certain band under ERM II thereby eliminating the possibility of competitive devaluations once and for all following unfavourable cost shocks. Further, ERM II requires that the currency should not be subject to serious pressures within that system for two years before entry into the euro area becomes possible. Consequently, there are institutional setups that are not compatible with ERM II: Crawling pegs, independent floats or managed floats without a mutually agreed central rate

[3] In a two-sector economy, domestic inflation is a weighted average of traded and non-traded inflation and the broadly supported idea of price stability has to be assessed in light of sector specific inflation rates.

Country	*Current regime*	*ERM II status of regime*	*Intentions for euro adoption*
Visegrad States			
Czech Republic	Managed float	Not compatible	2012
Hungary	Crawling peg	Not compatible	No target date
Poland	Free float	Not compatible	2012
Slovak Republic	Managed float	Member since May 2005	2009
Baltic States			
Estonia	Currency board	Member since June 2004	As soon as possible
Latvia	Fixed exchange rate	Member since May 2005	As soon as possible
Lithuania	Currency board	Member since June 2004	2010
2007 Entrants			
Bulgaria	Currency board	Compatible	2010-2015
Romania	Crawling peg	Not compatible	2014

Table 3.2: ERM II status of new EU members. Table modified from van Poeck et al. (2007, p. 462) and Geeroms (2007, p. 291). Intentions for euro adoption are based on official statements by central banks, ministries of finance, parliament, prime ministers or other official sources.

and pegs to anchors other than the euro, see European Commission (2000).

ERM II participation is already reality for the Baltic countries, as table 3.2 shows. Among the Visegrad countries, only Slovakia participates. The case of Slovakia makes clear that not only currency boards with the euro area (as in case of Estonia and Lithuania) or a fixed peg to the euro area (as Latvia) can master access but also countries that followed a managed floating regime prior to ERM II. Concerning the remaining countries, only Bulgaria can be considered a current valid candidate for ERM II from an institutional perspective. The Czech Republic, Hungary, Poland, and Romania will need to adjust their regime. As a consequence, migrating to ERM II constitutes a compulsory transition for a number of the EU9 to an intermediate exchange rate regime between the two extreme cases of fixing the exchange rate (or giving up monetary policy completely in a currency board) or letting the exchange rate float freely. Eventually all countries have to adopt a 'Maastricht-constrained' managed float regime: Inflation has to be kept low enough and the exchange rate stable enough to meet both criteria (Lewis, 2007, p. 6).

Second, the transition to market economies and the need to cope with shocks in a currency area requires flexibilisation of markets regarding supply conditions and the liberalisation of domestic markets for goods, capital and labour. A highly competitive and very open economy with a large share of GDP that is open to trade might face lower overall inflation than a less competitive and rather closed economy. Markups can be lower and more output prices might be determined in the world market. Whereas labour markets are somewhat more flexible than those of the current euro area as found in von Hagen and Siedschlag (2005), competition among firms in new members is still significantly below the European average according to the index published by the Centre for International Competitiveness (2008). Regarding the evolution of capital markets, liberalisation has advanced where the CEECs have removed all impediments to capital mobility (van Poeck et al., 2007, p. 460).

Third, transition economies are subject to a range of volatile macroeconomic shocks of both transitory and permanent nature that hit the supply and demand side of GDP. Ongoing permanent shocks to the level of productivity of goods produced in sectors that became open to trade are considered the main driver of growth and real convergence to euro area average income levels, see European Central Bank (2007). At the same time, associated nominal convergence is expected to deliver a trade-off between fulfilment of the Maastricht inflation criterion and maintaining exchange rate stability: In the presence of appreciation of the real exchange rate, a fixed exchange rate regime or currency board (as in case of Bulgaria as well as the Baltic states) guarantees exchange rate stability at the cost of increases in domestic inflation. Under a flexible exchange rate regime (Poland), the impact on domestic inflation can be mitigated at the cost of trend appreciation of the nominal exchange rate. For intermediate regimes (the Czech Republic, Hungary, Slovakia, Romania), pressures can arise from both sources. These low-frequency (trend) developments are coupled with high frequency (business cycle) shocks like temporary price shocks to imports as well as shocks to fiscal spending that make conformance with the Maastricht criteria even harder.

The main objective of this chapter is to assess compliance with monetary Maastricht criteria for new EU members from the East in an estimated structural macroeconometric model taking into account short and long run factors. Compliance with the criteria is assessed according to the expected outcome as well as the variability in the outcome such that both structural factors (that determine the expected outcome) and cyclical factors (that determine the variability in the outcome) is accounted for. Our analysis thus respects that albeit a country might manage to fulfil the criteria on average, it might not do so with high probability: The criteria might be met once, but violations of the bounds might not be infrequent in subsequent periods as emphasised in the introduction. If the latter is true for a country, we consider conformance with the criteria not to be sustainable. Unsustainable compliance assessed this way implies according to our view that the country under consideration is not yet ready for introduction of the euro. The criteria are evaluated on a quarter-by-quarter basis instead of the official interpretation by the ECB and the EC based on average year-on-year assessments, as presented earlier. This assumption allows for consistency with the modeling structure to be presented in the following where the timing convention is one quarter. The costs from this approach derive from the fact that the assessment is stricter than required officially. As one benefit applies that quarterly national accounts data can be used in order to estimate the model for each country in turn.

Table 3.3 provides the empirical picture of the current state of compliance, based on quarterly data.[4] The upper panel illustrates whether monetary Maastricht criteria (the inflation criterion $\pi_t - \pi_t^*$, the nominal interest criterion $i_t - \tilde{\imath}_t^*$, and the criterion for the nominal exchange rate $\frac{S_t}{S} - 1$ where S denotes the central parity) are passed on average. Criteria are assessed averaging the last four available data points. It turns out that all countries fulfil the nominal exchange rate criterion for the time span examined. The average values range from -10.71% in the Slovak Republic to 10.81% in Latvia where boths values are within the bounds on fluctuation margins required by the criterion. The interest rate criterion is not met by both Hungary and Latvia where interest rates stand at 4.09% and 1.30% respectively, far away from the upper limit of the quarterly criterion at 0.50 percentage points. The quarterly inflation criterion is not met in most countries. There is evidence that all countries under a fixed exchange rate regime or a currency board fail in meeting that criterion, as expected. This is however also the case for Hungary that operates under a managed floating system. In case of Hungary, the levels of short run interest rates eased only recently, where they stood at two digit levels in the years before.

Assessing the criteria based on local trends implied by recent data[5] might however not provide enough information as to whether criteria could be passed sustainably, i.e. repeatedly in the future with low uncertainty. Given that four subsequent values are equally distributed above and below the upper bound of a criterion and assuming that the latest two realisations missed the target, the criterion is still met on average. Longer run developments might however be indicated by the latest two observations. Relying on the mean outcome only might therefore provide a distorted picture. Means can become uninformative about the actual evolvement in case variability in series is high or might increase again after the formal examination. The negative assessment for Lithuania, where it was emphasised by the European Central Bank (2006) that according to most recent information HICP inflation was on the rise again , affirms this point. Throughout this chapter we will therefore assess the success of meeting a criterion not only on the average outcome but also on achieving low variability in actual values.

In this vein, the lower panel in table 3.3 imposes the requirement that the criteria should not only be met on average but that there is also a high probability of this event occurring. We consider meeting the criteria in 90 out of 100 cases to be a reasonable assumption. The table thus presents bounds that must not be exceeded by the standard deviation in the historical

[4] Details on the calculation of quarterly criteria can be found in section 3.6.

[5] The Maastricht criteria in their official legal form will be discussed in detail in section 3.6. Considering the above mentioned example of Lithuania, treaty provisions regarding price developments applied by the ECB were based on the reference period from April 2005 to March 2006, see European Central Bank (2006).

Current fulfilment of monetary Maastricht criteria				
Average outcome				
Criterion	$c_t^{\pi} = \pi_t - \pi_t^*$	$c_t^i = i_t - \tilde{\imath}_t^*$	$c_t^S = \frac{S_{t+j}}{S} - 1$	Pass
Bound	$\mathcal{E}c_t^{\pi} \leq 0.37pp$	$\mathcal{E}c_t^i \leq 0.50pp$	$-15\% \leq \mathcal{E}c_t^S \leq 15\%$	criteria
Visegrad States				
Czech Republic	-0.10	-0.95	-11.30	Yes
Hungary	**1.54**	**4.09**	3.92	No
Poland	-0.02	-	-3.76	Yes/No
Slovak Republic	0.19	-	-10.71	Yes/No
Baltic States				
Estonia	**0.76**	0.10	0.00	No
Latvia	**1.34**	**1.30**	10.81	No
Lithuania	**0.56**	0.04	-2.68	No
2007 Entrants				
Bulgaria	**0.54**	0.35	0.19	No
Romania	0.33	-	6.94	Yes/No
Variability of outcome				
Criterion	$c_t^{\pi} = \pi_t - \pi_t^*$	$c_t^i = i_t - \tilde{\imath}_t^*$	$c_t^S = \frac{S_{t+j}}{S} - 1$	Pass
Bound	$\sigma\left(c_t^{\pi}\right) \leq 0.23pp$	$\sigma\left(c_t^i\right) \leq 0.30pp$	$\sigma\left(c_t^S\right) \leq 9.12\%$	criteria
Visegrad States				
Czech Republic	**0.86**	**0.91**	2.82	No
Hungary	**0.94**	**0.98**	3.00	No
Poland	**1.17**	**0.90**	6.53	No
Slovak Republic	**1.97**	**0.91**	1.68	No
Baltic States				
Estonia	**0.76**	**0.91**	0.00	No
Latvia	**0.98**	**0.91**	4.33	No
Lithuania	**0.84**	**0.90**	3.02	No
2007 entrants				
Bulgaria	**1.98**	**0.93**	0.14	No
Romania	**3.61**	**0.90**	6.70	No

Table 3.3: Current state of fulfilment of Maastricht criteria: quarterly data. Bold type values indicate that a criterion is not passed.

series for values of criteria, indicated by the operator $\sigma(\cdot)$, in order to meet the criteria with high probability.[6] Accordingly, the upper volatility limit on the inflation criterion is 0.23 percentage points, 0.30 percentage points in case of the interest rate criterion, and 9.1 percentage points for the exchange rate criterion. One observes that empirical exchange rate volatility will not prevent meeting the criteria with high probability. The bound of 9.1pp is not breached in any country by its historical volatility in exchange rates. Therefore, the exchange rate criterion can be expected to harm the upper bound on the criterion (15%) or the respective lower bound (-15%) only infrequently in the future. In other words, the average outcome presented in the upper panel of the table provides useful information about the likely future path of developments in the exchange rate. Whereas fulfilment of the interest rate criterion is secured on average for most countries as the upper panel shows, it turns out that meeting this criterion

[6]We make use of inequality conditions explained in detail below, see equations (3.68) - (3.70) as derived in section 3.7.2. It holds that $\sigma^2\left(c_t^x\right) \leq (K-1) \times \left(B(c_t^x)\right)^2$, such that the variability criterion reported is $\sigma\left(c_t^x\right) \leq (K-1)^{1/2} \times B(c_t^x)$ where c_t^x is the criterion for variable x.

is uncertain in all countries: The empirical fluctuation in the three month rate has historically been stronger than allowed to fulfil the criterion in many cases. Values range from 0.90pp to 0.98pp where only 0.30pp would satisfy sustainable development of interest rates in each country. The same reasoning carries over to values for the quarterly inflation criterion. All countries exhibit too high volatility in the CPI rate in order to pass that criterion sustainably. Values range from 0.76pp in Estonia to 3.61pp in Romania where the upper limit is provided by 0.23pp. In summary, based on the historical evidence there is a less than 90% probability in case of the inflation and the interest rate criterion that compliance will be possible in the near future.[7]

3.1.2 Research Questions

Motivated by these basic findings, this chapter addresses the following main research questions:

- *Will new member states be able to pass the Maastricht criteria in the long run given the current economic outlook and exchange rate regime under prolonged real convergence?*

  In a first step, we assess compliance with Maastricht criteria based on the current economic outlook taking into account real convergence within an estimated structural macroeconometric framework conditional on the country-specific exchange rate regime. Real convergence is incorporated by exposing the model to a permanent shock in total factor productivity in the tradable sector. Volatility in macroeconomic series about the trend, i.e. business cycles, provides a range of possible future outcomes. We thus also report the most likely 90% realisations for values of criteria. Reporting the average outcome together with the bands allows to assess, whether meeting the criteria is associated with low or rather high uncertainty.

- *Will appropriate monetary stabilisation policies increase the probability of compliance with criteria?*

  Under the presence of nominal rigidities, monetary policy can mitigate fluctuations in economic variables that are relevant for passing the criteria. Policy thus has the potential to make fulfilment of criteria more likely. One can therefore ask, if the range of undesirable outcomes of values for criteria can be limited once monetary policy is chosen optimally. This question can be answered for a range of assumptions about the conduct of policy. Accordingly, current policy under the given exchange rate regime is compared to policy

[7] For the interest rate criterion and the inflation rate criterion, only the upper bound is relevant in assessments.

that is chosen optimally in order to minimise deviations from policy objectives. Targeting Maastricht objectives directly will also be considered.

- *Which structural factors matter most for compliance/hinder compliance with Maastricht criteria in each country?*

  The preceding analysis has to be seen against the backdrop of structural shocks that new member states are currently and repeatedly exposed to. Our structural macroeconometric framework allows to recover structural shocks on a country-by-country basis that drive volatility in variables that determine the range of outcomes critical for Maastricht compliance. Consequently, a detailed analysis which sources of volatility drive overall fluctuations is possible. The macroeconomic setup of each country regarding openness, composition of gross value added, and price dynamics will determine which shocks (home/foreign, real/monetary, supply/demand) matter most and how they transmit through the economy.

3.1.3 Main Results

To answer these questions, we set up a two-sector DSGE model of a small open economy that is made applicable to each of the EU9 respecting the different exchange rate regime in place. We refrain from an ad hoc calibration whenever it is possible and let the data speak by estimating structural parameters for each country using Bayesian methods. We allow for a wide array of shocks in order to detect relative contributions in determining overall macroeconomic volatility. Parameters for policy rules, the determinants of market structure, and of price setting as well as structural shocks thus exhibit a country-specific interpretation. We obtain the following main findings:

- *EU members from the East face challenges in meeting the monetary Maastricht criteria, when real convergence is assumed to derive from a trend factor productivity increase of 30% in the industry sector in ten years time. Upward risks especially materialise in meeting the interest rate and the inflation criterion.*

- *Compliance with the exchange rate criterion and the inflation criterion at the same time seems however not to be hindered by Balassa-Samuelson type (trend-related) price pressures. Accordingly, the impact of the current exchange rate regime in shaping the prospects of fulfilment should not be overstated.*

- *Appropriate monetary stabilisation policies will be necessary but not sufficient in making compliance with the criteria less uncertain. According to our model, structural shocks* *need to moderate conceivably in all countries before fulfilling criteria can be secured in a sustainable way. Coordinated monetary and fiscal policies aimed at maintaining price stability might be of added value.*

3.1.4 Organisation of the Chapter

The chapter proceeds as follows. We review the related literature in the following section. In section 3.3 we set up our 'work-horse' model that guides the analysis. The model is based on the two-sector one-country model described in detail in chapter 4. It is modified here to allow for different institutional setups of countries regarding the formation of monetary policy under a certain exchange rate regime. Compared to the currency union model laid out in chapter 2, we now also assign an explicit role for physical capital in production in order to account for long run real convergence aspects properly. Under 3.4, we explore the main workings of the model under different assumptions about the exchange rate regime by investigating impulse responses. The small-open economy is exposed to domestic real, domestic monetary and foreign shocks. In order to obtain a reasonable assessment of Maastricht compatibility, we then estimate the model conditional on the current exchange rate regime for each country in turn. We employ Bayesian methods and use the Kalman filter to recover structural (unobservable) shocks from observable volatility in key macroeconomic time series. We then compare moments of the estimated model with empirical moments for each country in order to check if the model reproduces the volatility that is observed empirically.

Equipped with the estimated framework we assess compliance with Maastricht criteria by means of a forecasting experiment that takes into account real convergence explicitly. We then conduct counterfactual experiments in order to derive policy recommendations how the probability of meeting the criteria could be increased, i.e. how the volatility in series critical for fulfilment might be reduced. We check whether this task can be best accomplished under the current exchange rate regime, the current exchange rate regime with policy parameters chosen optimally, as well as the ERM II regime, i.e. the regime where the Maastricht criteria are additional targets in policymaker's loss function.

3.2 Related Literature and Own Contribution

The literature on potential difficulties of new EU members from Central and Eastern Europe in coping with Maastricht criteria and ERM II is extensive.[8] At the same time, it expires rapidly, as macroeconomic conditions within the countries under consideration change from

[8] Schadler et al. (2005) provide an overview of current challenges for euro adoption in Central Europe.

year to year. We identify at least three strands of literature that are relevant for our analysis and to which our results add to.

One strand of literature discusses the proper exchange rate regime choice for small open economies and the costs and benefits of giving up a certain exchange rate regime.[9] Small open economies are natural candidates for pegging or fixing the exchange rate. The gain in credibility from importing the monetary policy of the anchoring country for foreign investors and the irrelevance of hedging against currency risk are assumed to outweigh the loss that comes from loosing the exchange rate as stabilisation tool following foreign price shocks. de Haan et al. (2001, p. 4) discuss the choice between a currency board and an independent central bank under a fully flexible rate in case of the Baltic States. It is found that a currency board becomes more attractive when the imported monetary policy replaces a dependent central bank and a government that is primarily oriented towards output gap stabilisation. Berger et al. (2001) show that the flexibility loss associated with pegging the currency may be reduced if domestic and foreign shocks are correlated and more volatile. As a consequence, allowing for a plausible structural change after a peg, a flexibility gain may result. Ravenna (2005) adds to the 'credibility versus flexibility' debate where results are derived from a DSGE framework that is based on optimising behaviour of households and firms that are subject to foreign and financial shocks. The author finds that the credibility gain from permanently committing to a fixed exchange rate by joining the European Monetary Union can outweigh the loss from giving up independent monetary policy.

A further strand of literature focuses on the implications of real and nominal convergence in new EU member states for Maastricht criteria compatibility. Nominal convergence (the convergence in price levels towards euro area levels, i.e. a trend increase in new EU members' price levels) and real convergence (the trend increase in overall real GDP) can take a long time. The expected appreciation in the real exchange rate requires either an inflation differential (for a fixed regime or a currency board), an appreciating nominal exchange rate (for a floater), or a combination of the two (for a managed floater). New EU members might therefore face difficulties in simultaneously satisfying the exchange rate and the inflation criterion.[10] Lewis (2007) argues that authorities can use their monetary policy to 'hit' one criterion, but

[9] General research on exchange rate regimes and monetary policy is summarised in Amato et al. (2005). Benigno and Benigno (2001) analyse policy rules under differrent exchange rate regimes within the framework of the New Open Economy Macroeconomics.

[10] The basic argument simply derives from the definition of the real exchange rate as

$$R_t = S_t \frac{P_t^{EMU}}{P_t}$$

where the exchange rate is in direct ('price') quotation from the applicant's perspective (units of home currency for one Euro). Assuming that all goods have eventually the same price in period $t+j$ when expressed in same must essentially just 'hope' to satisfy the other one. Devereux (2003) provides an analytical discussion of the adjustment to EU accession for an economy under alternative assumptions about monetary policy rules. If EMU membership is combined with membership in the euro area, then the post-accession period exhibits excessive foreign borrowing, high wage inflation, an excessive stock market boom, and much too rapid growth in the non-traded sector at the expense of the exportable goods sector. Therefore, the best policy attending EU accession is found to be flexible inflation targeting with some weight on exchange rate stability.

Ravenna and Natalucci (2008) develop a general equilibrium model of an emerging market economy where productivity growth differentials between tradable and non-tradable sectors result in an equilibrium appreciation of the real exchange rate, the so-called Balassa-Samuelson effect, see Balassa (1964) and Samuelson (1964). The authors show that the real exchange rate appreciation limits the range of policy rules that, with a given probability, keep inflation and exchange rate within predetermined numerical target ranges. The real exchange rate appreciation due to the Balassa-Samuelson effect shifts the output gap/inflation variance trade-off, increasing the cost of managing or fixing the exchange rate. As a consequence, the Maastricht criteria constrain the policy choice while providing no additional benefit to countries credibly committed to joining the euro area.

A third strand aims at determining the scope of fiscal and monetary policy stabilisation in improving economic outcomes. Properly assessing policy choices to cope with euro area accession requires a description of optimal domestic policies. Analysing welfare effects following different policy rules has become a standard tool in DSGE frameworks allowing for rigidities in the response of nominal entities, starting - at least - with the work by Rotemberg and Woodford (1997). Under quite general conditions, welfare costs for households can be assessed by evaluating volatility in macroeconomic aggregates the policymaker targets. Lipinska (2008) calculates monetary policy that is constrained to satisfy the (cyclical) Maastricht criteria in a two-sector DSGE framework and finds that the policy is characterised by a deflationary bias. Such a policy leads to additional welfare costs amounting to 30% of the optimal, unconstrained, monetary policy loss. In a related framework, Lopes (2006) finds that the EU9 may experience sizable welfare losses as a result of joining the euro area.

However, both authors disregard the role of real convergence (the role of permanent shocks) in shaping the prospects of these economies in meeting the criteria. We instead argue that

currency, R will converge to one (from above), i.e. eventually purchasing power parity will hold. Then

$$P_{t+j}^{EMU} = S_{t+j}^{-1} P_{t+j} \ .$$

Hence the nominal convergence in the home economy triggered by real convergence (the decrease in R) can either be established by an appreciation of the nominal exchange rate or by an increase in the price level.

new EU members are concerned about fulfilment of criteria in levels (i.e. regarding the average outcome of variables, the upper panel of table 3.3) *and* the mitigation of fluctuations in output, goods prices, and exchange rates by policy (i.e. reducing the range of possible outcomes, the lower panel of 3.3). Even if all volatility could be eliminated by appropriate policies, the analysis based on stationary developments would not provide guidance as to whether the criteria would be passed at all (as the average outcome might be outside the limits stipulated by the criteria due to real convergence). We thus argue that checking compliance with criteria is first and foremost a task of analysing trend developments in variables over the longer horizon. Having accomplished that task, one could then move on to analysing appropriate policy measures to reduce the uncertainty of outcomes.

Regarding the supply-side description of the model, we follow Ravenna and Natalucci (2008) and Lipinska (2008). As in their works, and as in the preceding chapter, gross value added is decomposed in tradables (a proxy for industry production) and non-tradables (a proxy for services production). Incorporating non-tradables is expected to explain fluctuations in aggregates and real exchange rate movements more appropriately, see also Benigno and Thoenissen (2003) and Corsetti and Dedola (2003). It can be assumed that the tradable (industry) sector responds differently to shocks than the non-tradable sector, which is also supported by empirical work for new EU members (see Mihaljek and Klau (2004) and Lipinska (2007)) and also highlighted in chapter 2. Contrary to these authors, we prefer 'a strong econometric interpretation' of the framework used, in the sense of Geweke (1999). Thus, the model is estimated for each country in turn based on system-wide, Bayesian estimation. We let the data speak in order to assess the relevance of sectoral contributions instead of calibrating the model using values that are not country-specific and therefore ad hoc.

Our contribution to the literature is therefore threefold. First, and as a novelty, we develop a comparative study where criteria can be assessed on a country-by-country basis. As a result, our framework can provide empirically minded and country-specific insights regarding economic developments in the EU9 and the prospects for euro area entry. Second, we properly take into account the role, real convergence triggered by permanent shocks might have in meeting the criteria on average whereas the related literature mainly focusses on the cyclical aspects only. Third, building on these results, we analyse the impact appropriate policies might have in increasing the probability of success where country-specific transitory shocks might otherwise render compliance with criteria an unlikely event.

3.3 The Two-Sector Small Open Economy

3.3.1 Households and Firms

Households derive utility from consuming a basket C_s^j of home and foreign tradable goods (industry goods) and non-tradable goods (services). They derive liquidity services from holding real money balances $\frac{M_s^j}{P_s}$ and receive disutility from providing labour L_s^j for production of sectoral gross value added. Lifetime utility U_t^j for an arbitrary household j is then given by

$$U_t^j = \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{(s-t)} \left\{ \mathcal{U}\left(C_s^j\right) + \mathcal{N}\left(\frac{M_s^j}{P_s}\right) - \mathcal{V}\left(L_s^j\right) \right\} \tag{3.1}$$

where

$$\mathcal{U}\left(C_s^j\right) = \ln C_s^j \tag{3.2}$$

$$\mathcal{N}\left(\frac{M_s^j}{P_s}\right) = \chi \ln \frac{M_s}{P_s} \tag{3.3}$$

$$\mathcal{V}\left(L_s^j\right) = \vartheta \frac{\left(L_s^{S,j}\right)^{\kappa}}{\kappa} \tag{3.4}$$

Unlike as in chapter 2 when modelling lifetime utility under (2.1) on page 31, we now provide explicit functional forms for $\mathcal{U}$ and $\mathcal{V}$. Also, utility from holding real money balances $\mathcal{N}$ is taken into account. Again the role of money is endogenously determined as monetary policy is conducted by setting the policy rate, as in the preceding chapter, however limited here by the choice of the exchange rate regime. As such, inclusion of money would not provide further insights. However, we aim at describing the linkage between the volatility in net foreign assets and the home economy more formally later on where changes in the domestic money stock will move one-for-one with changes in net foreign assets. This link is important here, as foreign shocks can be considered as one important driving force behind uncertainty in meeting criteria.

Private households are homogenous and provide hours of work $L_s^{S,j}$ to both sectors. This assumption differs from the approach in the preceding chapter where agents are consumer-producers that decide whether to produce in the H or N sector. Households are risk averse and we assume that risk aversion is equal to one, $\rho = 1$. Accordingly, we obtain logarithmic preferences. The intertemporal elasticity of substitution of consumption today for consumption tomorrow equals one, also. χ scales the utility of holding real money balances, so does ϑ for providing labour. κ denotes the inverse of the labour supply elasticity. A more detailed discussion of these assumptions is provided in chapter 4. Without loss of generality, we set

$s = t$ in the following.

A firm's objective in the industry sector H or services sector N is to maximise the expected present discounted value of cash flows generated from producing industry goods Y_H or services Y_N. Each firm $z \in [0,1]$ in the tradable and non-tradable sector combines capital $K_{t-1}^J(z)$ and labour $L_{J,t}(z)$ to produce output $y_{J,t}(z)$ according to the Cobb-Douglas production function

$$y_{J,t}(z) = A_{J,t}(z)(K_{t-1}^J(z))^{\alpha_J}(L_{J,t}(z))^{1-\alpha_J} \tag{3.5}$$

where $K_{t-1}^J(z) = \int_0^1 K_{t-1}^{J,j}(z)dj$ is capital rented out by households and $L_{J,t}(z) = \int_0^1 L_{J,t}^j(z)dj$ is labour supplied. We assume that total factor productivity is same for all firms in a sector, such that $A_{J,t}(z) = A_{J,t}$. The industry sector is open to trade whereas services are only sold domestically. Production technology (the level of total factor productivity) used in the tradable and non-tradable sectors, $A_{H,t}$ and $A_{N,t}$, forms a structural time series. Hence in both sectors J, technology is composed of a trend component $E_{J,t}^T$ and a cyclical component $E_{J,t}^C$. Both objects are stochastic processes and form a multiplicative time series such that

$$A_{J,t} = E_{J,t}^T E_{J,t}^C \tag{3.6}$$

where

$$\begin{aligned} E_{J,t}^T &= E_{J,t-1}^T \exp\left[x_{J,t}\right] && (3.7)\\ E_{J,t}^C &= \exp\left[v_{J,t}\right] && (3.8) \end{aligned}$$

which is the same structure as proposed in Lindé (2004). The shock $x_{J,t}$ determines the behaviour of the trend $E_{J,t}^T$ and we assume that trend productivity growth $x_{J,t} = \ln \frac{E_{J,t}^T}{E_{J,t-1}^T}$ can be described by

$$x_{J,t} = (1-\rho_{x,J})\mu_{E_J^T} + \rho_{x,J}x_{J,t-1} + \eta_{J,t} \qquad \eta_{J,t} \sim \left(0, \sigma_{\eta_J}^2\right) \tag{3.9}$$

$\eta_{J,t}$ denotes a random innovation to the trend of technology and its impact is therefore permanent. As a consequence, it also has lasting effects on the level of output $Y_{J,t}$ and labour productivity $Y_{J,t}/L_{J,t}$ as well as capital productivity $Y_{J,t}/K_{J,t}$. $\eta_{J,t}$ is the source of unexpected lasting shocks (e.g. the installment of better machines in sector J). For the persistence parameter $\rho_{x,J} = 0$, the trend in (the log of) total factor productivity $E_{J,t}^T$ is characterised as a random walk with drift where the drift parameter equals deterministic long-run growth $\mu_{E_J^T}$ adjusted for an error $\eta_{J,t}$. For $0 < \rho_{x,J} < 1$, trend-productivity growth is persistent as past innovations add up to the effect in an albeit muted way. When $\rho_{x,J} = 1$, trend productivity growth follows a pure random walk such that the (log of) productivity follows a pure random walk.

The structural shock process $v_{J,t}$ in E^C_t causes stationary fluctuations about trend E^T_t

$$v_{J,t} = \rho_{A_J} v_{J,t-1} + \varepsilon_{A_J,t} \qquad\qquad \varepsilon_{J,t} \sim \left(0, \sigma^2_{\varepsilon_{A_J}}\right) \tag{3.10}$$

$\varepsilon_{A_J,t}$ denotes an innovation in $v_{J,t}$ that occurs at random and that has only transitory effects on the overall level of technology (e.g. a shock to the utilisation of machines). For assessing compliance with Maastricht criteria, real convergence is explicitly incorporated in the model. It is then assumed that the trend of technology $E^T_{J,t}$ is exposed to structural shocks for a prolonged period of time. When assessing the sustainability of fulfilment of criteria (i.e. evaluating the uncertainty of outcomes), the trend term in technology will be assumed to be constant $\frac{E^T_t}{E^{T,ss}_t} = 1$.

The production structure is heterogeneous in both sectors $J = H, N$ in the sense that goods produced within a sector are imperfect substitutes to each other, which gives a producer pricing power over its product and introduces monopolistic competition. As in the preceding chapter, firms are not able to adjust prices every period such that output prices will be sticky in the short-run. Consequently nominal variables will affect real activity thereby introducing a non-trivial role for monetary policy. Like in section 2.2.2 on page 44, firms take into account that the set price needs to be optimal given that there is no possibility of resetting the price for their product in subsequent periods. As firms are owned by domestic households, profits are discounted by the discount factor used to evaluate their consumption streams. Accordingly, the stochastic discount factor $m_{t+k} \equiv \beta^k \frac{\mathcal{U}_C(\mathcal{C}_{t+k})}{\mathcal{U}_C(\mathcal{C}_t)}$ derived from the consumption asset pricing model (CCAPM) is employed for evaluating profits, as it was the case in the preceding chapter. Price setting of a forward-looking firm z in sector J can then be written as

$$\frac{p^o_{J,t}(z)}{P_{J,t}} = \frac{\rho_J}{\rho_J - 1} \frac{\mathcal{E}_t \sum_{s=0}^{\infty} (\theta_J \beta)^s (C_{t+s})^{-1} MC^J_{t+s} (\frac{P_{J,t+s}}{P_{J,t}})^{\rho_J} Y_{J,t+s}}{\mathcal{E}_t \sum_{s=0}^{\infty} (\theta_J \beta)^s (C_{t+s})^{-1} (\frac{P_{J,t+s}}{P_{J,t}})^{\rho_H - 1} Y_{J,t+s}} \tag{3.11}$$

$\frac{p^o_{J,t}(z)}{P_{J,t}}$ denotes the price set by firm z relative to the producer price index $P_{J,t}$ in sector J. Symmetry of firms (the weight of firms in sectoral gross value added is equal and negligible) yields that the average sectoral price coincides with the aggregate sectoral price index. Further, there exists a share ϖ_J in each sector of firms that cannot optimise in a forward-looking way when allowed to reset prices. They are backward-looking in the sense that they set prices by observing last periods gross sectoral inflation rate and the aggregate index of optimally set prices in that sector.

Log-linearising about a zero inflation steady state and following the steps as described in appendix C.9 of chapter 4 on page 321, one obtains that sectoral inflation rates evolve according to

$$\pi_{J,t} = \lambda_J^b \pi_{J,t-1} + \lambda_J^{mc} \widehat{MC}_{J,t} + \lambda_J^f \mathcal{E}_t \pi_{J,t+1} \tag{3.12}$$

The deep parameters are $\lambda_J^b = \frac{\omega_J}{\theta_J + \omega_J(1-\theta_J(1-\beta))}$, $\lambda_J^{mc_J} = \frac{(1-\omega_J)(1-\theta_J)(1-\theta_J\beta)}{\omega_J(1-\theta_J+\theta_J\beta)+\theta_J}$, $\lambda_J^f = \frac{\beta\theta_J}{\omega_J}\lambda_J^b$ where $J = H, N, F, N^*$. For the share of backward-looking firms ω_J attaining zero, we obtain the forward-looking New Keynesian Phillips curve.[11] Unlike the Phillips curve derived under (2.79) in chapter 2, constant returns to scale in the production technology (3.5) prevent that the price elasticity of demand ρ_J enters (3.12) explicitly. Hence, higher competition between firms will not reduce the effect of stickiness in the real marginal cost on the sectoral producer price inflation rate directly.

3.3.2 Goods and Financial Market Integration

There are factor markets for labour and capital as well as goods markets for the output produced in the H and N sectors. Further there are real and nominal asset markets where households and firms can invest in corporate bonds, money and sectoral physical capital. Financial markets are incomplete such that there are only available home and foreign bonds but no state-contingent claims. Unlike as in chapter 2, insurance of a specific household against idiosyncratic risk is thus possible only against average risk but not against state-contingent risk. This assumption seems justified here as the focus is on a small-open economy model of an emerging market economy where financial market integration with the world economy is less from complete compared to the situation of members within a currency union.

Home households buy home and foreign nominal corporate bonds (commercial paper). Foreign bonds can only be purchased at a risk premium, as in Ravenna and Natalucci (2008). Uncovered interest rate parity (UIP) holds in the model which arises as an arbitrage condition between holdings of home and foreign bonds. By maximising (4.1) with respect to $\{B_{H,.t}, B_{F,t}\}$ in the budget constraint (omitted here), one obtains

$$\mathcal{E}_t \left[\lambda_{t+1}^C \frac{P_t}{P_{t+1}} \left\{ (1+i_t) - (1+i_t^*) \frac{S_{t+1}}{S_t} \right\} \right] = 0 \tag{3.13}$$

[11] Also note that due to the constant returns to scale property of the production function, real marginal cost of the individual firm equal average real marginal cost in the sector. The case of labour being the only factor of production in a general production function is investigated in detail in chapter 2.

S_t denotes the nominal exchange rate. Accordingly, it is the price of foreign currency in home currency units that equilibrates the demand and supply of foreign and domestic currency. i_t is the three month money market rate at home and i_t^* denotes the foreign nominal interest rate at which home households can borrow. As explained in detail in the following chapter under 4.2.6, i_t^* will include a risk premium over the foreign nominal interest rate $\tilde{i}_t^*$ that increases in net real foreign liabilities.[12] Using that from the Euler equation $\mathcal{E}_t[\lambda_{t+1}^C \frac{P_t}{P_{t+1}}] = \frac{\lambda_t^C}{\beta(1+i_t)}$, one obtains that

$$\begin{aligned} 0 &= \mathcal{E}_t\left[\frac{\lambda_t^C}{\beta(1+i_t)}\left\{(1+i_t)-(1+i_t^*)\frac{S_{t+1}}{S_t}\right\}\right] \\ 1+i_t &= (1+i_t^*)\mathcal{E}_t\frac{S_{t+1}}{S_t} \end{aligned} \tag{3.14}$$

As an arbitrage condition, increases in the home nominal interest rate need to be accompanied by an expected depreciation of the home currency such that the home household is indifferent between holding the home or foreign corporate bond.

Openness in goods markets and asset structure makes international relative prices matter for domestic inflationary developments. On the goods markets, home agents face two relative prices that reflect relative scarcities of various goods, namely the domestic and the international relative price for goods. The same arguments as presented in chapter 2 under 2.2.1 on page 37 carry over. The consumption-based internal real exchange rate Q_t becomes relevant when deciding how to optimally mix tradables and non-tradable goods in the consumption basket C_t defined under (2.4), where again

$$Q_t \equiv \frac{P_{T,t}}{P_{N,t}} \tag{3.15}$$

Q_t denotes the domestic currency price of one unit of the tradables basket in units of non-tradables and reflects the relative price of tradables. A decrease in Q_t (an appreciation) implies that non-traded goods have become relatively more expensive.

International movements in the market for home and world industry goods are expressed by the terms of trade as already set out in chapter 2. T_t is given by the relative price of imported goods $P_{F,t}$ in terms of home produced tradables $P_{H,t}$, both expressed in domestic currency

$$T_t \equiv \frac{P_{F,t}}{P_{H,t}} \tag{3.16}$$

T_t therefore represents the relative price of foreign and domestic produced tradable goods and indicates international relative price movements: A decrease in T_t (an improvement in the terms

[12] The risk premium ensures the stability of the model. It prevents home households from borrowing infinite amounts of foreign debt.

of trade) implies that the purchasing power of the home currency increases such that the value of net wealth (net foreign assets) of the home economy (the expected present discounted value of all future trade surpluses and foreign reserve holdings) increases. We assume that the law of one price holds in the tradables (industry) sector, hence the domestic price of imports can be obtained by $P_{F,t} = P^*_{F,t} S_t$ where the exchange rate is in direct quotation (units of domestic currency needed to purchase one unit of foreign currency). It further follows that foreign (euro area) exporters cannot price discriminate across borders and that imports need to be invoiced in producer currency. However, as we allow for a heterogenous production structure in the H sector, the price of a good is not determined in the world market. The law of one price therefore (only) implies that a firm has to charge the same price at home and abroad, once expressed in same currency which leaves no room for price discrimination across borders. Accordingly, there is no scope for local currency pricing strategies to prevent exchange rate movements from affecting the price of goods in the importing country.

The assumption of the law of one price seemed to be straightforward in the currency union case where the nominal exchange rate is irrevocably fixed between regions. For the law of one price to hold here, we need to assume full pass-through from exchange rate fluctuations to the domestic currency price of imports. Hence exchange rate movements must be reflected in fluctuations in the price level of imports one-for-one, otherwise the price of imports would be different when expressed in the same currency. This requires flexibility in S_t and $P_{F,t}$. Hence the importing firms need to be fully competitive and have no pricing power over their respective imports when selling imported consumption goods to households and take $P_{F,t}$ as given. The assumption of full pass-through may appear somewhat strong, as there is some evidence that pass through from exchange rate changes to the domestic inflation rate might be limited depending on the type of good considered.[13]

In the open economy, the difference between total income and domestic consumption is defined as the current account. By employing the same steps as laid out in detail in chapter 4, we can write the balance of payments (BOP) as

$$B_{H,t-1} + S_t B_{F,t-1} + S_t B^C_{F,t-1} + NX_t + (-\frac{B_{H,t}}{1+i_t} - \frac{S_t(B_{F,t} + B^C_{F,t})}{1+i^*_t}) = Z_t - Z_{t-1}$$

We can denote net nominal foreign assets by $F_t = \frac{B_{H,t}}{1+i_t} + \frac{S_t(B_{F,t}+B^C_{F,t})}{1+i^*_t}$. F_t represents assets acquired at the end of period t which mature at beginning of period $t+1$. By using the uncovered interest parity condition (3.14) we obtain the evolution of real foreign assets F^r_t over

[13] Holtemöller (2007) finds a rather weak correlation of the change in the exchange rate and the domestic inflation rate.

time

$$F_t^r = \frac{1+i_{t-1}}{1+\pi_t} F_{t-1}^r + \frac{NX_t}{P_t} - (Z_t^r - Z_{t-1}^r \frac{P_{t-1}}{P_t}) \tag{3.17}$$

As borrowing cannot be redeemed by new borrowing indefinitely, we impose the no Ponzi-game condition, namely that the present value of real assets/liabilities far in the future is zero

$$\lim_{T\to\infty} \prod_{j=t}^{T} (\frac{1+i_{j-1}}{1+\pi_j})^{-1} F_T^r = 0$$

Hence the the economy remains solvent if

$$F_{t-1}^r = \mathcal{E}_t \sum_{s=t}^{\infty} (\prod_{j=t}^{s} (\frac{1+i_{j-1}}{1+\pi_j})^{-1}) \{Z_s^r - \frac{Z_{s-1}^r}{1+\pi_s} - NX_s^r\} \tag{3.18}$$

Further details on the derivation can be found in chapter 4 under (4.56) on page 227. A similar set of conditions was obtained in chapter 2 under (2.52).

3.3.3 Domestic Monetary and Fiscal Policy

Monetary Authority

The monetary authority follows an interest rate rule of the form

$$\frac{1+i_t}{1+i_t^{ss}} = \left(\frac{1+i_{t-1}}{1+i_{t-1}^{ss}}\right)^k \left(\left(\frac{1+\pi_t}{1+\pi_t^{ss}}\right)^{\varpi_\pi} \left(\frac{S_t}{S_t^{ss}}\right)^{\varpi_s} \left(\frac{GDP_t^r}{GDP_t^{ss,r}}\right)^{\varpi_{GDP}}\right)^{1-k} \exp\left[\varepsilon_{i,t}\right] \tag{3.19}$$

i_t denotes the monetary policy instrument of the central bank, where open market operations directly influence the three month money market rate. $\varepsilon_{i,t}$ denotes an innovation in the domestic interest rate (a monetary policy shock) that can be due to policy errors. The commitment to this rule is assumed to be from a timeless perspective, in the sense that constraints for the policymaker at date zero are the same as at any date before or after. Hence the constraints (the structure of the economy) are valid from the infinite past and there is no incentive to renege on implementing the policy, see Woodford (2003) for a discussion. Monetary policy is assessed in respect to deviations from trend/steady state. Accordingly, policy cannot influence trend developments such as real convergence - e.g. permanent shocks to $A_{J,t+j}$ defined in (3.6) - that puts ‘low-frequency’ pressure on prices.

Further, comparing (2.46) in chapter 2 with (3.19) one observes that the rule proposed here does not differentiate between efficient and inefficient fluctuations in target variables. The main justification can be seen in the task to be accomplished by monetary policy here, namely assisting the economy in fulfilment of the monetary Maastricht criteria. These criteria are not microfounded either. In consequence, there will be no micro-founded loss function the policy maker seeks to minimise. (3.19) is set up in a way that specific arrangements regarding the exchange rate can be derived for policy response coefficients (elasticities) ϖ_π, ϖ_s, and ϖ_{GDP} taking specific values. This approach for distinguishing regimes is common in the literature, see Ravenna and Natalucci (2008).

Currency Board or Fixed Exchange Rate Regime The currency board and fixed exchange rate regime represent cases where the nominal interest rate i_t is either tied to a foreign currency or the exchange rate S_t is under strict target, against a central parity. Both regimes are modeled as targeting the nominal exchange rate about its long-run equilibrium value, which is represented by a high penalty coefficient on an exchange rate target. As there is no control about the domestic inflation rate, the weight on this term is zero. As a consequence, under this regime the rule (3.19) exhibits $\varpi_\pi = 0$ and $\varpi_s \to \infty$. These assumptions apply for all Baltic countries (Estonia, Latvia, and Lithuania) and Bulgaria. The exchange rate does not enter the policy rule in first differences (the growth rate of S_t, when steady state growth is absent) but relative to its long-run equilibrium value $\frac{S_t}{S_t^{ss}}$.

Managed Floating Regime Under the managed float regime, the monetary authority also has concerns about the evolution of the domestic inflation rate. Thus both inflation and exchange rate terms have positive weights, such that $\varpi_\pi > 0$ and $\varpi_s > 0$. We classify the Czech Republic, the Slovak Republic and Hungary as managed floaters in line with the exposition in table 3.2. Romania, that follows a crawling (an adjustable) peg, is also classified as a managed floating regime.

Floating Regime Under the pure float, the monetary authority has full control over the home CPI inflation rate π_t. Accordingly, only the inflation coefficient has positive weight and there is no concern for other monetary objectives. As a consequence $\varpi_\pi \to \infty$ and $\varpi_s = 0$. We therefore would obtain *strict* inflation targeting in case there is no output weight $\varpi_{GDP} = 0$.[14] Floating exchange rates are expected to reduce macroeconomic and financial volatility. The nominal exchange rate equilibrates supply and demand for the home currency on the forex market freely that arises from exchange of corporate bonds B_H, B_F as well as exported/imported goods Y_H^*, Y_F. Only Poland is considered as having a flexible exchange rate regime, in line with the exposition under 3.2.

[14] If there is a positive weight for the output gap, we would obtain flexible inflation targeting.

Central Bank Balance Sheet Domestic money supply M_t^S is endogenously determined as monetary policy is conducted by setting the policy instrument i_t via the instrument rule (3.19). The nominal value in domestic currency can be derived from the balance sheet identity of the central bank. Therefore, the stock of net foreign assets F_t^S and the stock of domestic loans to private banks C_t^S (and potentially to the government) on the asset side have to equal the stock of base money (high powered money) on the liability side

$$\underbrace{M_t^S}_{\text{liabilities}} \equiv \underbrace{F_t^S + C_t^S}_{\text{assets}} \tag{3.20}$$

As the credit channel from the central bank to corporate banks is not our focus of analysis, we set $C_t^S = 0$ throughout. By definition, nominal and real money supply growth are linked by

$$\frac{M_t^S}{P_t} / \left(\frac{M_{t-1}^S}{P_{t-1}} \right) = M_t^S / M_{t-1}^S \left(\frac{P_t}{P_{t-1}} \right)^{-1} = \frac{1+\theta_t}{1+\pi_t} \tag{3.21}$$

where $M_t^S / M_{t-1}^S \equiv 1 + \theta_t$ is the (time-varying) gross growth rate of money and $\left(\frac{P_t}{P_{t-1}}\right) = 1 + \pi_t$ is the gross inflation rate.[15] It follows that in the open economy, nominal money supply (its growth rate) is endogenous and determined by the stock of nominal foreign assets F_t^S which are on their part depending on the domestic interest rate i_t via the uncovered interest rate parity (3.13).

Government

We assume that the government budget constraint needs to be balanced every period such that there cannot arise any deficits. As a consequence, no issuing of domestic nominal government bonds is allowed

$$T_t^{\#} + v_t = P_{H,t} G_{H,t} + P_{N,t} G_{N,t} + Q_t^{\#} \tag{3.22}$$

Current government expenditures (which 'go into the ocean', i.e. they are just consumptive) and lump sum transfers $Q_t^{\#} = \int_0^1 Q_t^j dj$ (social benefits) to the households have to be financed by lump sum taxation $T_t^{\#} = \int_0^1 T_t^{\#,j} dj$ and transfers v_t from the central bank (seigniorage earnings, interest income on foreign bond holdings). Unlike as in chapter 2 in (2.48), we assume that the government can collect lump-sum taxes $T_t^{\#}$ that will not distort labour supply decisions of households. This assumption will also lead to equalisation of gross nominal wages in the economy.

[15] If θ_t could be exogenously set, monetary policy would be conducted by a money growth rule similar to Walsh (2003, p. 83).

Further, we assume that government purchases only fall on domestic goods and services. Decisions on government spending $G_{J,t}$ follow simple rules of the form

$$\frac{G_{J,t}}{G_J} = \left(\frac{G_{J,t-1}}{G_J}\right)^{\rho_{G_J}} \left(\frac{Y_{J,t}}{Y_J}\right)^{a_J} \exp\left[\epsilon_{G_J,t}\right] \tag{3.23}$$

As government spending 'goes into the ocean', it is of no utility for the private sector, so there is a direct crowding out of private through public consumption (households are Ricardian). In accordance with the monetary rule, the output gap $\frac{Y_{J,t}}{Y_J}$ again will not distinguish between potential and natural rate movements in output where the same explanation applies. When analysing the prospects of meeting the Maastricht criteria, we abstract from output gap stabilisation, $a_J = 0$. Active, counter-cyclical policies, $a_J < 0$ will however be an issue when discussing options how the likelihood of meeting the criteria can be increased. $\epsilon_{G_J,t}$ denotes the innovation in government spending which is assumed to be white noise. Eventually, government spending as share of GDP is given by

$$\frac{G_t}{GDP_t} \equiv \frac{P_{H,t}G_t^H}{GDP_t} + \frac{P_{N,t}G_t^N}{GDP_t} = \frac{P_{H,t}G_t^H}{P_{H,t}Y_{H,t}}\frac{P_{H,t}Y_{H,t}}{GDP_t} + \frac{P_{N,t}G_t^N}{P_{N,t}Y_{N,t}}\frac{P_{N,t}Y_{N,t}}{GDP_t} \tag{3.24}$$

The assumptions about fiscal policy make clear that the fiscal criteria cannot be failed in this framework.

3.3.4 Market Equilibrium

A dynamic stochastic general equilibrium is a competitive market equilibrium with a sequence of prices

$$\left\{P_{H,t}, P_{N,t}, P_{T,t}, P_{F,t}, P^*_{F,t}, S_t, i_t, R_{J,t}, Q_{J,t}, T_t\right\}_{t=0}^{\infty}$$

and a sequence of allocations

$$\left\{\begin{array}{c} Y^S_{J,t}(z), Y^{D,j}_{J,t}, L^{S,j}_{J,t}, L^D_{J,t}(z), , B^{S,j}_{H,t}, B^{D,j}_{H,t}, B^{S,j}_{F,t}, B^{D,j}_{F,t}, G_{J,t}, W_t, \\ C^j_{J,t}, Q_t, T_t, \Pi_{J,t}(z), I^{S,j}_{J,t}, I^{D,j}_{J,t}, K^{S,j}_{J,t}, K^D_{J,t}(z), M^S_t, M^{D,j}_t, C^S_t, C^D_t, Z^S_t, Z^D_t \end{array}\right\}_{t=0}^{\infty}$$

where $J = H,N,\ j,z \in [0,1]$ for given exogenous structural shocks

$$\left\{\epsilon_{A_H,t}, \epsilon_{A_N,t}, \epsilon_{G_H,t}, \epsilon_{G_N,t}, \epsilon_{C^*_H,t}, \epsilon_{\tilde{\imath}^*,t}, \epsilon_{P^*,t}, \epsilon_{Z,t}, \epsilon_{i,t}\right\}_{t=0}^{\infty}$$

an initial stock of physical capital $K_{J,-1}$, initial home and foreign issued corporate bond holdings $B_{H,-1}$, $B_{F,-1}$ as well as fiscal and monetary policies such that

- any household j of the home economy maximises utility by choosing $C_{J,t}^{j}$, labour supply $L_{J,t}^{S,j}$, investment $I_{J,t}^{S,j}$, capital supply $K_{J,t}^{S,j}$ and by demanding home and foreign corporate bonds $B_{H,t}^{D,j}$, $B_{F,t}^{D,j}$ as well as money balances $M_{t}^{D,j}$.

- any firm z in each sector maximises profits by using labour input $L_{J,t}^{D}(z)$ as well as capital input $K_{J,t}^{D}(z)$. It takes prices of labour W_t and capital $R_{J,t}$ as given. It sets prices of its output good relative to the average price $p_J(z)/P_{J,t}$, but cannot influence the producer price index $P_{J,t}$ which it takes as given.

- the government meets the per period budget constraint

$$T_t^{\#} + v_t = P_{H,t}G_{H,t} + P_{N,t}G_{N,t} + Q_t^{\#}$$

  and commits to implementing the fiscal rule (3.23).

- the monetary authority commits to following the monetary policy rule (3.19).

- all markets clear, so we can omit superscripts representing supply S and demand D to indicate that the value of the variable denotes an equilibrium relationship.

- the balance of payments identity holds and real net foreign assets are sustainable

$$F_{t-1}^{r} = E_t \sum_{s=t}^{\infty} \left(\prod_{j=t}^{s} \left(\frac{1+i_{j-1}}{1+\pi_j} \right)^{-1} \right) \left\{ Z_s^r - \frac{Z_{s-1}^r}{1+\pi_s} - NX_s^r \right\}$$

  As a consequence, initial real net foreign asset holdings F_{t-1}^{r} must equal the expected discounted income stream generated by foreign reserve holdings and future trade surpluses.

3.4 The Impact of the Exchange Rate Regime on the Business Cycle

In this section we explore the basic mechanics of the model when exposed to shocks and the response of monetary policy under differing assumptions about the exchange rate regime in place. In section 3.5 we move on to estimating the model for each EU9 country prior to the Maastricht criteria check under 3.6. As in the preceding chapter, there is no explicit (analytical) solution to the original non-linear model. For small enough stochastic disturbances around the non-linear deterministic steady state, we can however restrict attention to analysing impulse responses and joint dynamics in the log-linearised model. For any variable X_t in levels, the percentage deviation of the variable from its period t steady state X_t^{ss} is by definition

$\hat{X}_t \equiv \frac{X_t}{X_t^{ss}} - 1$ such that $X_t \equiv X_t^{ss}\left(1 + \hat{X}_t\right)$. Then for $\hat{X}_t$ small, we can approximate $\hat{X}_t$ by the log-difference $\ln X_t - \ln X_t^{ss}$ such that $X_t \simeq \left(1 + \ln \frac{X_t}{X_t^{ss}}\right) X_t^{ss}$.[16] In order to make the model as empirically plausible as possible, the small open economy is exposed to a range of structural (hence unobservable) shocks. Shocks can be classified as follows:

- *Domestic real shocks*: Supply side shocks arising from cyclical innovations ϵ_{A_J} in the level of technology A_J as well as from permanent innovations η_J. Variability in technology in the tradables sector that stems from new technology employed is an important source of overall fluctuations in output in new EU members. Long-run shocks describe the long-run development in the level of production which reflects real catch up processes. Furthermore, we include demand side shocks arising from innovations ϵ_{G_J} in sectoral government spending G_J. Fiscal spending shocks are a proxy for infrastructure investments undertaken and the necessary reorganisation of social security systems on the path of transition to a market-based economy.

- *Domestic nominal shocks*: Innovations ϵ_i in the monetary policy rule that summarise variability in the policy rate not explainable by endogenous responses to policy targets as well as shocks to the growth rate of official foreign reserve holdings dZ from innovations ε_Z.

- *Foreign (euro area) shocks*: Shocks to the foreign inflation rate π^*, foreign interest rate i^*, and exports C_H^*, triggered by innovations ϵ_{π^*}, ϵ_{i^*}, and $\epsilon_{C_H^*}$. Booms in export demand will cause the economy to be affected by terms of trade shocks. Shocks in the foreign inflation rate will impact on external competitiveness of the country. Movements in the interest rate differential will put the domestic nominal exchange rate under pressure.

Some details on the log-linearisation seem worth noting. For technology in the services sector $A_{N,t}$, by using (3.6) one can write

$$\hat{A}_{N,t} = \hat{E}_{N,t}^C \tag{3.25}$$

such that $\hat{E}_{N,t}^T = 0$ throughout. The cyclical component $\hat{E}_{N,t}^C$ follows a stationary autoregressive process which induces stationary fluctuations in factor productivity about its steady state. For

[16] The log-linear approximation is robust to assumptions about the time path of X_t^{ss} (e.g. X_t^{ss} could be a trend-stationary variable or follow a random walk with drift). The decomposition is a typical multiplicative trend-cycle decomposition, known from structural time series models. Once X_t^{ss} is time-invariant (when steady state growth is absent), X_t^{ss} coincides with its initial steady state value and time subscripts can be omitted $X_t^{ss} = X_0^{ss} = X$. In the absence of steady state growth, $X_t^{ss} = X^{ss}$, period on period growth rates coincide with period on period cyclical log-deviations, $\ln X_t - \ln X_{t-1} \simeq \hat{X}_t - \hat{X}_{t-1}$.

tradable productivity, we can decompose

$$\hat{A}_{H,t} = \hat{E}^{T}_{H,t} + \hat{E}^{C}_{H,t} \tag{3.26}$$

Real convergence is brought about by the permanent component $\hat{E}^{T}_{H,t}$ being exposed to shocks such that technology is pushed on higher levels permanently (the deviation from the initial steady state A_J will be permanent). The fiscal rules can be written in log-linear form as

$$\hat{G}_{J,t} = \rho_{G_J} \hat{G}_{J,t-1} + \epsilon_{G_J,t} \tag{3.27}$$

by using that $\hat{G}_{J,t} \equiv \ln G_{J,t} - \ln G_J$. Fiscal policy follows AR(1) processes such that fiscal policy is credibly implemented by the government. The rules abstract from any other target variables, like output gap targeting.[17] As a consequence, the aim of fiscal policy is to deviate little from a constant spending path through time.

For the other exogenous processes we obtain accordingly

$$\begin{aligned}
\hat{Z}_t - \hat{Z}_{t-1} &= \rho_Z \left(\hat{Z}_{t-1} - \hat{Z}_{t-2} \right) + \epsilon_{Z,t} && (3.28)\\
\hat{\pi}^*_t &= \rho_{\pi^*} \hat{\pi}^*_{t-1} + \epsilon_{\pi^*,t} && (3.29)\\
\widehat{\imath}^*_t &= \rho_{\widehat{\imath}} \widehat{\imath}^*_{t-1} + \epsilon_{i^*,t} && (3.30)\\
\hat{C}^*_{H,t} &= \rho_{C^*_H} \hat{C}^*_{H,t-1} + \epsilon_{C^*_H,t} && (3.31)
\end{aligned}$$

where $\hat{Z}_t - \hat{Z}_{t-1}$ denotes quarter-on-quarter growth in foreign reserve holdings[18], $\hat{\pi}^*_t$ is the foreign (euro area) inflation rate, $\widehat{\imath}^*_t$ denotes the foreign three months rate and $\hat{C}^*_{H,t}$ is import demand by the world economy (the euro area). All series are in deviations from steady state. Further details on the log-linearisation are provided in appendix B.4 on page 298. The complete log-linear model can be found in appendix C.2 on page 304. The model is solved with `dynare`, see Griffoli (2007) and Juillard (2001).

[17] However, this assumption can be relaxed easily, as will be the case when discussion policy measures that can increase the probability of mastering the Maastricht criteria. Especially in the Baltic States, the very successful public finance performance cannot be considered windfall profits, see Geeroms (2007). Adjustments resulted from sometimes painful restrictive and active fiscal stabilisation policies.

[18] Note that we assume that foreign reserves growth $\hat{Z}_t - \hat{Z}_{t-1}$ is following an exogenous process. If both the current account CA_t and the financial account FA_t were given, it would be directly determined and hence be endogenous also. However we can argue that demand for home money arises from speculative behaviour in the financial account FA_t, which is prone to shocks. Also note, that we neglected the role of FDI shocks. FDI shocks would affect the stock of capital and subsequently sectoral production. This would add a further indirect channel that is affected by capital adjustment costs. This channel is incorporated in the exposition of the model in chapter 4.

The calibration of the following impulse responses is based on a 'hypothetical average country' aimed at reflecting the situation of the average new EU member state. Average values of country-specific steady state values are taken as presented in table 3.4 below. Deep parameters guiding inflation dynamics and structural shocks resemble those presented in chapter 2, see the discussion of scenario 4 under 2.4. We restrict attention to an innovation in technology in the home tradable sector, $\epsilon_{A_H,t}$, government spending that falls on non-tradables $\epsilon_{G_N,t}$, an innovation in the home nominal interest rate $\epsilon_{i,t}$ and a shock to the foreign inflation rate (a terms of trade shock), $\epsilon_{\pi^*,t}$. We investigate impulse-responses in the model under different exchange rate regimes by means of a single on-off innovation in exogenous processes such that $\epsilon_{x,s} > 0$, for $s = t$ and zero otherwise. The transmission of any of the shocks makes visible differences how the exchange rate regime might determine the response of key economic variables along the business cycle.

3.4.1 Domestic Real Supply Shocks

Temporarily higher productivity in the industry sector brought about by a cyclical increase in $A_{H,t}$ raises industry output about trend equally under all three regimes. This becomes clear from the upper panel of figure 3.1 on page 156. Gross value added in the industry sector Y_H is about one percent above trend for the first five periods following the shock. Also, the response of Y_N is roughly equal and accordingly, the response of real GDP is equal as well, as can be seen in the lower panel of 3.2. As firms have pricing power over their product due to heterogeneity in product markets, goods prices are lowered by firms that are able to do so such that the sectoral inflation rate declines temporarily.[19] Due to producer currency pricing - goods are invoiced in currency of the origin of production and there is no price discrimination across borders - the foreign price of home produced tradables will decline as well. The overall effect however depends on the development of the exchange rate such that the law of one price is ensured. The assumed presence of backward-looking firms essentially stretches the response in variables over time. The effect is thus more muted than in case of both a high price-resetting probability and a low share of price-indexing firms.

Under the managed float and the fixed exchange rate regime, the internal exchange rate appreciates (Q_t declines) implying that non-tradable goods have become relatively more expensive, whereas a lasting depreciation will occur under the floating regime. At the same time, home produced tradables can be sold at lower prices which leads to a worsening in the terms of trade T_t (an increase) under all regimes. This effect is strongest in case of the flexible ex-

[19] If prices would be fully flexible (the case where $\theta_J, \varpi_J \to 0$) and in the absence of persistence in the shock process, the inflation rate would immediately return to its equilibrium value following the unexpected shock.

change rate regime. The purchasing power of the home currency deteriorates, as one unit of home currency buys less imported goods than before the shock. Simultaneously, demand for home exports will increase. These developments put pressure on the nominal exchange rate S_t. Eventually however, a depreciation will show up under the floating and managed floating regime, which is strongest in case the exchange rate can vary freely. Under the fixed exchange rate regime, the exchange rate needs to be stabilised. Monetary policy can accommodate the productivity shock by decreasing its policy rate under the floating and managed floating regime whereas it remains silent under the fixed rate. CPI inflation therefore experiences the sharpest decrease in case of the fixed regime and is rather muted if there is flexibility in the exchange rate.

3.4.2 Domestic Real Demand Shocks

Following a fiscal shock that falls on services (which also serves as a proxy for a general demand shock), production in the N sector increases. As higher demand for N goods is not accompanied by an equal increase in supply possibilities (the supply curve has not moved), prices of services need to rise in order to meet the higher demand. This results in positive sectoral inflation where fluctuations in the sectoral price level are triggered by firms that can adjust prices. The effect is equal under all exchange rate regimes. Whereas the internal real exchange rate (the relative price of non-tradables) and the terms of trade (the relative price of tradables) moved in opposite directions following a productivity shock, now both relative prices decrease (see the lower panel of figure 3.3 on page 158). As non-tradable goods have become relatively more expensive, an appreciation of the internal exchange rate builds up and the terms of trade improve such that a positive wealth effect results. Both effects are strongest in case of the flexible exchange rate regime. Accordingly, home exports become relatively more expensive. As net exports contribute to the change in the net real foreign asset position, a decline in the current account shows up as depicted in the lower panel of figure 3.4.

The twin increase in industry and services inflation rates, $\pi_{H,t}$ and $\pi_{N,t}$, causes an increase in the CPI rate π_t. There is a direct effect from $\pi_{H,t}$ as the foreign inflation rate remains unaffected by the home fiscal shock and the home economy cannot influence the price of foreign produced goods as it is small in economic size. The effect is strongest under the fixed exchange rate regime. The monetary authority will in response dampen domestic demand by increasing the nominal interest rate in the flexible regime and the managed float regime strongly, and in a rather muted fashion in case of the fixed exchange rate regime. The increase in the policy rate causes a drop in the level of overall consumption. Forward-looking agents can increase lifetime

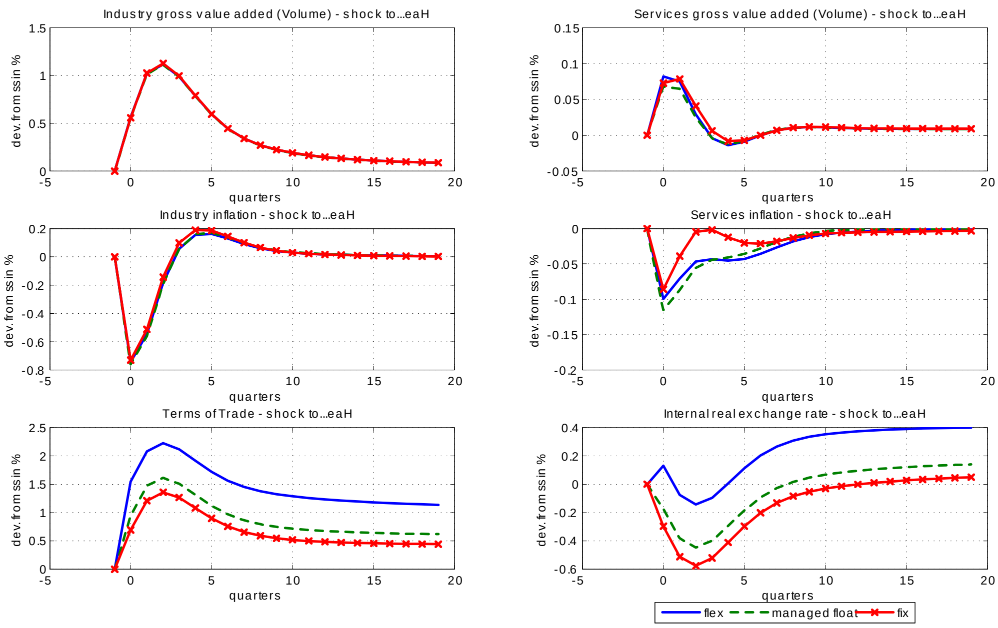

Figure 3.1: Improvement in industry productivity: response of gross value added and relative prices.

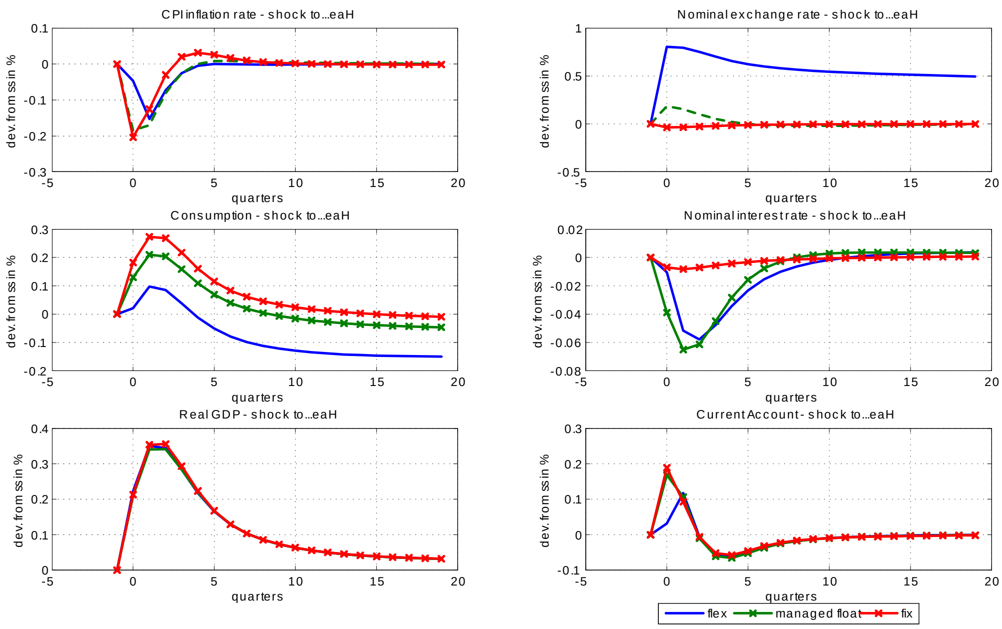

Figure 3.2: Improvement in industry productivity: response of the CPI inflation rate, real GDP, the nominal exchange rate, and the nominal interest rate.

consumption by saving (purchasing bonds) and realise higher returns on bonds which will yield higher consumption streams in the future. In addition, the decrease in current consumption is also influenced by the increased government spending that crowds out private consumption expenditures directly.

3.4.3 Domestic Monetary and Foreign Shocks

In the presence of nominal rigidities where prices do not respond immediately to shocks and where backward-looking firms generate intrinsic inflation persistence in each sector, monetary policy is non-neutral. The innovation in the home interest rate appreciates the internal real exchange rate and improves the terms of trade which becomes clear from figure 3.5. Thus, domestic assets are revalued and home wealth increases. Accordingly, consumption increases. Under the flexible exchange rate regime, the nominal exchange rate will appreciate via the UIP channel as home bonds have now become relatively more attractive for foreigners. Under the other regimes, the effect on consumption is rather muted. Higher asset demand cannot appreciate the nominal exchange rate under the fixed regime and only slightly under the managed float regime. Accordingly, pressures from the terms of trade and the internal real exchange rate will increase the domestic consumer price inflation rate. Accordingly, monetary tightening will be required in theses cases such that eventually a large increase in the nominal interest rate results.

An increase in the foreign inflation rate (a price increase in imported goods $\pi^*_{F,t}$) translates to the home economy depending on the stabilisation strategy. The gain in home external competitiveness is supported by a depreciation in the home exchange rate S_t initially by more than 2% in case of consumer price targeting and about 0.5% under the managed float regime, as the upper panel of figure 3.8 on page 160 reveals. As a result, the gain in competitiveness translates into a twin depreciation in the internal real exchange rate Q_t and in the terms of trade T_t, which in turn increases production in both sectors in case of the flexible exchange rate regime. Negative wealth effects in all three regimes result as the purchasing power of the home currency has decreased. Accordingly, overall consumption declines in all cases as households want to smooth consumption over time.

3.5 Estimation

We estimate the log-linearised model on a country-by-country basis for each EU9 member conditional on its regime. Among other things, estimation can clarify the relative importance

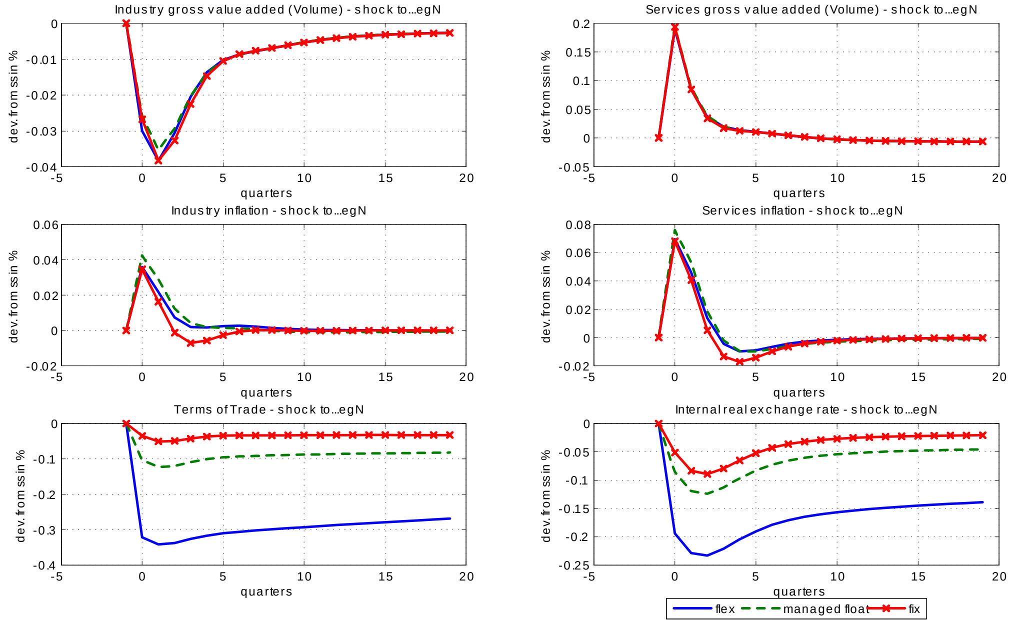

Figure 3.3: Fiscal spending shock: response of gross value added and relative prices.

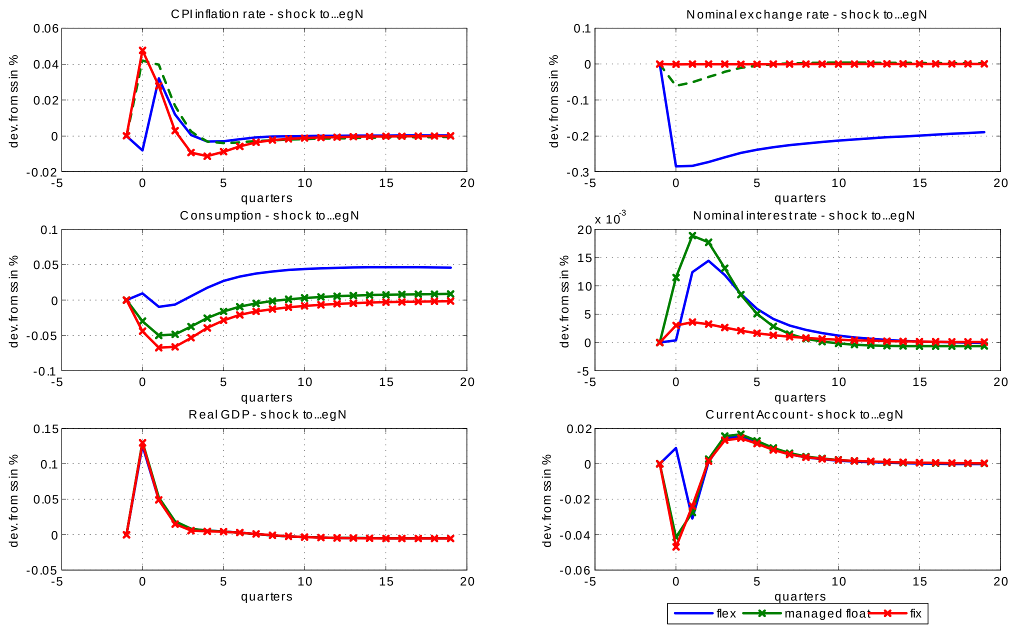

Figure 3.4: Fiscal spending shock: response of the CPI inflation rate, real GDP, the nominal exchange rate, and the nominal interest rate.

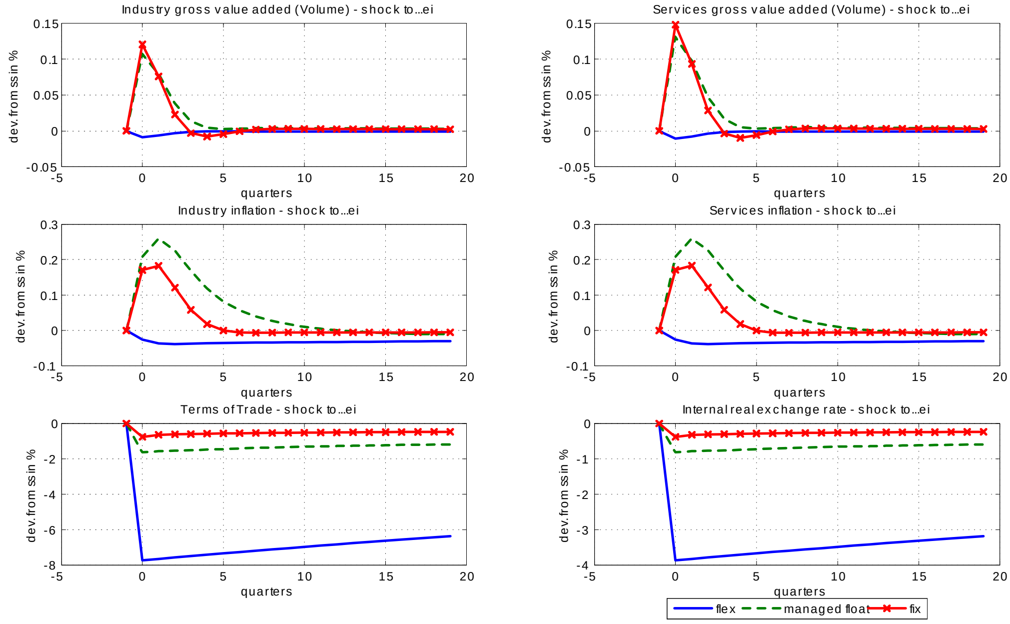

Figure 3.5: Monetary policy shock: response of gross value added and relative prices.

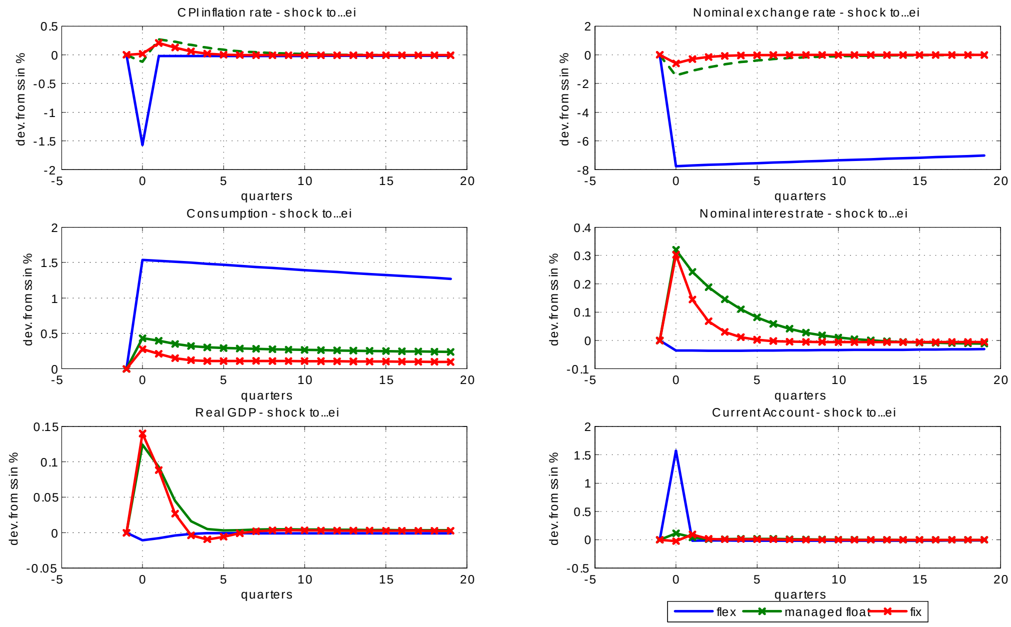

Figure 3.6: Monetary policy shock: response of the CPI inflation rate, real GDP, the nominal exchange rate, and the nominal interest rate.

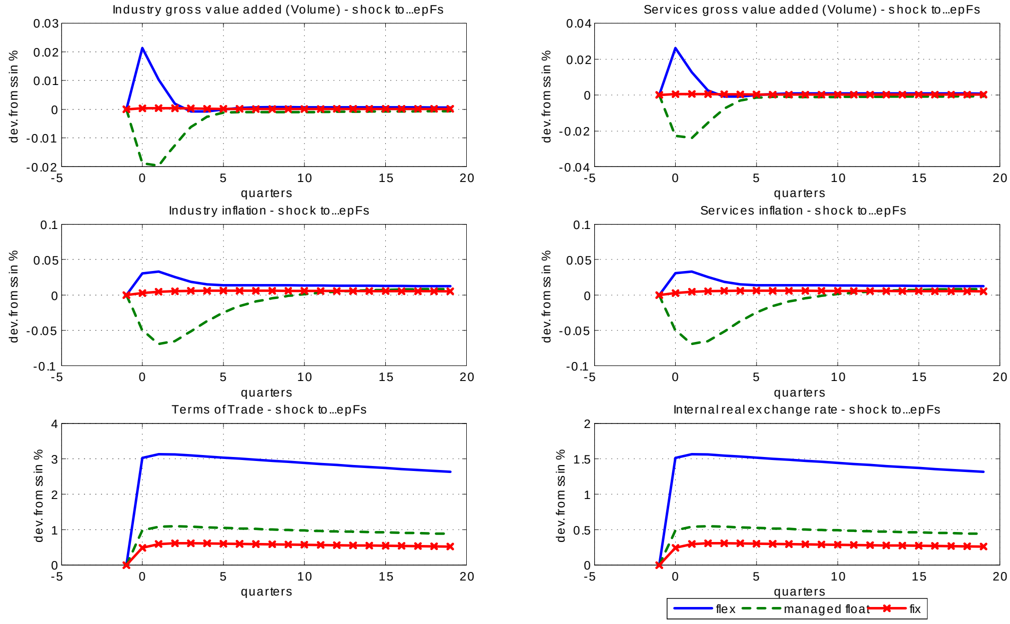

Figure 3.7: Foreign price shock: response of gross value added and relative prices.

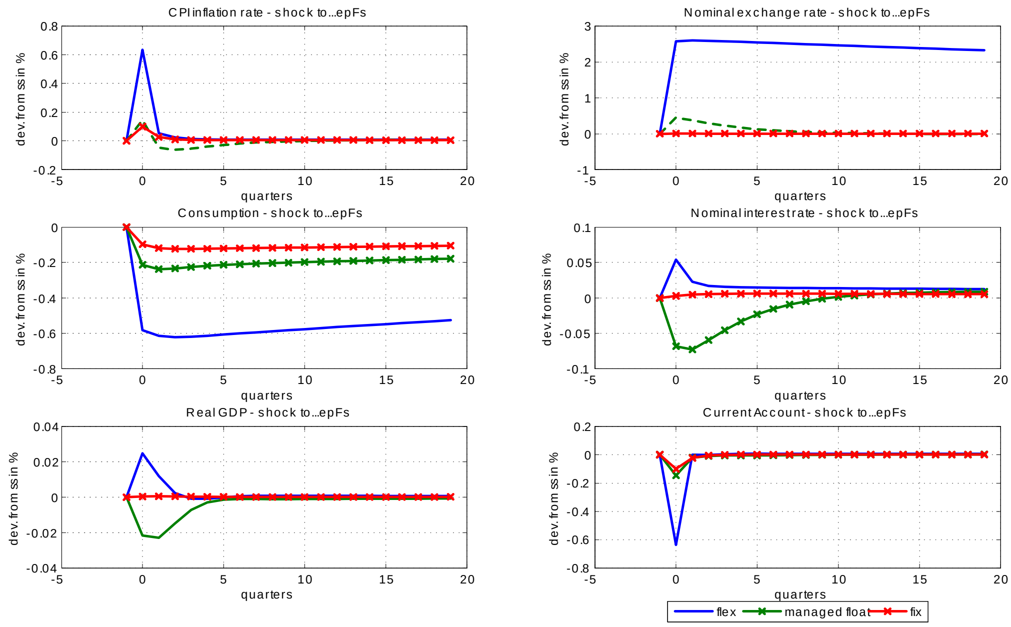

Figure 3.8: Foreign price shock: response of the CPI inflation rate, real GDP, the nominal exchange rate, and the nominal interest rate.

of structural shocks in each country which is essential before approaching the main objective of the chapter, i.e. checking the fulfilment of Maastricht criteria by new EU members. Structural macroeconometric frameworks can suitably be approached by Bayesian estimation of the log-linearisation of the original non-linear model, see DeJong (2007, chapter 9). We first highlight some features of that technique, before we discuss data-related issues. We then explain, which model parameters are calibrated and which can be taken from the data directly as they apply to steady state relationships. Eventually, we present key estimated parameters.

Contrary to classical inference, Bayesian analysis does not need to rely on asymptotic theory for inference but Bayes' rule to judge the validity of the economic model, see Block and Thams (2007). In Bayesian estimation, the true parameter is considered as stochastic and data is assumed to be fixed, exactly opposite to classical inference. Prior beliefs are updated with the log-likelihood function of the model in its state-space form to get the multivariate parameter posterior density.[20] Choosing priors (inspired by economic theory) can be considered analogous to identification in time series models. Bayesian analysis does therefore not suffer from the degrees of freedom problem associated with classical inference: The imposition of prior distributions makes the method applicable for small sample sizes, the case we face for new EU members. Quarterly national accounts data can be considered reliable only from 1999 onwards.

Bayesian estimation of DSGE models has three main characteristics (Schorfheide and An, 2007, p. 123). First, unlike GMM estimation based on equilibrium relationships such as the consumption Euler equation, Bayesian analysis is a full-information method. It fits the solved DSGE model to a vector of aggregate time series, like Maximum-likelihood methods.[21] Second, the estimation is based on the likelihood function generated by the DSGE model rather than minimising the distance between impluse responses in the DSGE model and VAR impulse responses, as proposed by Christiano et al. (2005). Third, prior distributions can be used to incorporate additional information available from related work into parameter estimation. As all estimation and evaluation methods, Bayesian estimation is confronted with the challenges of potential model misspecification and possible lack of identification of parameters of interest.[22]

The estimation strategy pursued here is inspired by the estimation of DSGE models in

[20] With uniform priors, Bayesian estimation becomes Maximum Likelihood (ML) estimation. However, then it is possible that the optimiser will hit the boundaries. Loose, but curved priors will yield better results, see Griffoli (2007).

[21] A system-wide estimation delivers more efficient estimates of the structural model parameters. It provides a consistent estimate of the structural shock processes driving economic developments, as argued in Smets and Wouters (2004).

[22] We do not discuss model misspecification here. Identification however will become an issue when we try to estimate the deep parameters that guide the inflation dynamics.

Schorfheide (2000), Negro et al. (2007) and Smets and Wouters (2003). The estimation routine is carried out using `dynare`, see Adjemian et al. (2007, p. 19). First, the linearised version of the rational expectations model is solved which allows to represent the dynamics in a state-space representation (non-linear in the deep parameters). Second, the posterior kernel of the model (i.e. the log-prior densities and the log-likelihood of the model obtained by running a Kalman filter) is evaluated and maximized. The posterior mode and the approximate covariance matrix, based on the inverse Hessian evaluated at the mode, are obtained by numerical optimisation on the log posterior density. Third, once the posterior mode and the approximate covariance matrix is found, the entire posterior distribution is subsequently explored by generating draws using the Metropolis-Hastings algorithm. The proposal distribution is taken to be the multivariate normal density centered at the previous draw with a covariance matrix proportional to the inverse Hessian at the posterior mode.[23]

3.5.1 National Accounts Data

We use twelve time series on domestic developments for each of the EU9 as well as two foreign (euro area) series. As quarterly data is employed, data is seasonally adjusted and adjusted by working days. In detail, we use the following data:

- PY_t: gross value added in total (in millions of euro at 1995 prices and exchange rates)
- $P_H Y_{H,t}$: gross value added in total industry (excluding construction, in millions of euro at 1995 prices and exchange rates)
- $\triangle defl_t$: percentage change in deflator of gross value added for all sectors over previous period
- $\pi_{H,t}$: percentage change in deflator in total industry (excluding construction) over previous period
- π_t: quarter on quarter CPI inflation rate
- i_t: domestic three month money market interest rate
- $P_C C_t$: final consumption expenditures of households (in millions of euro at 1995 prices and exchange rates)
- $P_G G_t$: final consumption expenditure of general government (in millions of euro at 1995 prices and exchange rates)

[23] See Adolfson et al. (2007) for a detailed description of the steps involved.

- $P_F C_{F,t}$: imports of goods and services
- $P_H^* C_{H,t}^*$: exports of goods and services
- S_t: nominal exchange rate in direct quotation (home currency / 1 euro)
- Z_t: official foreign reserves
- π_t^*: percentage change in price index in total industry (excluding construction) in the euro area over previous period
- $\tilde{i}_t^*$: nominal interest rate in the euro area (three month money market rate)

Note that constant price levels in data measurement $\{P, P_H, P_H^*, P_C, P_G, P_F\}$ correspond to steady state price levels in the model. Gross value added in the manufacturing sector is taken as a proxy for tradables production in volumes, $P_H Y_{H,t}$. All other elements of gross value added are treated as non-tradables/services $P_N Y_{N,t}$, as we use these terms interchangeably. $P_N Y_{N,t}$ thereby presents an unobservable (latent) variable. This classification is chosen, acknowledging that public spending and social security spending will not be determined by market prices. As a result, gross value added at constant prices, $P_H Y_{H,t} + P_N Y_{N,t}$ adds up to real GDP, GDP_t^r and gross value added at market prices $P_{H,t} Y_{H,t} + P_{N,t} Y_{N,t}$ yields nominal GDP_t.[24] We proceed accordingly in case of price changes in deflators, where $\triangle defl_t$ and $\pi_{H,t}$ are given by the data and $\pi_{N,t}$ is obtained residually.[25] The exchange rate S_t is excluded from the estimation in case of fixed exchange rate countries/currency boarders as it does not contain any information on variability in that cases. π_t^* denotes the price change in the production of euro area industry goods instead of the change in the euro area harmonised consumer price index. This assumption is implied by the model structure where all goods are traded at foreign. Foreign fluctuations in prices therefore represent price developments in the foreign industry sector.

Graphical inspection of the data shows, that all inflation series were characterised by a disinflation period in the nineties following the transition phase in all new eastern EU members. Therefore, using data from 1995 onwards would not be appropriate. This phase seems to have seized in 1999. We therefore use only data from 1999 onwards, loosing 16 observations in case of countries for which data from 1995 onwards was available. We hence have 32 observations in total for each country.[26] To balance the panel, also data for the euro area is used from 1999

[24] See equation (B.5) in the appendix.

[25] In case of Bulgaria and Romania, there are no historical series available on sectoral data. We nevertheless estimate the model conditional on the remaining available macro series.

[26] We observed a great improvement in fit when using the shorter sample by inspecting the kernel of the log posterior. The kernel decreased largely in value.

Steady States	$\frac{Z}{F}$	$\frac{P_H Y_H}{GDP}$	$\frac{NX}{P_H Y_H}$	$\frac{P_H C_H^*}{P_H Y_H}$	$\frac{L_H}{L}$	$\frac{G}{GDP}$	γ	$\frac{R^H P_H K^H}{P_H Y_H}$	$\frac{R^N P_N K^N}{P_N Y_N}$
Visegrad									
CZ	−0.502	0.35	−10.0	79.6	30.5	21.8	0.40	0.41	0.37
HU	0.034	0.30	−1.0	80.0*	26.5	22.1	0.42	0.39	0.37
PL	0.107	0.30	−0.1	31.3	22.2	17.8	0.42	0.38	0.41
SK	0.088	0.35	−3.1	76.7	28.5	19.4	0.44	0.49	0.46
Baltics									
EE	0.029	0.24	−14.8	72.1	26.1	17.9	0.41	0.44	0.40
LV	0.013	0.22	−13.4	37.6	19.4	19.3	0.38	0.51	0.47
LT	0.024	0.28	−25.3	53.1	21.0	18.8	0.38	0.49	0.41
2007									
BG	0.033	0.24	−25.3	60.0*	22.8	17.9	0.36	0.40	0.40
RO	0.023[29]	0.34	−25.5	47.0	24.1	17.0	0.40	0.40	0.40
Av.	−0.02	0.29	−13.17	61.56	24.57	19.11	0.40	0.43	0.41

Table 3.4: National accounts data, capital shares and CPI weights. For values with an asterisk, please refer to annotations in appendix B.5.

onwards. This is especially reasonable for interest rates, as the third stage of EMU began in 1999q1 and a 'great moderation' in $\tilde{\imath}^*$ was observed during the years before.[27]

3.5.2 Calibrated Parameters and Steady State Shares

To obtain empirically plausible values for deep parameters, we keep the amount of parameters that are calibrated 'ad hoc' to a minimum. As a consequence, only the risk premium ξ, the depreciation rate on physical capital δ, the discount factor β, and the inverse of labour supply elasticity κ are calibrated directly, inspired by values common in the RBC literature that obtain for a model for quarterly data. We therefore set $\xi = 0.02$, capital depreciates at a quarterly rate of $\delta = 2.5\%$, the discount factor for households is $\beta = 0.99$, and $\kappa = 2$. All country-specific parameters are either taken from means implied by averaging historical data or estimated within the Bayesian framework. Generally, steady state shares are obtained by averaging historical series. As real convergence will be taken into account, these values represent the behaviour of the economies in the initial equilibrium. To calculate these measures, we employ annual national accounts data on NACE 6 digits level provided by *Eurostat* and average for the years 2000 to 2007.[28] Table 3.4 presents the results. A detailed description of the methodology to calculate other ratios illustrated can be found in section B.5 in the appendix on page 299.

[27] For Hungary, data is only balanced from 2000q2 onwards.

[28] Data comes from the quarterly national accounts aggregates and also using employment by branch (according to NACE), provided by *Eurostat*.

3.5.3 Measurement Equations

Observable variables described under 3.5.1 are linked to stationary model variables $\widehat{\mathbf{Y}}_t$ by

$$\mathbf{Y}_t^{obs} = \boldsymbol{Z}\,\widehat{\mathbf{Y}}_t + \bar{\mathbf{Y}}_t^{obs} + \boldsymbol{q} + \boldsymbol{\varepsilon}_t \tag{3.32}$$

(3.32) represents a linear trend-cycle decomposition (i.e. in logs) where $\mathbf{Y}_t^{obs}$ are observable variables in logs (accordingly log differences for prices or variables in nominal terms). $\widehat{\mathbf{Y}}_t$ is the vector of log-linearised endogenous variables of the DSGE model, $\bar{\mathbf{Y}}_t^{obs}$ are the steady state values of observables (that can possibly be time varying), and $\boldsymbol{q}$ is the log-deterministic trend in observables. We can summarise the deterministic trend in the steady state, $\bar{\mathbf{Y}}_t^{obs'} \equiv \bar{\mathbf{Y}}_t^{obs} + \boldsymbol{q}$.[30] $\boldsymbol{\varepsilon}_t$ is a vector of measurement errors.[31] $\boldsymbol{Z}$ is a selection matrix that aligns empirical with model variables. In our case, the measurement equations are then given by

$$\mathbf{Y}_t^{obs} - \bar{\mathbf{Y}}_t^{obs'} = \boldsymbol{Z}\,\widehat{\mathbf{Y}}_t + \boldsymbol{\varepsilon}_t \tag{3.33}$$

We further demean all data prior to estimation and therefore remove any non-zero steady state values from the data.[32] That operation also removes any non-zero steady state values in the inflation rates and interest rates.

Together with the structural model written in its state-space form given by

$$\widehat{\mathbf{Y}}_t = \boldsymbol{\Gamma}\left(\boldsymbol{\theta}\right)\widehat{\mathbf{Y}}_{t-1} + \boldsymbol{\Psi}\left(\boldsymbol{\theta}\right)\boldsymbol{u}_t \tag{3.34}$$

the state-space system of the solution to the model is formed. $\boldsymbol{\Gamma}$ and $\boldsymbol{\Psi}$ are matrices of the deep parameters of the model, $\boldsymbol{\theta}$. The likelihood function of the DSGE model $\mathcal{L}\left(\boldsymbol{\theta}|\,\mathbf{Y}_t^{obs} - \bar{\mathbf{Y}}_t^{obs'}\right)$ is obtained from the Kalman filter recursion that is employed on the state-space system described by (3.33) and (3.34). The posterior kernel $\mathcal{K}$ is then given by the likelihood function, weighted by the prior distribution for structural parameters $p\left(\boldsymbol{\theta}\right)$. Therefore, in logs

$$\ln\mathcal{K}\left[\boldsymbol{\theta}|\left(\mathbf{Y}_t^{obs} - \bar{\mathbf{Y}}_t^{obs'}\right)\right] = \ln\mathcal{L}\left[\boldsymbol{\theta}|\,\boldsymbol{Y}\left(_t^{obs} - \bar{\mathbf{Y}}_t^{obs'}\right)\right] + \ln p\left(\boldsymbol{\theta}\right) \tag{3.35}$$

[30] When there is a stochastic trend present, q_i is the drift parameter, and the steady state follows a unit root process.

[31] In `dynare`, measurement errors are uncorrelated with errors of the state equations (the structural errors in the DSGE).

[32] To match the model variables in log-deviations from trend, all macroeconomic series besides foreign reserves, are in logs and then detrended by means of the Hodrick and Prescott (1997) filter with smoothing parameter for quarterly data (1600), prior to estimation. For foreign reserves, the model variable is matched by the quarterly growth rate in reserves $d\ln Z_t$. Variables that are in log-differences $(\pi_H,\, \pi_N,\, \pi^*,\, i^*)\times\frac{1}{100}$ cannot be detrended. Note that in case of the inflation rates $\hat{\pi}_J \equiv \ln P_{J,t} - \ln P_{J,t-1} - \left(\ln P_{J,t}^{ss} - \ln P_{J,t-1}^{ss}\right) = \pi_{J,t} - \pi_{J,t}^{ss}$. Demeaning $\pi_{J,t}$ implies that $\pi_{J,t}^{ss}$ drops out and the observable variable corresponds to the model variable $\hat{\pi}_J$.

Maximising the log-posterior kernel $\ln \mathcal{K}\left[\boldsymbol{\theta}|\left(\mathbf{Y}_t^{obs} - \bar{\mathbf{Y}}_t^{obs'}\right)\right]$ with respect to $\boldsymbol{\theta}$ using numerical methods, one obtains estimates for the vector of deep parameters $\boldsymbol{\theta}$. As the posterior distribution is generally non-linear and there is no analytical solution available, the distribution is explored by means of the Metropolis-Hastings algorithm, i.e. a procedure of stochastic simulation over different proposals for the parameter vector.[33]

In Bayesian estimation, there cannot be more time series used for estimation than shocks, otherwise stochastic singularity occurs. We have available fourteen macro series for the Visegrad States and thirteen series for the Baltic States.[34] For Bulgaria and Romania, there is no quarterly national accounts data available but data on other variables is provided. As there are nine structural shocks in the model (shocks to sectoral productivity, shocks to sectoral fiscal spending, two domestic monetary shocks, and two foreign shocks), we need to add (at least) five measurement errors such that all stochastic volatility is attributed to identified structural shocks and measurement errors. Therefore, five measurement errors are added in case of Visegrad States (14 - 9) and accordingly for other countries. We assume that measurement errors are normal about zero and estimate its standard error. For the distribution of the standard error, we impose inverse gamma distributions.[35] In our framework, the following variables are assumed to be measurable only up to a random error[36]

$$\pi_t^{obs} = \pi_t + \varepsilon_{\pi,t}^{obs} \tag{3.36}$$

$$i_t^{obs} = i_t + \varepsilon_{i,t}^{obs} \tag{3.37}$$

$$C_{H,t}^{obs*} = C_{H,t}^{*} + \varepsilon_{C_H^*,t}^{obs} \tag{3.38}$$

$$C_{F,t}^{obs*} = C_{F,t}^{*} + \varepsilon_{C_F^*,t}^{obs} \tag{3.39}$$

$$\triangle Z_t^{obs} = \triangle Z_t^{obs} + \varepsilon_{Z,t}^{obs} \tag{3.40}$$

3.5.4 Discussion of Priors and Estimated Parameters

Micro-evidence on prior distributions for parameters governing structural inflation dynamics is hardly available. We employed the range of estimates in Lendvai (2005). In case of Hungary, the overall share of backward-looking firms is between 0.3 and 0.55 whereas price-resetting probabil-

[33] Further details on the procedure can be found in the `dynare` manual, see Griffoli (2007), on pages 85 to 87.

[34] We excluded the nominal exchange rate series S_{t-j} in case the country is currently under a fixed exchange rate regime or currency board.

[35] Note also that adding measurement errors improves the fit of the model as more shocks can capture variability in endogenous variables. It also helps in smoothing shocks. The choice of measurement errors can drive the information content the data has for the model. We observe that measurement errors might soak up the variability such that the data becomes uninformative for estimating the deep parameters.

[36] Note that measurement errors are treated like structural errors in the Bayesian estimation, i.e. exogenous variables in the nomenclature of `dynare`.

ity ranges between 0.45 and 0.6. We therefore assume beta distributed prior probability density functions for θ_J and ϖ_J that center at 0.55 and 0.40 respectively, i.e. at means of the estimated range.[37] Despite the fact that these parameters enter the inflation equations in non-linear form, they can be estimated, as we in fact perform a weighted Maximum-Likelihood estimation (see also Schorfheide and del Negro (2007)).[38] Priors for policy parameters $\{\varpi_{GDP}, \varpi_\pi, \varpi_k\}$ in the monetary policy rule (3.19) were taken from Smets and Wouters (2003). However, we allowed for more variability in the distribution of the true parameters, as we are less certain about their location. For the exchange rate coefficient ϖ_S, we assumed that for managed floaters the true parameter is beta distributed centered at 0.5, whereas for exchange rate targeters, the parameter is assumed to have mean 1.7.[39] As explained under 3.3.3, we identify a certain exchange rate regime by imposing zero restrictions on certain policy parameters. A fixed exchange rate/currency board regime is thus identified by setting $\varpi_\pi = 0$ and a flexible regime by $\varpi_S = 0$. These parameters are thus not estimated.

For exogenous processes (government expenditures, export demand, and foreign reserves growth as well as the euro area money market rate and the euro area inflation rate), we fit AR(1) processes for the series. The estimated persistency parameters obtained serve as priors for the location measures of the 'true' persistency coefficients $\left\{\rho_{G_J}, \rho_{C_H^*}, \rho_{dZ}, \rho_{\pi^*}, \rho_{\tilde{\imath}^*}\right\}$.[40] The standard deviations of the estimated residuals are likewise treated as priors for the volatility in the structural/'true' innovations $\left\{\epsilon_{G_J}, \epsilon_{C_H^*}, \epsilon_{dZ}, \epsilon_{\pi^*}, \epsilon_{\tilde{\imath}^*}\right\}$. Hence, if the AR process represented the real data generating process, the structural error would be directly observable.[41] We obtain that euro area inflation on a quarter-to-quarter basis is not very persistent with a coefficient of .257 and 'structural' error of .479. Analogously, we obtain high persistence in the three-months money market rate[42] with an autocorrelation coefficient of 0.89 and standard deviation in the dependent variable of 1.18 percentage points. As domestic government expenditures are

[37] The structural parameters governing inflation dynamics, namely θ_J and ω_J can also be recovered from reduced form coefficients that guide overall inflation dynamics, i.e. λ_J^b, λ_J^{mc}, and λ_J^f. With these estimates and the identifying assumptions implied by the model structure, one can obtain θ_J and ω_J by solving the non-linear three equation system in the two unknowns θ_J and ω_J. However, to obtain valid priors for the reduced form parameters λ_J^b, λ_J^{mc}, and λ_J^f (that might be estimated by instrumental, single equation methods) in case of new EU members is hard to impossible. Single equation results are provided by Franta et al. (2007), but with only inconclusive and unreliable results for selected new EU members. However, they find that their selection of new EU members tends to have a higher share of backward looking firms compared to euro area countries. The sensitivity of results with respect to different assumptions about priors (especially their curvature and looseness) can be investigated by the result they deliver when minimising the objective function (the negative of the log posterior kernel).

[38] We also observed that when using structural parameters as priors we got better fit than using the reduced form inflation dynamics parameters.

[39] Changing the parameter assumption from beta to uniform brought a great improve in fit.

[40] We use the `vare` option of the Econometric Toolbox provided by LeSage (2005).

[41] In case of Romania, the standard deviation of the error would be arround 137%. We therefore calibrate the volatility to 10%, a reasonable value compared to estimates for other countries.

[42] The rate at which business banks exchange base money supplied by the central bank.

not endogenous in the sense that they are not explained by macroeconomic behaviour in the model neither, and therefore exogenous, we exclude the spending processes from the system estimation.

Table 3.5 presents key estimates regarding inflation dynamics, markups, and the policy rule for each of the nine countries.[43] We observe that the sectoral price-resetting probability $1-\theta_J$ tends to be higher in the sector that is open to international trade. This result is an argument in favour of the expectation that globalisation causes prices to be increasingly determined in the world market such that local price rigidity declines and Phillips curves become flatter, see also Sbordone (2007) and Razin and Binyamini (2007). It is also in line with the finding in the previous chapter that small-open economies tend to have rather forward-looking price-setting dynamics. Further, a high export share 'tends' to be associated with a higher price-resetting probability which becomes clear when comparing Slovakia and Poland concerning their $P_H C_H^* / P_H Y_H$ share. Openness therefore contributes to lower price duration also. There is some indication that countries under a managed floating system prior to ERM II entry (the Czech Republic, Hungary, Slovakia, and Romania) have a lower share of backward-looking firms in the sector that is open to trade and comparatively higher persistence in the non-tradables sector. At the same time, price-resetting probabilities are rather low in these countries in that sector. Markups in the industry sector (the tradable sector) tend to be higher than in the services sector. Hence there is little evidence that product market integration reduces markups in the sector that is open to trade. Whereas markups tend to be low or nearly inexistent for production in Latvia and Lithuania, they are especially high in the Czech Republic, Hungary, Poland, and Romania. Further, the data is rather uninformative when it comes to estimating the coefficients of the policy rule. Values are often close to their priors.

Table 3.6 illustrates that the data is very informative about the volatility and persistence of shocks. We obtain that volatility in total factor productivities A_H and A_N as well as in nominal shocks is large. Also, there is no general pattern observable that non-tradables fluctuations are of smaller magnitude than tradable ones. Government spending volatility $\sigma_{\epsilon_{G_J}}$ is lower than supply side variability $\sigma_{\epsilon_{A_J}}$. As government spending shocks cannot be identified on a sectoral basis (there is no data on sectoral purchases), estimates of volatilities result in same values in both sectors. As a consequence of often too high volatility in structural shocks (volatility often exceeds 4 percentage points for certain innovations in structural shocks), we generally introduced upper bounds on given estimates for innovations and persistence in the exogenous processes. Values imposed were based on calibrations of these processes in related work by

[43] Markups are calculated from steady state relationships together with estimates for α_J, see (B.13) in the appendix on page 301. If estimation implied markups were smaller than one, we set them equal to one.

Structural Parameters	Regime	Inflation dynamics ω_H	θ_H	ω_N	θ_N	Mark ups $\frac{\rho_H}{\rho_H-1}$	$\frac{\rho_N}{\rho_N-1}$	Policy rule ϖ_π	ϖ_s	k
Visegrad States										
CZ	M. float	0.40	0.55	0.48	0.60	1.26	1.23	2.00	0.11	0.81
HU	M. float	0.38	0.53	0.47	0.68	1.32	1.22	2.00	0.11	0.82
PL	Float	0.47	0.62	0.41	0.50	1.30	1.13	1.99	**0**	0.79
SK	M. float	0.33	0.47	0.47	0.78	1.14	0.98	2.00	0.11	0.82
Baltic States										
EE	Fixed	0.41	0.56	0.30	0.43	1.23	1.22	**0**	2.00	0.81
LV	Fixed	0.40	0.55	0.40	0.56	1.00	0.97	**0**	2.00	0.99
LT	Fixed	0.39	0.54	0.39	0.53	1.05	1.11	**0**	2.00	0.99
2007 Entrants										
BG	Fixed	0.40	0.55	0.40	0.55	1.23	1.23	**0**	2.00	0.81
RO	M. float	0.39	0.55	0.39	0.55	1.30	1.13	**0**	2.00	0.80

Table 3.5: Key estimated structural parameters. Parameters are the modes of the multivariate posterior. Values in bold type indicate zero restrictions for identifying the current exchange rate regime.

Structural shocks	Domestic real $\sigma_{\epsilon_{A_H}}$	$\sigma_{\epsilon_{A_N}}$	$\sigma_{\epsilon_{G_H}}$	$\sigma_{\epsilon_{G_N}}$	Domestic monetary σ_{ϵ_Z}	σ_{ϵ_i}	Foreign $\sigma_{\epsilon_{C_{H^*}}}$	$\sigma_{\epsilon_{\tilde{\imath}^*}}$	$\sigma_{\epsilon_{\pi^*}}$
Visegrad									
CZ	4.2	7.7	2.3	2.3	7.2	2.7	5.7	1.5	0.7
HU	3.2	5.1	1.5	1.5	11.2	2.7	2.8	0.3	0.5
PL	0.3	3.9	0.6	0.6	7.6	0.7	7.4	0.5	0.4
SK	4.4	6.8	2.6	2.6	11.9	5.2	5.2	0.4	0.5
Baltics									
EE	4.5	1.5	1.8	1.8	7.5	4.3	8.9	0.5	0.5
LV	4.1	2.1	1.4	1.4	14.9	2.3	4.3	0.7	0.4
LT	7.7	5.2	2.0	2.0	18.5	1.4	4.8	0.3	0.2
2007									
BG	2.0	2.0	2.0	2.0	5.8	2.0	2.0	0.3	0.5
RO	2.0	2.0	2.0	2.0	10.0	2.0	2.0	2.0	0.5
Literature	2.0	2.0	2.0	2.0	5.8	2.0	2.0	1.8	0.5

Table 3.6: Estimated volatility in structural innovations. Volatility of structural shocks in percentage points. Measurement shocks omitted.

Ravenna and Natalucci (2008) and Lipinska (2008), insofar same processes were included.[44] We then obtained an improved fit of reproducing the empirical volatility in series of Y_J as well as of Y and π.

For persistency in domestic exogenous processes, we imposed an upper bound such that $\rho_X \leq 0.8$ for $X \in \{A_J, G_J, C_H^*, \triangle Z\}$. We kept the estimates for persistence in $\tilde{\imath}^*$ and π^*. Volatility in total factor productivity in the industry sector H was restricted to not exceed two and a half percentage point a quarter such that $\sigma_{\epsilon_{A_H}} \leq 0.025$. For non-tradables productivity we could match empirical series more closely by imposing that $\sigma_{\epsilon_{A_N}} \leq 0.005$. For remaining structural innovations we assumed

$$\sigma_{\epsilon_{G_J}} \leq 0.02, \qquad \sigma_{\epsilon_{C_H^*}} \leq 0.02, \qquad \sigma_{\epsilon_i} \leq 0.005, \qquad \sigma_{\epsilon_Z} \leq 0.05$$

which is in line with values proposed in Natalucci and Ravenna (2005) or estimates in Smets and Wouters (2003). The value for σ_{ϵ_Z} derives from the fact that volatility in innovations in reserves growth ϵ_Z should not exceed ten percent at an annualised rate, which is a good proxy for actual volatility in that measure in the EU9.[45] Concerning volatility in measurement errors, we imposed that the standard error should not exceed one percentage point. These imposed restrictions on structural volatility and persistence in shocks limit the informative value of the estimation results to a certain extent. However, more reliable estimates might be obtained as more data becomes available.

3.5.5 Comparing Theoretical and Empirical Moments

Equipped with estimated parameters and data-founded steady state relationships for each of the countries under consideration, one can simulate developments in activity and prices in any of the nine EU countries. Whereas impulse responses focus on the effect a single structural shock has on business cycle dynamics, now all shocks will contribute to dynamics. We simulate time series where the model is taken as data generating process and compare results with time series characteristics of historical series. Table B.2 on page 295 in the appendix presents the results. Historical (left panel) and model generated series (right panel) regarding reduced-form (overall) volatility, persistency and cross-correlation in output and inflation rates Y_J, π_J, π, Y are compared. We make three main observations: The estimated model tends to produce higher

[44] One could alternatively assign bounds for estimates prior to estimation which allows to perform a kind of constrained kernel maximisation. Parameters are then in the range of best guesses but at the cost of lower fit of the model.

[45] The condition requires that $\sigma_{\varepsilon_Z}^{upper,y} \leq 0.1$. As approximately $\left(\sigma_{\varepsilon_Z}^{y}\right)^2 \simeq 4\left(\sigma_{\varepsilon_Z}^{q}\right)^2$ it follows that $\sigma_{\varepsilon_Z}^{upper,q} \leq 0.05$.

volatility than observed historically. Whereas the model tracks persistence in the developments of tradables Y_H and non-tradables production Y_N reasonably, it overstates the persistence in the developments of sectoral inflation rates. The estimated framework delivers negative cross-correlations between output and prices. Negative correlation is much stronger for tradable output than for non-tradables. This result is in line with the insight that the sector open to trade faces competition from abroad such that price changes will be associated with stronger output responses.

A word of caution regarding the explanatory power of table B.2 should nevertheless be pointed out here: The focus of the analysis is on a comparative study that aims at a 'strong econometric interpretation' in the sense of Geweke (1999) (impaired to a certain extent by the requirement to limit structural volatilities in shocks obtained in the estimation). Accordingly, our analysis is based on estimations of all moments of endogenous variables as the log-data density is provided instead of fitting first or second order moments only. The table therefore is not representative concerning the overall quality of the estimation.

3.5.6 Variance Decomposition

With the estimated model calibrated with the modes of the posterior distribution, variances of key series can be decomposed in order to answer which structural shocks represent the main sources of fluctuations in key aggregates in each country.[46] We therefore decompose reduced form volatility in the CPI inflation rate (π_t), real GDP (GDP_t^r), and private consumption (C_t) according to the contribution each estimated structural shock has. We can further observe whether inflation is driven by real or monetary factors that originate at home or foreign. Also, it can be investigated whether country groups show similar patterns and whether the exchange regime matters. Results are shown in table 3.7 on page 173.[47]

For currency boarders or fixed exchange rate countries, developments in the euro area short run rate $\tilde{\imath}_t^*$ explain most of fluctuations in home CPI inflation π_t. This comes at no surprise, as the exchange rate cannot adjust and UIP holds. For countries under the managed float regime, the effect is smaller and for the floater (Poland) the effect is negligible. Foreign shocks

[46] Note that as quarter-on-quarter rates are used, there is much more volatility in the series than if comparing year-on-year (which is equal to an averaged quarter-on-quarter in the quarter compared to a year earlier). Observe that there is great volatility in services inflation which supports the view that only a two-sector framework can reasonably assess the compatibility with Maastricht criteria.

[47] Given the model in state space form is $y_t = Ay_{t-1} + Bu_t$, then $\sum_y = A\sum_y A' + B\sum_u B'$ is a Lyapunov equation in $\sum_y$ which can be solved in `dynare` by an algorithm specialised for this type of equation. We obtain the conditional variance decomposition from the state space representation of the model. Measurement errors are of no role for the volatility decomposition. The decomposition is instead influenced by the size of volatility in innovations and persistence of shocks. Alternatively, we could simulate data by the model (as in the preceding section) and evaluate the covariance matrix generated by the data.

$\{\epsilon_{C_{H^*}}, \varepsilon_{\tilde{\imath}^*}, \varepsilon_{\pi^*}\}$ are important for countries with a high openness to trade as described in table 3.4 on page 164. Demand shocks affect π_t mainly if they originate at foreign whereas domestic fiscal spending shocks have no impact. Real GDP is mainly driven by domestic supply and foreign export demand shocks. Especially, one observes the relevance of the non-tradable sector in driving overall fluctuations in the Baltic States and the 2007 entrants. Accordingly, for countries with a high content of non-tradable production, innovations in non-tradable productivity matter more for aggregate activity than in other countries. The lower panel illustrates the driving forces underlying households' overall consumption C_t. Domestic real supply shocks and variability in the EMU interest rate as well as export demand shocks are detected as main factors. The dependence on domestic real supply shocks and export demand shocks reflect the fact that consumption depends on labour income that fluctuates with factor productivity and demand shocks from abroad. The dependence on the foreign interest rate makes clear that intertemporal consumption-savings decisions via the Euler equation depend not only on the domestic but also on the foreign rate in case the economy is open.

3.6 Compliance with Monetary Maastricht Criteria

Equipped with the model that describes the empirical picture in each country reasonably well, compliance with Maastricht criteria can be approached. As a first step towards the Maastricht check, criteria need to be transformed in a model-consistent way concerning timing conventions. It seems useful to restate the criteria in their original legal form first. Article 109 j of the Maastricht Treaty of 1992 stipulates the criteria regarding nominal convergence.[48] The criteria are at the heart of the convergence reports published by the European Central Bank and the European Commission and aim at "[...] the achievement of a high degree of sustainable convergence by reference to the fulfilment by each Member State".[49] Reports are compiled at least once every two years or at the request of a member state with a derogation and are submitted in parallel to the Council of the EU, see European Central Bank (2006) for an example. Criteria are clarified quantitatively in the protocols amending the treaty. Further, there are no exceptional clauses provided such that new EU members from the East have to fulfil the same criteria as current euro area members.

The inflation rate criterion requires "[...] the achievement of a high degree of price stability;

[48] In the consolidated version of the treaty as of 2002, the criteria are stipulated in article 121(1) and 122(2), in the version as of 2008, the criteria are formulated in article 140, see `http://eur-lex.europa.eu/en/treaties/index.htm`.

[49] Current convergence reports by the European Central Bank are available from `http://www.ecb.int/pub/convergence/html/index.en.html` and from `http://ec.europa.eu/economy_finance/publications/specpub_list9259.htm`. in case of the European Commission.

Volatility decomposition	Contribution of volatility in innovations								
CPI inflation rate π_t	Domestic real				Domestic monetary		Foreign		
	ϵ_{A_H}	ϵ_{A_N}	ϵ_{G_H}	ϵ_{G_N}	ϵ_Z	ϵ_i	$\epsilon_{C_{H^*}}$	$\epsilon_{\tilde{\imath}^*}$	ϵ_{π^*}
Visegrad States									
CZ	11.0	2.1	0.0	0.1	45.1	3.5	9.9	26.8	1.6
HU	41.5	0.3	0.0	1.6	0.6	1.0	69.5	0.4	0.0
PL	6.9	1.6	0.0	0.0	0.5	0.0	92.9	0.0	0.0
SK	34.9	0.9	0.0	0.0	3.5	6.9	27.8	24.1	1.9
Baltic States									
EE	8.2	5.1	0.0	0.2	0.8	2.7	6.8	70.5	5.8
LV	4.0	5.1	0.0	0.4	0.0	14.1	2.0	68.8	5.4
LT	5.7	5.1	0.0	0.2	0.6	10.7	2.9	71.6	3.3
2007 Entrants									
BG	3.2	4.6	0.0	0.7	1.2	3.0	1.1	84.4	1.7
RO	2.8	4.4	0.0	0.4	0.6	34.7	2.7	48.3	6.2
Real GDP GDP_t^r	Domestic real				Domestic monetary		Foreign		
	ϵ_{A_H}	ϵ_{A_N}	ϵ_{G_H}	ϵ_{G_N}	ϵ_Z	ϵ_i	$\epsilon_{C_{H^*}}$	$\epsilon_{\tilde{\imath}^*}$	ϵ_{π^*}
Visegrad States									
CZ	18.2	7.6	0.0	0.4	49.3	0.3	23.8	0.3	0.0
HU	16.8	10.0	0.0	1.6	0.6	1.0	69.5	0.4	0.0
PL	4.9	1.6	0.0	0.0	0.5	0.0	92.9	0.0	0.0
SK	55.0	6.3	0.1	1.6	4.9	1.2	30.3	0.6	0.1
Baltic States									
EE	39.0	34.0	0.0	1.4	1.6	0.7	21.0	1.4	0.0
LV	35.5	35.6	0.0	2.7	0.2	4.0	20.5	1.5	0.0
LT	29.7	33.3	0.0	0.7	1.0	3.2	30.7	1.3	0.0
2007 Entrants									
BG	17.2	55.3	0.2	9.1	3.6	2.0	9.6	3.1	0.0
RO	12.3	46.3	0.1	6.3	1.5	6.0	25.7	1.6	0.2
Consumption C_t	Domestic real				Domestic monetary		Foreign		
	ϵ_{A_H}	ϵ_{A_N}	ϵ_{G_H}	ϵ_{G_N}	ϵ_Z	ϵ_i	$\epsilon_{C_{H^*}}$	$\epsilon_{\tilde{\imath}^*}$	ϵ_{π^*}
Visegrad States									
CZ	5.2	2.9	0.0	0.5	58.6	1.6	28.1	2.1	1.1
HU	14.0	1.5	0.0	0.2	0.3	4.7	75.4	2.6	1.3
PL	6.2	0.0	0.0	0.0	0.1	0.4	93.0	0.0	0.2
SK	10.0	2.8	0.0	0.7	7.3	9.0	61.2	6.1	3.0
Baltic States									
EE	12.7	14.0	0.0	1.2	1.8	3.6	40.0	23.6	3.4
LV	8.3	15.5	0.0	4.5	0.2	19.8	25.4	23.1	3.3
LT	10.4	18.5	0.0	1.7	1.4	16.0	32.1	17.5	2.4
2007 entrants									
BG	7.8	27.9	0.1	12.8	4.6	6.6	17.0	20.8	2.4
RO	3.4	11.8	0.0	4.3	1.5	34.1	24.2	13.1	7.6

Table 3.7: Variance decomposition of the CPI inflation rate, real GDP, and real consumption.

this will be apparent from a rate of inflation which is close to that of, at most, the three best performing Member States in terms of price stability". The protocol to the treaty clarifies in article I that "[...] a Member State has a price performance that is sustainable and an average rate of inflation, observed over a period of one year before the examination, that does not exceed by more than $1\frac{1}{2}$ percentage points that of, at most, the three best performing Member States in terms of price stability. Inflation shall be measured by means of the consumer price index". Consequently, compliance with the inflation criterion $c_t^{\pi^y}$ is obtained if

$$\mathcal{E}_t c_t^{\pi^y} = \mathcal{E}_t \left[\pi_t^y - \pi_t^{y*}\right] \leq B(c_t^{\pi^y}) = 1.5\text{pp}$$

where $\mathcal{E}_t \pi_t^y$ is the average year-on-year inflation rate in the country under consideration conditional on the most recent information. The average inflation rate in the three EU countries with the lowest inflation rate is proxied by π_t^{y*}. The upper bound for the inflation criterion $c_t^{\pi^y}$ is provided by $B(c_t^{\pi^y})$.[50] Reformulating the criterion for quarterly data, we obtain

$$\mathcal{E}_t c_t^{\pi} = \mathcal{E}_t \left[\pi_t - \pi_t^*\right] \leq B(c_t^{\pi}) = \left(1 + B(c_t^{\pi^y})\right)^{0.25} - 1 = 1.015^{0.25} - 1 = 0.373\text{pp} \tag{3.41}$$

where $\pi_t = (1+\pi_t^y)^{0.25} - 1$, $\pi_t^* = \left(1+\pi_t^{y*}\right)^{0.25} - 1$ and the quarterly bound reads $B(c_t^{\pi}) = \left(1 + B(c_t^{\pi^y})\right)^{0.25} - 1$, see also Lipinska (2008). Note that π_t denotes the inflation rate in its level, not in its deviation from steady state. It turns out that the quarterly criterion is generally tighter than the original criterion, as its average value has to be below the bound four times a year, not only once a year. Also, yearly inflation rates might be less volatile (short-run effects might be washed out) such that quarter-on-quarter rates might overstate average volatility present in the economy in a year-on-year assessment. The reference value for the inflation rate is therefore obtained as $\mathcal{E}_t \pi_t \leq B(c_t^{\pi}) + \mathcal{E}_t \pi_t^*$ such that the reference value is time-variable, see e.g. the recent convergence report by the European Commission (2008).

Regarding interest rates, the protocol clarifies in article IV that "[...] observed over a period of one year before the examination, a Member State has had an average nominal long-term interest rate that does not exceed by more than 2 percentage points that of, at most, the three best performing Member States in terms of price stability. Interest rates shall be measured on

[50] Note that according to the official application of the treaty by the ECB, the inflation rate for the country under consideration is calculated using the change in the latest available 12-month average in the HICP over its previous 12-month average. Methodologically, this measure is the latest available 12-month moving average, see also European Commission (2008). The reference value π_t^{y*} is obtained officially by taking the unweighted arithmetic average of inflation rates in the three EU countries with the lowest inflation rate, based on the same reference period. See European Central Bank (2006) for further details, also on the formal application of the following criteria.

the basis of long term government bonds or comparable securities". Therefore

$$\mathcal{E}_t \left[i_t^y - \tilde{\imath}_t^{*y} \right] \leq B_{i-\tilde{\imath}}^y = 2\text{pp}$$

where i_t^y is the annualised return on a long-run government bond till maturity in the home economy. $\tilde{\imath}_t^{*y}$ denotes the average annual interest rate for holding a comparable government bond up to maturity in the three countries of the EU with the lowest inflation rate. The expectations hypothesis holds for the term structure of interest rates in our model implicitly as domestic tradable financial assets only cover a (corporate) bond B_H traded on the money market. Also, we consider the corporate bond a comparable security. The long run interest rate is then generated by a sequence of short rates by $(1 + i_t^y) = (1 + i_t)^4$ and likewise for $\tilde{\imath}_t^{*y}$. Therefore, the short rate safely replaces the long rate in the model and a premium for holding longer-run bonds is absent. On a quarterly basis, the criterion reads[51]

$$\mathcal{E}_t c_t^i = \mathcal{E}_t \left[i_t - \tilde{\imath}_t^* \right] \leq B(c_t^i) = 0.496\text{pp} \tag{3.42}$$

where $i_t = (1 + i_t^y)^{0.25} - 1$, $\tilde{\imath}_t^* = \left(1 + \tilde{\imath}_t^{*y}\right)^{0.25} - 1$ and $B(c_t^i) = \left(1 + B(c_t^{i^y})\right)^{0.25} - 1$.

For the fiscal criteria, the treaty defines that the sustainability of the government financial position is "[...] apparent from having achieved a government budgetary position without a deficit that is excessive". The protocol on the excessive deficit procedure clarifies that the reference values referred to in article 104c(2) of the treaty are "[...] 3% for the ratio of the planned or actual government deficit to gross domestic product at market prices", and "[...] 60% for the ratio of government debt to gross domestic product at market prices". Hence the two criteria can be stated as

$$\mathcal{E}_t c_t^G = \mathcal{E}_t \frac{G_t + Q_t^{\#} - \left(T_t^{\#} + v_t\right)}{GDP_t} \leq B(c_t^G) = 0.03 \tag{3.43}$$

$$\mathcal{E}_t c_t^{G^d} = \mathcal{E}_t \frac{G_t^d}{GDP_t} \leq B(c_t^{G^d}) = 0.6 \tag{3.44}$$

The fiscal criterion is met in our framework by assumption as deficits are not possible, such that always $G_t + Q_t^{\#} = T_t^{\#} + v_t$. Given the absence of initial debt and no possibility for fiscal deficits in any period, the debt criterion is met throughout, $G_t^d = 0$. Hence, the framework presented here remains silent about the question whether the public finance criteria could be

[51] In case there is no quarterly domestic interest rate data available (the case of PL, SK, and RO), we assume that the differential between home and foreign nominal interest rates is 0.50 initially, such that the criterion would be met if no other pressures build up over time.

met, conditionally on the fact that monetary criteria had been met.[52]

The exchange rate criterion requires the "[...] observance of the normal fluctuation margins provided for by the Exchange Rate Mechanism of the European Monetary System, for at least two years, without devaluing against the currency of any other Member State". The operating procedures for ERM II have been laid down in an agreement between the ECB and the non-euro national central banks and were released jointly with stage three of EMU, i.e. on 1st January 1999. It states that "[...] for the currency of each Member State participating in the mechanism, a central rate against the euro and a standard fluctuation band of ±15% are defined, in principle supported by automatic unlimited intervention at the margins, with very short-term financing available". Declaring the central parity S as the steady state value of S_t, we can write

$$-B\left(c_t^S\right) \leq \mathcal{E}_t\left[\frac{S_t}{S}-1\right] \leq B\left(c_t^S\right) \tag{3.45}$$

where $B\left(c_t^S\right) = 15\%$ and $c_t^S = \frac{S_t}{S} - 1$, analogously to Lipinska (2007, p. 13).

Using the model to forecast developments into the future, where real convergence is triggered by prolonged trend growth in tradable technology under the given exchange rate regime, it becomes possible to assess compliance in the longer run.[53] We follow Natalucci and Ravenna (2008) in assuming that real convergence is induced by factor productivity growth in the tradable sector of 30% in ten years time. This assumption seems a raw guess but prevents making arbitrary assumptions regarding the long-run growth prospects for an individual economy under consideration. According to our specification of total factor productivity $A_{H,t}$ provided by (3.6), real convergence therefore implies that $\frac{E^T_{H,t+40}}{E^T_{H,t}} = \exp\left[40 \times x_{H,t}\right] \stackrel{!}{=} 1.30$. Quarterly steady state growth in total factor productivity $A_{H,t}$ is then $x_{H,t} = \ln(1.3)/40 = 0.656\%$. In order to produce a sequence $\{x_{H,t+s}\}_t^{t+40}$ with the desired properties, we assume that $\rho_{x,H} = 1$ in (3.9) such that the sequence is generated by positive structural shocks $x_{H,t+s} = \eta_{H,t+s} = 0.66\%$ that last for 40 periods. Rational expectations of households make this forecasting exercise non-trivial, but increase its empirical plausibility: As the shock is deterministic, it is expected such that households and firms will adapt to the new situation right from the beginning. Accordingly, the simulation is computed conditionally on agents knowing the future values of the deterministic exogenous variables. This procedure also leads to robustness to the Lucas (1976) critique in the sense that households cannot be surprised by deterministic (systematic) movements in the economy.[54] Further, we assume that the central bank implements monetary

[52] Also, the introduction of a richer fiscal side would require additional assumptions regarding the optimising behaviour of households (Ricardian versus non-Ricardian).

[53] We employ the `forecast` option in `dynare`.

[54] See also the discussion of the `forecast` option in Juillard (2001).

policy conditional on the current exchange rate regime given by (3.19). The fiscal authority implements (3.23).

Initial conditions for Maastricht criteria c^{π}_{t+s}, c^{i}_{t+s}, c^{S}_{t+s} are based on averages over the last four quarters in the sample that was latest available, i.e. 2006q3-2007q2 which yields consistency with the examination of the current situation in table 3.3 in the introduction. S, the equilibrium nominal exchange rate that provides the reference value about which fluctuations are assessed (the entry parity for ERM II), is proxied by the 30-period historical average of S_t. We can interpret $c^S_t = \frac{\bar{S}_t}{S} - 1$, c^i_t, and c^{π}_t as initial 'add factors' where

$$\begin{aligned} \hat{S}_{t+0} &\simeq \frac{\bar{S}_t}{S} - 1 = c^S_t \\ (i^{ss}_t - \tilde{\imath}^{*ss}_t) &= c^i_t \\ (\pi^{ss}_t - \pi^{*ss}_t) &= c^{\pi}_t \end{aligned}$$

For $1 < s \leq 40$, forecasts conditional on these values can then be calculated. The deviation of X_{t+40} about the initial steady state X^{ss}_{t+0} will be large in the end as real convergence implies non-stationarity of the model. It thus no longer holds that the logarithmic deviation $\ln \frac{X_{t+40}}{X^{ss}_{t+0}}$ obtained in the simulation is by itself a good approximation of the precentage deviation of X_{t+40} about its starting value in $t + 0$. Therefore, the following adjustment is necessary

$$\hat{X}_{t+40} \equiv \frac{X_{t+40}}{X^{ss}_{t+0}} - 1 \simeq (1 + \frac{1}{40} \underbrace{\ln \frac{X_{t+40}}{X^{ss}_{t+0}}}_{\text{provided by model}})^{40} - 1 \tag{3.46}$$

in order to obtain growth rates properly.[55] Also note that as we account for differences in steady state values in the initial period t and as we take deviations from the initial steady state, subsequent steady state deviations can be neglected. Eventually, we obtain the terminal value for a criterion, say the inflation criterion as

$$c^{\pi}_{t+40} \equiv \pi_{t+40} - \pi^{*}_{t+40} = \hat{X}^{\pi}_{t+40} + c^{\pi}_{t+0} \tag{3.47}$$

which is just the endpoint of the sequence $\left\{c^{\pi}_{t+s}\right\}^{s=40}_{s=1}$. Note that our procedure secures that obtained values describe the *criteria in levels* (the criteria in the original sense) instead of log-deviations of criteria about the steady state. Initial steady state differentials in variables

[55] To recover levels from the log-linearised model, we note that for any variable $X_{t+s} = X^{ss}_{t+s}(1 + \hat{X}_{t+s})$, where $\hat{X}_{t+s} \equiv X_{t+s}/X^{ss}_{t+s} - 1$ and more generally $X_{t+s} = X^{ss}_t(1 + g)^s$, where $g \equiv (X_{t+s}/X^{ss}_t)^{1/s} - 1$. As the model is log-linearised about its initial steady state, $X^{ss}_{t+s} = X^{ss}_t$. For g small, when can hence write $g \simeq \frac{1}{s} \ln (X_{t+s}/X^{ss}_t) \simeq \frac{1}{s}\hat{X}_{t+s}$ such that eventually $X_{t+s} = X^{ss}_t(1 + \frac{1}{s}\hat{X}_{t+s})^s$. Provided X^{ss}_t, we can calculate the terminal value X_{t+s}.

are eliminated at the endpoint.

Besides the mean outcome, we also provide lower and upper bounds for values for criteria in order to contrast the mean value with the range of likely values. Volatility in potential paths for S_{t+j}, i_{t+j}, π_{t+j} derives from innovations in (stationary) structural shocks that cause business cycles along the convergence path for any given conduct of monetary and fiscal policies. This yields a probabilistic interpretation of fulfilment of Maastricht criteria which will be analysed in detail in the sections to come. In line with results underlying the current outlook in table 3.3, the confidence interval is based on the central 90% of possible values.

Table 3.8 reveals the prospects of passing the monetary Maastricht criteria according to our model. The columns report the current values for the criteria, the most-likely values for the criteria in 40 periods time (the mean outcomes), as well as the lower and upper bounds curtailing the most likely 90% of outcomes. According to our measures, all countries currently, i.e. for $s = 0$, meet the nominal exchange rate criterion, such that model results are consistent with the current empirical outlook presented in table 3.3 on page 134. However, prospects seem to worsen for Poland and Hungary in case the current exchange rate regime would remain in place in the future. According to our forecasting exercise, a strong *devaluation* of the exchange rate might materialise. In both countries, the exchange rate criterion is missed and values settle far outside the admissible range. In case of Poland, developments of the exchange rate can be explained by the assumption of CPI targeting that leaves the exchange rate unbounded in the long run. Hungary seems to be exceptionally vulnerable under its managed floating regime compared to other countries under the same regime. Fulfilment of the exchange rate criterion appears however not to be critical in all other countries. This holds - nearly - by definition for currency board countries and fixing countries that constitute the majority of remaining countries. However, estimated policy coefficients in these countries could indicate that actual policy obeys a different weighting scheme in the policy rule than officially communicated. But also other Visegrad States and Romania will not face problems. It is tempting to conclude that the current exchange rate regime is not as crucial in shaping the prospects of euro area accession as assumed in the related literature by Lewis (2007) and Ravenna and Natalucci (2008).

Turning to the inflation criterion, one observes that average inflationary pressures on the consumer price level ease over time in all countries under consideration besides Poland. Balassa-Samuelson type inflationary pressures where the trend increase in A_H is expected to lead to price pressures in the nontradable sector seem not to be essential in general. As the deterministic series of shocks is rationally expected by firms in the tradable sector, prices can be lowered permanently whereas price pressures in the non-tradables sector are only transitory. For a long enough period of time, decreases in the sectoral price levels of the industry sector transmit fully to the CPI level. Therefore, determining the degree of pricing power of tradable goods producing firms empirically turns the implications of results in the literature that assume determination of H prices in the world market at its head. The developments in Poland's inflation rate also rather point to demand-side price pressures that derive from the long-run non-stationarity that is brought about its flexible exchange rate regime. The continuing devaluation of the home currency triggers an export demand boom that leads to overheating. Whereas currently, the quarterly inflation criterion is not met by Hungary, Estonia, Latvia, Lithuania, and Bulgaria, only Poland and Latvia will face problems in this respect in the longer run on average in our model. However, for Poland, the Baltic States and 2007 entrants, upward price risks remain visible. More than 5 out of 100 realisations will materialise above the bound of the inflation criterion (<0.37pp). Inflation developments can therefore be expected to remain volatile in these countries rendering compliance with this criterion uncertain.

Visegrad States are especially susceptible to failing compliance with the interest rate criterion. Most likely 90% of outcomes are completely beyond the limit imposed by the criterion (<0.50pp), except in case of the Czech Republic. The same holds true for Latvia. Again, developments in Hungary and Poland are of key concern, given that the current regime remains in place over the forecasting horizon. Results for Hungary might also reflect characteristics of the data used in the estimation. Empirically, three month interest rates have remained mostly on two digits levels till the end of 2004 such that the forecast extrapolates these developments into the future. In Poland, high rates can be explained by the stance of monetary policy. As the exchange rate is flexible, monetary policy counteracts the upward pressures on the CPI inflation rate by appropriate increases in its policy rate causing a one-for-one increase in the three month money market rate. We further observe that risks in breaching the upper bound of the interest rate criterion materialise in nearly all countries.

Altogether, it follows that meeting the monetary Maastricht criteria is at risk in many new EU members from Central and Eastern Europe given that the current exchange rate regime would remain in place. Meeting the inflation and exchange rate criterion at the same time seems to be less of a problem, though. Compliance with the inflation and the interest rate criterion jointly is critical instead. Some countries nevertheless turn out to master the criteria in the longer run on all accounts in the average outcome: The Czech Republic, Estonia, Lithuania and the 2007 entrants. Accordingly, most countries that fix the exchange rate are successful in mastering the criteria. However price and interest rate risks remain visible in nearly all countries indicated by a high probability of breaching the criteria's upper bounds in case of the inflation and the interest rate criterion. The outlook for meeting the criteria jointly thus remains uncertain in most countries. In consequence, only the Czech Republic would fulfil the requirements with high probability conditional on current monetary and fiscal settings. Reducing fluctuations as well as adjustments in the policy regimes might prove essential in order to make compatibility with criteria more likely. The following section will be concerned to fathom the scope of policies to accomplish this goal.

Compliance with monetary Maastricht criteria												
Variable	Nominal exchange rate				Nominal interest rate				CPI inflation rate			
Criterion	$c^S_{t+j} = \frac{S_{t+j}}{S} - 1$				$c^i_{t+j} = i_{t+j} - \tilde{i}^*_{t+j}$				$c^\pi_{t+j} = \pi_{t+j} - \pi^*_{t+j}$			
Critical range	$-15\% \leq c^S_{t+j}100\% \leq 15\%$				$c^i_{t+j} \leq 0.50$ pp				$c^\pi_{t+j} \leq 0.37$ pp			
Projections	c^S_t	c^S_{t+40}	$c^S_{t+40,low}$	$c^S_{t+40,high}$	c^i_t	c^i_{t+40}	$c^i_{t+40,low}$	$c^i_{t+40,high}$	c^π_t	c^π_{t+40}	$c^\pi_{t+40,low}$	$c^\pi_{t+40,high}$
Visegrad												
CZ	-11.30	-5.41	-9.53	-1.12	-0.95	-0.95	-2.02	0.14	-0.10	-2.58	-4.65	-0.46
HU	3.92	**52.83**	**42.05**	**64.43**	**4.09**	**11.13**	**9.70**	**12.57**	**1.54**	-10.91	-13.99	-7.73
PL	-3.76	$\rightarrow\infty$	$\rightarrow\infty$	$\rightarrow\infty$	0.50	$\rightarrow\infty$	$\rightarrow\infty$	$\rightarrow\infty$	-0.02	$\rightarrow\infty$	$\rightarrow\infty$	$\rightarrow\infty$
SK	-10.71	3.68	-0.14	7.62	0.50	**2.75**	**1.69**	**3.82**	0.19	-5.45	-7.23	-3.63
Baltics												
EE	0.00	0.04	-1.56	1.66	0.10	0.39	-0.40	**1.19**	**0.76**	-0.41	-1.64	**0.85**
LV	10.81	10.74	9.07	12.43	**1.30**	**1.50**	**0.65**	**2.36**	**1.34**	**0.60**	-0.61	**1.83**
LT	-2.68	-2.87	-4.59	-1.12	0.04	-0.02	-0.85	**0.82**	**0.56**	-0.34	-1.67	**1.00**
2007												
BG	0.19	-0.13	-1.59	1.36	0.35	0.12	-0.52	**0.77**	**0.54**	-0.66	-2.02	**0.72**
RO	6.94	10.05	2.14	**18.61**	0.50	0.20	-1.14	**1.56**	0.33	-0.69	-2.15	**0.80**

Table 3.8: Compliance with monetary Maastricht criteria. Bold: value for criterion does not respect the required bound(s).

3.7 Reducing Uncertainty in Meeting Criteria

Provided that meeting criteria jointly is uncertain in a range of countries, we now explore the scope of monetary stabilisation policies to improve upon previous results. The success of policy in decreasing uncertainty in future values for criteria c^{π}_{t+s}, c^{i}_{t+s}, c^{S}_{t+s} affords a probabilistic interpretation of the criteria as stressed in the introduction. The main idea of sustainable fulfilment is that criteria should not only be met on average but also with high probability such that a high number of realised values for criteria is below the imposed upper bound/within the bounds. The average outcome alone will on the contrary be insufficient to judge as to whether meeting the criteria can be sustained in the future. We argue that this requirement is implicitly underlying the country examination by the ECB and the European Commission in their convergence reports. Whereas we assume that policy cannot influence the path of real convergence (it is triggered by real supply shocks only), policy may contribute nevertheless to narrowing the range of 'undesirable' possible paths in order to make conformance more likely. Before moving on, it seems important to stress that the following analysis will remain silent about how to improve the average outcome. The average outcome however seems to be especially relevant for countries that have not managed compliance in that respect according to table 3.6 (this concerns Hungary, the Slovak Republic, Latvia, and Poland). The further analysis might nevertheless provide useful for these countries as well. It allows to assess whether these countries would have the scope of meeting the criteria in the absence of trend-related pressures but given all other factors remain in place. Cyclical sources of risk to meeting the criteria can thus be identified.

In the following, monetary policy will be described as being set actively by a policymaker that seeks to minimise a loss objective in order to reduce the uncertainty in meeting the criteria. We will conduct counterfactual experiments and simulate the model under the current exchange rate regime, the current exchange rate regime with optimally chosen weights on policy objectives, and the Maastricht-constrained optimal policy problem where Maastricht criteria enter the loss objective explicitly. The policy-objective will not be based on the intertemporal utility loss experienced by the household, in contrast to chapter 2. It can be argued that the Maastricht criteria have no utility foundation neither such that they cannot suitably be connected to a microfounded policy problem. Authors that introduce a microfounded objective - assuming a stationary economy - like Lipinska (2008) also miss the point that welfare might crucially hinge on real convergence such that the welfare objective might be flawed once these factors are left out. An ad-hoc objective can be rationalised as being delegated by society to an (independent) central bank that can vary relative weights of policy objectives without being tied to the will of society, see Söderström (2005). As a further advantage, an ad hoc loss function allows to keep the policy problem to be solved relatively tractable which is of added value when incorporating the Maastricht criteria in the loss function in the following. The analysis will show that policy alone, even when targeting Maastricht criteria explicitly, just provides the necessary condition for increasing the probability of mastering the criteria. Further adjustments in the economy will be necessary, as will be explained below.

3.7.1 Objective Function under the Current Regime

The policymaker is assumed to conduct policy by evaluating fluctuations in activity and prices by means of an intertemporal loss function $\mathcal{L}$ that summarises the expected present discounted value of period losses L_t. Contributions to losses derive from deviations of policy objectives from target levels, where objectives are weighted by their relative importance. Therefore

$$\mathcal{L} \equiv (1-\beta)\,\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t L_t = (1-\beta)\,\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \widehat{\boldsymbol{Y}}_t' \boldsymbol{W} \widehat{\boldsymbol{Y}}_t \tag{3.48}$$

$\widehat{\boldsymbol{Y}}_t$ denotes the (column) vector of all endogenous variables in the model (their deviations from the initial steady state) and $\boldsymbol{W}$ is a quadratic weighting matrix that assigns the relative importance of objectives, similar to (2.121) in chapter 2. Covariance terms (off-diagonal elements in the weighting matrix $\boldsymbol{W}$) are of no stabilisation concern and set to zero, which is a common assumption. In the baseline case, we assume that the policymaker seeks to minimise gaps in sectoral output, $\hat{Y}_{J,t}$, and the CPI inflation differential, $\hat{\pi}_t - \hat{\pi}_t^*$. Accordingly, $\hat{Y}_{J,t} = \ln \frac{Y_{J,t}}{Y_{J,t}^{ss}}$ denotes the deviation of actual sectoral gross value added $Y_{J,t}$ about its steady state/natural rate value $Y_{J,t}^{ss}$. $\hat{\pi}_t - \hat{\pi}_t^* = \ln \frac{1+\pi_t}{1+\pi_t^{ss}} / \frac{1+\pi_t^*}{1+\pi_t^{ss*}}$ is the differential of home CPI inflation π_t and foreign (euro area) inflation π_t^* about their respective steady state levels π_t^{ss} and π_t^{ss*}. Deviations in sectoral output in the H sector and the N sector from targets as well as deviations in the CPI inflation differential from target determine stabilisation costs from the policymaker's perspective. The relative importance of objectives is assessed by weights μ_1 (for $\hat{Y}_{H,t}$), μ_2 (for $\hat{Y}_{N,t}$), and μ_3 (for $\hat{\pi}_t - \hat{\pi}_t^*$) such that

$$\boldsymbol{W} = \begin{bmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{bmatrix}$$

(3.48) takes into account that in the open economy, the home central bank wants to minimise fluctuations in the inflation differential $\hat{\pi}_t - \hat{\pi}_t^*$ (fluctuations in the real exchange rate), instead of solely concentrating on dynamics in the home inflation rate $\hat{\pi}_t$. Such a target seeks to provide stability in the external real exchange rate that stabilises the nominal values of exports and imports. Also it is acknowledged that the CPI incorporates fluctuations that arise from foreign shocks. Sectoral output gap targeting reflects the fact that the policymaker cares about real activity also, as was obtained in chapter 2 within the microfounded setting, see (2.110) on page 92. The specification of objectives makes clear that the policymaker is assumed to stabilise deviations from trend, where the trend $\left\{Y_{J,t+j}^{ss}, \pi_{t+j}^{ss}, \pi_{t+j}^{ss*}\right\}$ is taken as given. As a result, an inflationary bias of the Barro and Gordon (1983) type is absent in the model. Also, as the objectives in (3.48) are symmetric, positive and negative fluctuations of same magnitude are associated with equal stabilisation costs.

In chapter 2 it was argued that the policy maker should concentrate on stabilising fluctuations that derive from nominal rigidities only such that efficient shocks (shocks that alter relative prices under price flexibility) are of no policy concern. Only the difference between inefficient fluctuations and efficient shocks should be curtailed. In (3.48) there is no such assumption underlying, as the potential rate of output is not distinguished explicitly from the natural rate of output. We argue that the Maastricht criteria do not differentiate between efficient and inefficient fluctuations in criteria neither as they lack an (explicit) foundation in consumer theory of the household: Even if all variability derived from efficient reallocations and given the economy was fully flexible in adapting to shocks, but the variability too high, the criteria would still not be met with high probability. Hence, a policymaker in EU9 is forced to contend with other objectives than those derived from maximising households' welfare only. As a result, we did not opt for a microfounded loss-function here.

The period loss function L_t in (3.48) that respects output and inflation targets can be written as

$$L_t = \mu_1 \hat{Y}_{H,t}^2 + \mu_2 \hat{Y}_{N,t}^2 + \mu_3 \left(\pi_t - \pi_t^*\right)^2 \qquad (3.49)$$

We set output weights in $\boldsymbol{W}$ in accordance to relative contributions of Y_H and Y_N to overall gross value added in the data such that $\mu_1 = \frac{P_H Y_H}{GDP}$ and $\mu_2 = 1 - \mu_1$. Developments in the sector with larger share in overall gross value added thus receive a higher weight. μ_3 is set to one in the baseline case as in loss functions for closed economy models, see Walsh (2003, chapter 11). Note that these preferences will remain invariant to the actual policy regime chosen which allows for relative comparisons between policy outcomes. (3.49) forms the objective behind a prototypical Taylor (1993) rule adjusted for the open economy. In order to determine the costs that can truly be attributed to requirements from Maastricht compliance, we first identify the minimum loss attainable under the current exchange rate regime before incorporating Maastricht targets in L_t explicitly. We determine the optimal simple rule under the current policy regime by minimising

$$\mathcal{L}^{opt} \equiv \min_{\{\boldsymbol{p}_i\}} \left\{ \mathcal{L} = (1-\beta)\, \mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \mu_1 \hat{Y}_{H,t}^2 + \mu_2 \hat{Y}_{N,t}^2 + \mu_3 \left(\pi_t - \pi_t^*\right)^2 \right\} \right\} \tag{3.50}$$

subject to the linearised economy, the policy rule (3.19) and conditional on the conduct of fiscal policy (3.23). $\boldsymbol{p}_i$ denotes the policy parameter vector, i.e. the weights on objectives (elasticities) in (3.19) that minimise $\mathcal{L}$ for given preference weights $\{\mu_1, \mu_2, \mu_3\}$. Under the current policy regime, the possibility to adjust the relative importance of objectives in the instrument rule is conditional on the characteristics of the specific exchange regime in place. Therefore $\boldsymbol{p}_i = \{k, \varpi_\pi, \varpi_S, \varpi_{GDP}\}$ is at hand for a managed floater, $\boldsymbol{p}_i = \{k, \varpi_S, \varpi_{GDP}\}$ for a currency boarder/fixed exchange rate regime and $\boldsymbol{p}_i = \{k, \varpi_\pi, \varpi_{GDP}\}$ in case of a floating exchange rate regime. We will argue below that under the Maastricht regime (the loss function L_t augmented for Maastricht criteria), all countries have at hand all instruments in the monetary policy rule. The role of optimally choosing parameters in the fiscal rule (3.23) (such that fiscal policy assists monetary policy in minimising $\mathcal{L}$) is deferred to the subsequent section 3.8. We are thus able to explore the efficacy of 'active' monetary policy in reducing uncertainty in outcomes first, for given 'passive' fiscal policies.

As in chapter 2 under 2.6, we evaluate the loss function under the limiting condition that $\beta \to 1$. In the class of optimal simple rules, the rescaled intertemporal loss function then boils down to the unconditional expectation of the period loss function

$$\lim_{\beta \longmapsto 1} (1-\beta)\, \mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t L_t = \mathcal{E}\left[L_t\right] \tag{3.51}$$

and therefore

$$\mathcal{E}\left[L_t\right] = \mu_1 V\left[\hat{Y}_{H,t}\right] + \mu_2 V\left[\hat{Y}_{N,t}\right] + \mu_3 V\left[\pi_t - \pi_t^*\right] \tag{3.52}$$

$\mathcal{E}[\cdot]$ applied to average period loss L_t can be regarded as the long run average value of the random variable L_t. $V\left[\hat{X}\right] = \mathcal{E}\left[\hat{X}_t^2\right]$ is the unconditional variance operator (see also Woodford 2003, p. 431). Fluctuations in consumption and output were shown to cause losses in utility for the household about trend, as explained in chapter 2 under (2.110).[56] Accordingly, $\mathcal{E}[L_t]$ can be understood as representing average losses each household faces in any period (any quarter) that arise from the fact that due to trade-offs between competing objectives, fluctuations cannot be eliminated fully. As a result, it is not possible that $\hat{Y}_{H,t} = \hat{Y}_{N,t} = \pi_t - \pi_t^* = 0$ at the same time in case of positive weights on all objectives. (3.52) does not provide a direct welfare

[56] Further note that $\mathcal{E}$ is applicable (only), as we abstracted from steady state growth in this section. Otherwise we would have infinite variances and a date zero zero perspective needs to be taken, see Obstfeld (1994).

interpretation, such that costs should be referred to as stabilisation costs instead. In a closed economy setting however, an objective like (3.52) can be derived from a utility-maximising framework, see Walsh (2003, chapter 11). Finding the optimal path for the policy rate i_t in (3.19) that minimises this objective eventually boils down to minimising the expected loss by minimising the unconditional, reduced form variances of the variables in the objective function

$$\mathcal{L}^{opt} = \min_{\{\varpi_\pi, \varpi_s, \varpi_{GDP}, k\}} \mathcal{E}\left[L_t\right] = \min_{\{\varpi_\pi, \varpi_s, \varpi_{GDP}, k\}} \left[\text{trace}\left[\boldsymbol{W} \sum\nolimits_{yy}\right]\right] \tag{3.53}$$

by choosing policy parameters $\{\varpi_\pi, \varpi_s, \varpi_{GDP}, k\}$ appropriately. $\sum_{yy}$ denotes the unconditional variance-covariance matrix of all endogenous variables, obtained from the state-space form (3.34).[57]

3.7.2 Maastricht-Constrained Loss Function

Instead of choosing policy optimally under the given exchange rate regime, one could think of targeting criteria directly. Woodford (2003, p. 427-435) performs a related exercise. There, monetary policy is concerned about the positive probability of hitting the zero lower bound of the nominal interest rate. Based on the framework in Woodford (2003), Lipinska (2008) determines the set of inequality constraints that must be met for welfare-maximising policy to succeed in meeting the monetary Maastricht criteria. The derivation of constraints here will follow the exposition proposed therein whereas our implementation will differ, as explained below.

Conditions for admissible variability of criteria concern the maximum average deviation of values for a Maastricht criterion c^x from its bound $B(c_t^x)$. It will be required that $B(c_t^x) - c_t^x \geq 0$ will occur often which is our interpretation of sustainable compliance with a certain criterion. Provided $B(c_t^x) - c_t^x \geq 0$, one could determine conditions for the variability in c^x such that the criterion does not harm the constraint in 90 out of 100 cases.[58] Therefore, the upper bound on this condition reads

$$P(c_t^x \leq B(c_t^x)) = 0.95 \tag{3.54}$$

Then also

$$P\left(Z \leq \frac{B(c_t^x) - \mathcal{E}(c_t^x)}{\sigma\left(c_t^x\right)}\right) = 0.95 \tag{3.55}$$

[57] The covariance matrix is obtained by simulating the estimated model for a long period of time where innovations in each of the nine structural shocks and in measurement shocks occur at random.

[58] We use the central 90% of realisations in case of all criteria to obtain comparability across criteria. As the inflation criterion and the interest rate criterion are asymmetric in nature, it would also be possible to disregard the lower bound.

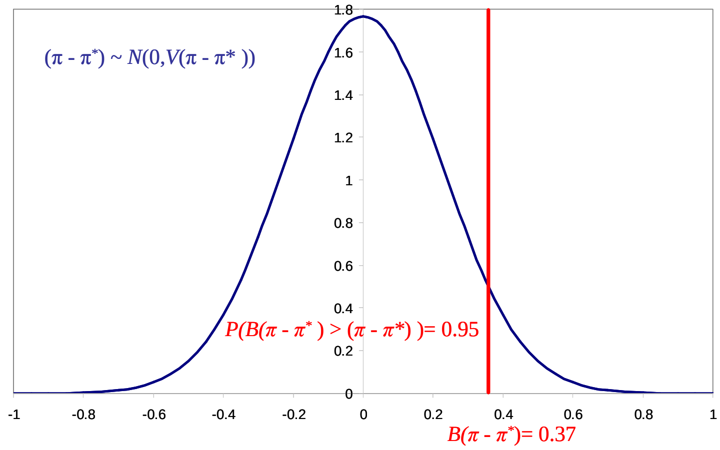

Figure 3.9: Sustainable compliance with the inflation criterion.

Assuming c_t^x is a normally distributed random variable with constant mean $\mathcal{E}(c_t^x)$ and constant variance $\sigma^2(c_t^x)$, Z will be standard-normal and $Z = 1.645$ solves the condition. The criterion is therefore met with high probability in case values for c_t^x respect $\mathcal{E}c_t^x \leq B(c_t^x) - 1.645\sigma(c_t^x)$. Chart 3.9 illustrates the reasoning for sustainable fulfilment of the inflation criterion. The criterion is met with 95% probability, if a realisation c_t^π materialises to the left of the upper bound 0.37pp.

As a necessary condition, the policymaker then seeks to solve

$$\mathcal{E}_0(1-\beta)\sum_{t=0}^{\infty}\beta^t\left(B(c_t^x)-c_t^x\right)\geq 0 \tag{3.56}$$

where discounting and a date zero perspective are introduced to conform with the assumptions about the setup of the loss function (3.48). (3.54) requires to think about second moments as well.[59] Thus,

$$\mathcal{E}_0(1-\beta)\sum_{t=0}^{\infty}\beta^t\left(B(c_t^x)-c_t^x\right)^2\leq K\left((1-\beta)\,\mathcal{E}_0\sum_{t=0}^{\infty}\beta^t\left(B(c_t^x)-c_t^x\right)\right)^2 \tag{3.57}$$

where $K = 1 + k^{-2}$, with $k = 1.645$. (3.56) and (3.57) directly apply for the inflation rate

[59]Note that from (3.55) and with $k = Z$

$$\begin{aligned}
\mathcal{E}_0 k\sigma(c_t^x) &\leq \mathcal{E}_0\left[B(c_t^x)-\mathcal{E}c_t^x\right]\\
\mathcal{E}_0 k^2\sigma^2(c_t^x) &\leq \left(\mathcal{E}_0\left[B(c_t^x)-\mathcal{E}c_t^x\right]\right)^2\\
\mathcal{E}_0\left[(B(c_t^x)-c_t^x)^2\right] &\leq \left(1+\frac{1}{k^2}\right)\left(\mathcal{E}_0\left[B(c_t^x)-c_t^x\right]\right)^2
\end{aligned}$$

using that $\sigma^2(c_t^x) = \sigma^2(B(c_t^x)-c_t^x) \equiv \mathcal{E}\left[(B(c_t^x)-c_t^x)^2\right] - \left(\mathcal{E}\left[B(c_t^x)-c_t^x\right]\right)^2$ and applying $\mathcal{E}_0$ to the latter expression.

criterion and the interest rate criterion, i.e. $c_t^x = \{c_t^\pi, c_t^i\}$. Unlike as in Lipinska (2008), we allow for variability in π_t^* and i_t^* also. We deem invariance in the euro area inflation rate and the euro area nominal interest rate not an appropriate assumption neither empirically nor in the modeling framework presented here. Terms of trade shocks could then not arise from foreign which would unnecessarily limit the role external competitiveness has for the domestic industry sector. For the nominal exchange rate criterion we define

$$(1-\beta)\,\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \left(B(c_t^S) - c_t^S\right) \;\; \geq \;\; 0 \tag{3.58}$$

$$(1-\beta)\,\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \left(B(c_t^S) + c_t^S\right) \;\; \geq \;\; 0 \tag{3.59}$$

and second moment conditions read

$$(1-\beta)\,\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \left(B(c_t^S) - c_t^S\right)^2 \;\; \leq \;\; K\left((1-\beta)\,\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \left(B(c_t^S) - c_t^S\right)\right)^2 \tag{3.60}$$

$$(1-\beta)\,\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \left(B(c_t^S) + c_t^S\right)^2 \;\; \leq \;\; K\left((1-\beta)\,\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \left(B(c_t^S) + c_t^S\right)\right)^2 \tag{3.61}$$

as in Lipinska (2008). Stabilisation costs that arise under (3.48) and (3.53) can be compared to the case where a 'Maastricht'-constrained period loss function L_t^M is in place that minimises deviations from criteria from their steady state values (also labeled the ERM II regime in the following). We state the problem as an augmented optimal policy problem of the form

$$\mathcal{L}^M = \min_{\{\varpi_\pi, \varpi_s, k\}} \mathcal{E}_0 \sum_{t=0}^{\infty} (1-\beta)\,\beta^t L_t^M \tag{3.62}$$

with

$$L_t^M = \mu_1 \hat{Y}_{H,t}^2 + \mu_2 \hat{Y}_{N,t}^2 + \mu_3 (\underbrace{\pi_t - \pi_t^*}_{c_t^\pi})^2 + \phi_1 (\underbrace{i_t - i_t^*}_{c_t^i})^2 + \phi_2 \underbrace{\hat{S}_t^2}_{c_t^S} \tag{3.63}$$

Inflation and output objectives from (3.49) remain in place, already comprising the inflation criterion, as $c_t^\pi \equiv \pi_t - \pi_t^*$. Additionally, the interest rate criterion c_t^i, and the exchange rate criterion c_t^S need to be respected. Note that contrary to Lipinska (2008), (3.62) is not derived by solving the original optimal policy problem augmented for the additional (non-linear) constraints imposed by the Maastricht treaty, i.e. subject to (3.56) - (3.57). If one allows for the latter, and in the presence of binding Maastricht constraints, "[...] the optimal [microfounded] monetary policy constrained by the Maastricht convergence criteria does not only lead to smaller variances of the Maastricht variables, it also assigns target values for these variables that are different from the steady state of the optimal monetary policy", see Lipinska (2008, p. 25).

We instead doubt the possibility of policy to affect target (steady state) values π_t^{ss}, π_t^{ss*}, i_t, $\tilde{\imath}_t^*$, $Y_{J,t}^{ss}, S_t^{ss}$ in the loss function (3.62). It will thus be the case that even in case the constraints (3.56) and (3.57) are binding, the policymaker cannot readjust targeted objectives such that criteria will still be missed. In case of Woodford (2003) and Lipinska (2008) successful stabilisation can instead *always* be secured as the policy problem entails one 'additional degree of freedom': In case stabilisation turns out to still produce a (too) wide range of outcomes, the target can be lowered, until values will no longer breach the bound in many cases (the mean and the variance of values can be influenced). In our setup, it can instead happen that even under the optimal stabilisation policy, criteria will still not be met with high probability such that compliance remains uncertain. Contrary to these authors, we are hence less optimistic about the possibility of policy to affect steady state outcomes: Steady state allocations arise from optimising behaviour of households and firms. Markets might discipline policymakers such that any deviation from steady state values cannot be sustained forever. Arbitrage possibilities will be exploited until the outcome favoured by the private sector is restored again (thus, a kind of 'Lucas-critique argument' applies).

The Maastricht regime (3.63) is minimised intertemporally subject to the constraints imposed by the structure of the economy and choosing optimal values for response coefficients in the policy rule (3.19).[60] As stressed above, the regime switch to (3.62) allows choosing all coefficients $\{\varpi_\pi, \varpi_s, \varpi_{GDP}, k\}$ in the policy rule optimally. Furthermore, the higher the value for a specific penalty coefficient in the policy function given by (3.62), the higher the cost of meeting the particular target. Again assuming that the discount factor attains its limit, one obtains that (3.62) can be expressed as the average per period loss[61]

$$\mathcal{E}\left[L_t^M\right] = \mathcal{E}\left[L_t\right] + \phi_1 V\left[c_t^i\right] + \phi_3 V\left[c_t^S\right] = \mu_1 V\left[\hat{Y}_{H,t}\right] + \mu_2 V\left[\hat{Y}_{N,t}\right] + \mu_3 V\left[c_t^\pi\right] + \phi_1 V\left[c_t^i\right] + \phi_3 V\left[c_t^S\right] \tag{3.64}$$

$\mathcal{E}\left[L_t^M\right]$ represents the weighted sum of unconditional variances in gross value added and values for monetary Maastricht criteria to be minimised.[62]

[60] The problem has the form of an n-dimensional grid-search that is performed within the framework of the simplex method, see the `fminsearch` documentation in Matlab.

[61] We again used that the conditional variance becomes unconditional in case of the limiting assumption about the discount factor, $\lim_{\beta \mapsto 1} \hat{V}\left[\hat{X}_t\right] = V\left[\hat{X}_t\right]$, as explained above.

[62] We use the optimal simple rule routine `osr` in `dynare` in the quantitative evaluation. Minimum, unscaled, lifetime loss can be recovered from per period average loss according to

$$\mathcal{L}^M \simeq \frac{1}{1-\beta} \min_{\{\varpi_\pi, \varpi_s, k\}} \mathcal{E}\left[L^M\right] \tag{3.65}$$

Assumptions about the limiting case concerning the discount factor β also have implications for the set of inequality constraints presented earlier. As first moments of log-linearised variables are all zero *about the trend path*, one obtains that

$$\mathcal{E}\left[\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \hat{X}_t\right] = 0 \tag{3.66}$$

such that the inequality constraint on the criteria's first moment (3.56) is always satisfied

$$\mathcal{E}\mathcal{E}_0\left[B(c_t^x) - c_t^x\right] = B(c_t^x) > 0 \tag{3.67}$$

Note that in the work of Lipinska (2008), this condition is underlying the Maastricht check right from the beginning, i.e. when it is assessed how optimal policy would look like that is not constrained to fulfil the criteria. In our case however, this condition is only valid on top of the evaluation of average outcomes performed in section 3.6. For the second moment condition (3.57), again applying the unconditional expectations operator, we obtain

$$\begin{aligned}
\mathcal{E}\left[\mathcal{E}_0\left(1-\beta\right)\sum_{t=0}^{\infty}\beta^t\left(B(c_t^x)-c_t^x\right)^2\right] &\leq \mathcal{E}K\left(\left(1-\beta\right)\mathcal{E}_0\sum_{t=0}^{\infty}\beta^t\left(B(c_t^x)-c_t^x\right)\right)^2 \\
V\left[c_t^x\right]+\left(B(c_t^x)\right)^2 &\leq \mathcal{E}\left[K\left(\mathcal{E}_0\left[B(c_t^x)-c_t^x\right]\right)^2\right] = K\left(B(c_t^x)\right)^2
\end{aligned}$$

where we also used (3.67). We arrive at the following set of inequality constraints that provide guidance for sustainable - low uncertainty - fulfilment of monetary Maastricht criteria[63]

$$V\left[c_t^\pi\right] \leq (K-1)\left(B(c_t^\pi)\right)^2 = 0.227^2 \tag{3.68}$$

$$V\left[c_t^i\right] \leq (K-1)\left(B(c_t^i)\right)^2 = 0.302^2 \tag{3.69}$$

$$V\left[c_t^S\right] \leq (K-1)\left(B(c_t^S)\right)^2 = 9.119^2 \tag{3.70}$$

Note that these inequality constraints do not become part of the policy program the policymaker minimises explicitly.

[63] The criteria were also already used in the introduction in assessing the variability in the current outlook in table 3.3 on page 134. Note that the same conditions can be obtained by applying the Woodford (1999, p. 45) measure of variability, namely $V(x) \equiv \mathcal{E}\left[\mathcal{E}_0\left\{(1-\beta)\sum_{t=0}^{\infty}\beta^t\hat{x}_t\right\}\right]$.

3.7.3 Meeting Criteria with High Probability

Table 3.9 on page 193 provides results on the efficacy of stabilisation policies in decreasing the uncertainty of compliance with the criteria. Unconditional variances of the Maastricht criteria, $V(c_t^\pi)$, $V(c_t^i)$, $V(c_t^S)$ are obtained by taking the estimated model as data generating process similar to section 3.6. However, the deterministic shock path underlying the analysis therein is shut down as the focus is on volatility generated by structural shocks and measurement errors about the trend path. Accordingly, we have $E_{t+s}^T = 1$ in the tradable technology process (3.6). The first row for each country in table 3.9 labeled 'historical' illustrates the development of volatility in the Maastricht variables when the current exchange rate regime prevails in the future such that policy takes coefficients in the instrument rule (3.19) as given and period loss is evaluated according to (3.52). The second row presents volatilities in criteria when policies are chosen optimally under the current regime, i.e. by choosing optimal weights in the instrument rule (3.19) to minimise (3.52). The last row for each country shows volatilities in the criteria when Maastricht constraints are included as additional targets the policymaker needs to stabilise. In that case, policy aims at minimising (3.64). As in the introduction in table 3.3, simulated volatilities in criteria are presented, such that $\sigma(c_t^x) \equiv \sqrt{V(c_t^x)}$ (in percentage points).

We obtain that meeting the criteria jointly remains unlikely, even after policy has taken effect by choosing appropriate stabilisation policies. There is no single case under any regime where variability in all criteria respects the upper bounds imposed by (3.68) to (3.70). The inflation criterion is only met in very few cases. The bound is satisfied under the optimal and Maastricht-constrained policy for Poland and under the optimal policy for Slovakia. Therefore, both countries could improve upon results by choosing policy optimally, already under the current regime. However, there is no general pattern concerning the effect of the current exchange rate regime in determining the success of compliance. Visegrad states continue having problems in meeting the interest rate criterion as found earlier. Countries that are currently under a currency board or a fixed exchange rate regime (the Baltic States and Bulgaria) do better in this respect. Consistent with the presentation in section 3.6, the exchange rate criterion is met under the current regime in all countries and also under the other regimes in most of the cases.

The economic costs that are associated with attempts to increase the likelihood of compliance can be assessed by the expected period loss $\mathcal{E}\left[L_t^o\right]$ associated with each strategy provided in the table. In the adjacent column labelled 'rank', the costs are ordered from highest to lowest under each strategy across countries where highest losses are indicated by the number 1.

The table reveals that Visegrad States incur highest losses and 2007 entrants lowest, whereas Baltic States range in between. We further obtain that choosing policy optimally under the current regime will bring a large reduction in costs. Migrating to a Maastricht-type loss function is 'Pareto efficient' in the sense that losses are lower than under the current policy for all countries, the exception being Slovakia. Consequently, policymakers will do better than under the current regime if they deliberately start to target Maastricht objectives supplementary to policy targets in the basic loss function. This would induce higher stability in output and the inflation rates and lead to a higher probability of meeting the criteria.

$\mathcal{E}\left[L_t^o\right]$ also provides information as to whether the loss rank changes when moving from the current to the optimal and to the ERM II regime. If there are only minor changes when moving from one regime to another, the current exchange rate regime appears to have little influence on the relative ranking of costs. We indeed observe that the ranking of losses remains rather invariant across country groups and considered regimes: Visegrad States that exhibit high losses under the current regime remain mostly high loss countries under the optimally chosen coefficients and also in case policy targets Maastricht objectives explicitly. The Baltic States remain in the midfield when moving from one regime to the other. 2007 entrants experience the lowest losses across regimes.

3.7.4 Meeting Criteria in Subsequent Periods

In our context, sustainable fulfilment of criteria as presented in the preceding section refers to securing low (business cycle) volatility in values along the convergence path. Based on our sustainability concept, number of periods in which criteria are passed subsequently can be determined. In order to do so, we need to take the trend paths analysed in section 3.6 again explicitly into account. In the real convergence excercise we obtained a sequence of mean values for each criterion along the projection horizon $\left\{c_{t+s}^x\right\}_{s=1}^{40}$. Augmenting the trend path by deviations about the path (business cycles) allows to detect period(s) where criteria are passed under a certain policy regime. It becomes clear that there is no one-to-one relationship between sustainable fulfilment of criteria and number of quarters criteria are passed. Nevertheless, the higher the number of subsequent periods where criteria are met jointly, the more sustainable developments are.

Figure 3.10 illustrates how values for criteria unfold over the projection horizon for one draw from the distribution of structural shocks and measurement errors.[64] For illustrative purposes, the chart shows the path for Romania under the current policy regime. Limits for

[64] Definitions of shocks are described in sections 3.4 and 3.5.3.

Monetary stabilisation for Maastricht compliance		Volatility in Maastricht criteria $\sigma\left[c_t^{\pi}\right]$	$\sigma\left[c_t^{i}\right]$	$\sigma\left[c_t^{S}\right]$	$\mathcal{E}\left[L_t^o\right]$ in $\%^2$	Rank	Pass
		≤ 0.227	≤ 0.302	≤ 9.119			
Visegrad States							
CZ	Historical	**0.86**	**0.91**	2.82			No
	Current	**1.30**	**0.68**	2.30	4.20	3	No
	Optimal	**0.66**	**2.26**	4.88	2.25	2	No
	ERM II	**1.21**	**0.39**	0.39	3.49	2	No
HU	Historical	**0.94**	**0.98**	3.00			No
	Current	**2.84**	**1.62**	8.10	10.49	2	No
	Optimal	**0.80**	**0.67**	3.72	2.67	1	No
	ERM II	**0.93**	**0.36**	0.21	3.04	3	No
PL	Historical	**1.17**	**0.90**	6.53			No
	Current	**13.7**	**13.4**	$\mapsto\infty$	$\mapsto\infty$	1	No
	Optimal	0.00	**1.10**	**46.2**	1.69	6	No
	ERM II	0.10	**1.10**	**62.5**	2.81	4	No
SK	Historical	**1.97**	**0.91**	1.68			No
	Current	**1.28**	**0.80**	2.21	3.72	4	No
	Optimal	0.06	**1.03**	**26.21**	2.01	3	No
	ERM II	**0.32**	**1.00**	**14.27**	201.5	1	No
Baltic States							
EE	Historical	**0.76**	**0.91**	0.00			No
	Current	**0.75**	**0.48**	0.86	2.06	7	No
	Optimal	**0.71**	**0.57**	1.15	1.64	7	No
	ERM II	**0.82**	**0.39**	0.19	1.94	8	No
LV	Historical	**0.98**	**0.91**	4.33			No
	Current	**0.76**	**0.57**	1.05	2.58	5	No
	Optimal	**0.73**	**0.52**	0.76	1.78	4	No
	ERM II	**0.79**	**0.37**	0.15	2.01	7	No
LT	Historical	**0.84**	**0.90**	3.02			No
	Current	**0.82**	**0.54**	1.03	2.11	6	No
	Optimal	**0.81**	**0.59**	1.12	1.78	4	No
	ERM II	**0.88**	**0.35**	0.16	2.04	5	No
2007 Entrants							
BG	Historical	**1.98**	**0.93**	0.14			No
	Current	**0.84**	**0.44**	0.90	1.43	9	No
	Optimal	**0.75**	**0.69**	3.43	0.94	9	No
	ERM II	**0.97**	**0.31**	0.06	1.42	9	No
RO	Historical	**3.61**	**0.90**	6.70			No
	Current	**0.91**	**0.80**	3.48	1.65	8	No
	Optimal	**0.84**	**0.63**	1.46	1.03	8	No
	ERM II	**0.94**	**0.31**	0.30	1.34	6	No

Table 3.9: Uncertainty of meeting the Maastricht criteria under various assumptions about the conduct of policy. Values in bold type indicate that a criterion is not passed under the specific regime. Results refer to the unconditional, simulated standard deviation in Maastricht criteria, in percentage points.

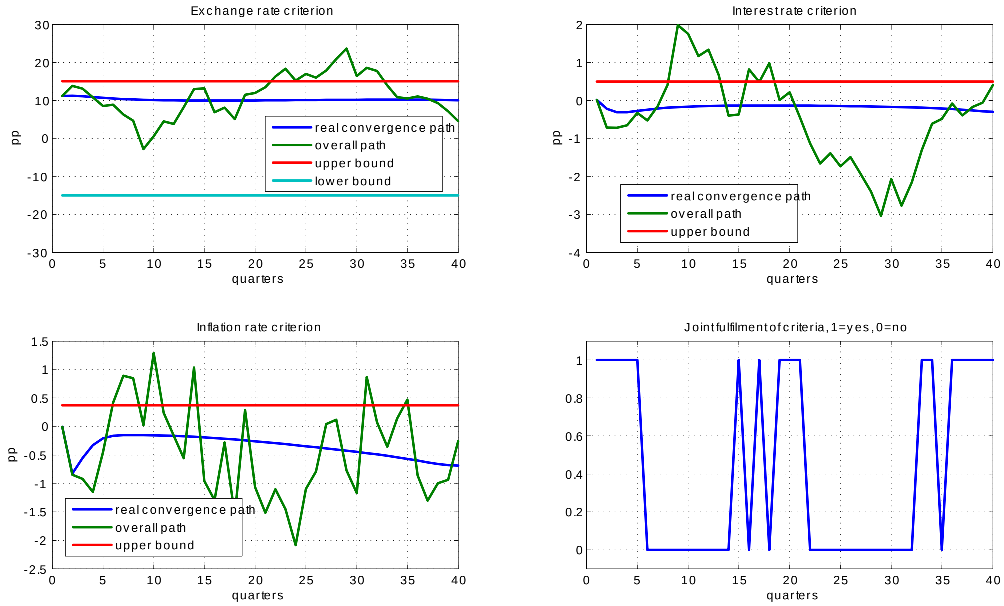

Figure 3.10: Time path for criteria: The case of Romania under the current policy regime.

criteria are shown also in order to assess the breaching of criteria in certain periods. Albeit Romania satisfies the criteria on average along the convergence path (blue line), as presented earlier in table 3.8, we find Romanian volatility in interest rates and the inflation rate to be too high to secure sustainable convergence: The overall path (green line) that combines the convergence path with deviations about the path breaches the imposed limits too often. Albeit criteria are met in some periods and there might even be joint fulfilment of criteria in one or more subsequent periods, these events are rare. Furthermore, fulfilling the criteria in adjacent periods does not become essentially more likely further in the future as the lower right panel of figure 3.10 makes clear. Albeit the criteria are met jointly after about 20 quarters in the future for some subsequent quarters, there follows a prolonged phase where criteria will again not be met jointly.

Evaluating the projection horizon in this fashion for all countries, we can determine the number of periods any criterion is met and also periods where all criteria are met jointly under current, optimal and Maastricht constraint policies. Barriers in fulfilling a criterion c_t^x materialise in a low number of periods in which the criterion is met. Accordingly, the joint fulfilment at all considered time spans becomes less likely. The analysis thus yields an additional interpretation of sustainability in terms of time. Table 3.10 presents the summary of results.[65]

[65] Note that the results are based on a certain number of draws in order to wash out effects of specific draws from the joint distribution of structural shocks.

The left panel displays the number of times *each* criterion is met based on averaging individual stochastic simulations of the forecasted path. As compliance with each criterion is necessary but not sufficient, the right panel shows time spans for which criteria are met *jointly* under a certain policy, ranging from one to eight subsequent quarters. As four and eight subsequent periods are evaluated too, it is possible to generalise our results to the year-on-year assessment as laid down in the original criteria for the inflation and interest rate criterion and the two year assessment in ERM II for the exchange rate criterion. For example, given that the quarterly inflation criterion is met in four subsequent periods, also the year-on-year inflation rate will pass the test, when assessed by the yearly inflation criterion. Accordingly, results from quarterly assessments carry over to yearly assessments for other criteria. Again considering the example of Romania under the current regime of managed-floating, we observe from table 3.10 that on average the inflation criterion is passed 32 times, the interest rate criterion 35 times, and the exchange rate criterion 36 times about the projection horizon of 40 quarters. Joint fulfilment of criteria for one period will happen on average 24 times, for two subsequent periods 13 times, for four following quarters 7 times, and for eight quarters 2 times.

For all countries, we obtain from 3.10 that the inflation criterion is met in many quarters in countries where trend-behaviour implied a steady decrease in the domestic price level following the permanent increase in domestic productivity. This holds for all countries but Poland and Latvia (see table 3.8). Again, the policy chosen has no essential impact on the results as stressed before. Turning to the interest rate criterion, one observes that countries that meet the criterion in few periods under the current regime (Hungary, Poland, and Slovakia) will fail also under optimally chosen policy and Maastricht-constrained policy. The exchange rate criterion under the current policy regime is however met in most periods and countries but Poland and Hungary. This result is analogous to results on volatility measures obtained for exchange rates presented in table 3.9 on page 193. There we obtained that $\sigma\left[c_t^S\right] \mapsto \infty$ in case of Poland and $\sigma\left[c_t^S\right]$ =8.1pp in case of Hungary. The result is again also dependent on trend path developments in the exchange rate where in both countries a continuing depreciation of the domestic currency was visible along the projection horizon. Results across all countries are again less affected by the policy chosen.

Turning to the right panel, one observes that results on joint fulfilment of criteria vary considerably across countries. The Czech Republic does best in jointly complying with criteria in subsequent quarters also over the longer horizon (four to eight quarters) under the current policy. We find that trend effects largely compensate for (too high) volatility along the convergence path in this country such that unsustainability derived from volatility measures is 'hidden' behind productivity related developments. Under the current policy, there is no single quarter where criteria are satisfied simultaneously in HU and PL. Under the optimally chosen policy, it turns out that fulfilling the criteria for a certain period of time is not a least-cost option for CZ. However, other countries maintain joint fulfilment for certain periods as under the current policy (SK, LT) or do better (RO), whereas all remaining countries do equally bad or worse than under the status quo. Following a Maastricht-constrained loss function mainly leads to decreases in losses as argued in the preceding section and shown in table 3.9. The number of times there is a joint pass of criteria in subsequent quarters increases again in most countries when moving from the optimal policy under the current regime to the Maastricht-constrained policy (in CZ, SK, EE, LT, BG, RO). This result again provides support that deliberately targeting Maastricht objectives under the current policy regime will improve prospects of sustainable compliance with the criteria.

Summarising the results of this section and the preceding section, we find that stabilisation policies under the current, optimal, and Maastricht-constrained formulation of policy are not effective in securing low enough uncertainty in values for criteria. Volatility in all criteria is too high along the real convergence path and leads to short time spans where criteria are met jointly. We thus conclude that joint fulfilment of criteria *remains at risk* in all countries. As we stick with the assumption that policy cannot affect mean outcomes, departing from assumptions in Woodford (2003) and Lipinska (2008) for the scope of optimal policy, we need to explore further sources of uncertainty that seem to overburden domestic monetary policies in securing a probable fulfilment of Maastricht criteria.

3.8 The Role of Adjustments in Markets, Structural Shocks, and Fiscal Policies

We observed that none of the members will qualify so far for a simultaneous fulfilment of the monetary criteria that is associated with high probability and correspondingly, a high enough number of quarters where criteria are met simultaneously. The inflation criterion turned out to be especially hard to meet for all countries. Cyclical variability remained high in the interest rate, even after optimal stabilisation policies were taken into account. Policy could therefore not yield a major improvement to the results presented in section 3.6. A detailed analysis of driving forces behind macroeconomic fluctuations that cause uncertainty enables us to explore whether sources of non-compliance are similar across countries and which factors matter most. Sources of excessive fluctuations in criteria will be grouped as to whether they are of a structural nature

Time dimension of sustainable compliance		# Maastricht criteria met c_t^π	c_t^i	c_t^S	# joint pass in ... subsequent quarters 1	2	4	8
Visegrad States								
CZ	Current	40	40	40	40	37	35	30
	Optimal	40	29	40	29	13	6	0
	ERM II	38	40	40	38	34	30	24
HU	Current	39	0	4	0	0	0	0
	Optimal	39	0	7	0	0	0	0
	ERM II	39	0	3	0	0	0	0
PL	Current	0	0	0	0	0	0	0
	Optimal	0	0	0	0	0	0	0
	ERM II	0	0	0	0	0	0	0
SK	Current	40	11	40	11	6	4	1
	Optimal	40	13	40	13	6	4	1
	ERM II	40	13	40	13	10	8	4
Baltic States								
EE	Current	31	33	40	24	15	9	3
	Optimal	28	34	40	22	12	6	1
	ERM II	34	35	40	29	22	16	7
LV	Current	13	1	40	0	0	0	0
	Optimal	9	0	40	0	0	0	0
	ERM II	16	0	40	0	0	0	0
LT	Current	26	38	40	25	14	9	3
	Optimal	31	34	40	25	14	9	3
	ERM II	23	40	40	23	16	11	6
2007 Entrants								
BG	Current	31	34	40	25	12	6	1
	Optimal	33	28	40	22	9	4	1
	ERM II	34	40	40	34	30	26	19
RO	Current	32	35	36	24	13	7	2
	Optimal	40	30	28	25	17	11	5
	ERM II	31	40	40	31	23	18	13

Table 3.10: The time dimension of sustainability under various assumptions about the conduct of policy. Entries in the table show the number of times, a criterion is met or criteria are met jointly in subsequent quarters.

(i.e. they concern the competitive structure of the economies determined by markups and inflation dynamics), whether they derive from the exposure to and magnitude of nominal and real structural shocks, or whether they are derived from an ineffective policy mix of monetary and fiscal policies. We address these measures in turn before we present the impact of measures conditional on the monetary policy regimes considered.

3.8.1 Alternative Objectives for Stabilisation Policy

As monetary policy is not effective in reducing uncertainty of outcomes substantially, one can think about replacing objectives in the policy rule (3.19) thereby altering factors that affect stabilisation outcomes. Accordingly, we evaluated forward-looking CPI inflation targeting, thereby replacing $\frac{1+\pi_t}{1+\pi_t^{ss}}$ with $\mathcal{E}_t\left[\frac{1+\pi_{t+1}}{1+\pi_{t+1}^{ss}}\right]$ in (3.19). We also checked sectoral inflation rate targeting (targeting non-traded goods inflation only, $\frac{1+\pi_{N,t}}{1+\pi_{N,t}^{ss}}$) as in Devereux et al. (2006). There, it is argued that with high pass-through, stabilising the exchange rate involves a trade-off between real stability and inflation stability and the best monetary strategy is to stabilise non-traded goods prices. We also replaced the CPI inflation by the change in the deflator for overall gross value added $\triangle defl_t$. In the deflator, imported goods inflation $\pi_{F,t}$ is of no role and volatility that originates in prices abroad and mainly affects the home consumer price level is removed.[66] However, simulations results omitted here made clear that these schemes did not yield any qualitatively different results from the ones presented in the preceding paragraph.

Further, one could think about a more prominent role of fiscal policy to assist monetary policy in controlling volatility in Maastricht relevant objectives. So far, fiscal policy was assumed to be passive, being implemented by passive fiscal spending rules that do not depend on the state of the business cycle. Consequently, one could think of a sectoral output gap target such that fiscal policy can provide an additional gear to bring down sectoral inflation rates that form a large part of the CPI inflation rate. Prudent fiscal policies of this sort have proven to be very successful in case of the Baltic States, see Geeroms (2007). Observed tensions between meeting various policy-objectives at the same time can then be expected to become less severe, as more instruments are made available but the number of objectives is left unchanged. We thus assume that $a_{Y_J} < 0$ in (3.27) such that in log-linearised form

$$\hat{G}_{J,t} = \rho_{G_J}\hat{G}_{J,t.-1} + a_{Y_J}\hat{Y}_{J,t} + \varepsilon_{G_J,t} \tag{3.71}$$

This rule belongs to a more general class of rules than the one proposed in Galí and Monacelli (2005). They find that $\hat{G}_{J,t} = -\hat{Y}_{J,t}$ turns out to be an optimal fiscal stabilisation policy from

[66] Another option would be to replace log deviations by quarter-on-quarter growth rates.

the perspective of maximising utility of the household.[67] For $a_{Y_J} < 0$ government expenditures will be decreased in good times (in case of positive output gaps) and increased in bad times (in case of negative output gaps). The rule therefore implements automatic stabilisation. In case monetary policy is implemented under the current regime where no optimisation takes place, the value for a_{Y_J} is set to $a_{Y_J} = -1$ which mimics the result in Gali and Monacelli (2005). In case of optimal conduct of fiscal policy, the parameter is chosen optimally, i.e. with the objective to minimise (3.52) in case monetary policy follows the optimal simple rule and (3.64) in case monetary policy targets the Maastricht objectives. This joint approach to monetary and fiscal policy assumes that there arise no coordination problems between the domestic monetary and fiscal authority in jointly minimising domestic loss.

3.8.2 Adjusting Market Structure

It is often argued that new EU members should not slow down in implementing further structural reforms that improve the economies in the long run, see International Monetay Fund (2008). Adjusting the structure of the economy can be interpreted as device to become more efficient. In our modeling framework this demands economies to move closer to the first best economy where nominal and real rigidities are absent by definition. As economies become more competitive and the price system less rigid, the degree of homogeneity across products increases and adjustments following shocks happen faster. Accordingly, we increase flexibility and remove real and nominal rigidities in the framework by assuming that

$$\rho_J \to \infty, \quad \theta_J \to 0, \quad \varpi_J \to 0, \quad J = H, N \tag{3.72}$$

For $\rho_J \to \infty$, markups vanish as goods become highly substitutable for each other. We therefore analyse to what extent market structure and competition policy can contribute to increase the likelihood of compliance. For $\theta_J \to 0$ and $\varpi_J \to 0$, prices are fully flexible and inflation persistence is removed. Prices are then set in a forward-looking manner in any period.

3.8.3 Moderating Structural Shocks

As time goes by, the exposure to structural shocks should moderate as the EU9 become more linked to the current euro area regarding trade in goods and financial assets. The magnitude of estimated structural error variances provided in table 3.6 on page 169 should then no longer provide valid descriptions of volatility in the economy. Inspecting the variance decomposition

[67] We obtain that specification for $\rho_{G_H} = 0$ and $a_Y = 0$ where we abstract from the role of efficient fluctuations throughout.

presented in table 3.7 on page 173, we can detect which shocks matter most in driving macroeconomic volatility. Consequently, we discard shocks from the system by removing volatility and persistence in innovations in exogenous processes (3.25), (3.26), and (3.28) to (3.31). Thus

$$\rho_X \to 0, \quad \sigma_{\epsilon_X} \to 0 \tag{3.73}$$

3.8.4 Results

Results are presented in table 3.11 for Visegrad States, in table 3.12 for Baltic States, and in table 3.13 for 2007 entrants. For any country group, we inspect the effectiveness of the proposed measures (adjustments in markets, mitigation in shocks, coordinated policies; in columns) conditional on the three monetary policy regimes discussed throughout (current policy in the top panel, optimal policy in the middle panel, Maastricht constrained policy in the lower panel). Scenario I represents the status quo in which we obtained that no country will meet the criteria jointly with high probability. Scenario II inspects adjustments in market structure, whereas III takes a look at the mitigation of structural shocks. IV evaluates fiscal measures by means of countercyclical output gap targeting. Scenarios V to VIII cover combinations of the discussed measures. Under V, shocks will mitigate and fiscal policies will be counter-cyclical thereby assisting monetary policy in dampening fluctuations. Under VI, markets adjust and shocks moderate. Under VII, markets adjust and policies will act contractionary whereas in scenario VIII all proposed measures are taken. Losses associated with each combination of measures under any monetary regime can be read off from the column labelled ‘Pass’ in each table. Losses are reported only in case a country meets all criteria considered. Further, minimum losses across measures are printed in bold.

We find that moderation of structural shocks concerning their volatility and persistence will be essential in order to fulfil the monetary Maastricht criteria sustainably (scenario III). The result proves robust across country groups considered. The contribution of moderation in shocks is also robust across monetary policy measures assumed, i.e. under the current policy, the optimal policy and the Maastricht constrained policy. This becomes clear from inspecting policy scenarios III, V, VI, and VIII across regimes and country groups.[68] Shock moderation even provides a *necessary and sufficient* condition for meeting the criteria with higher probability: Measures will be only successful in case they involve shock moderation and they are just as well successful if only shock moderation is included (but come at higher losses). Adjustment in market structure alone (scenario II) never generates enough adjustment in any

[68] Note that there are two exceptions: Poland and Latvia will not meet the criteria under the current exchange rate regime, even when structural shocks have eased.

country in order to simultaneously and sustainably fulfil the criteria. Similar results apply when focussing on contractionary fiscal policies only (scenario IV). The results further more emphasise that the current exchange rate regime is not essential for assessing the long-run prospects of Maastricht compliance. Given that shocks moderate, a more likely fulfilment of the criteria would also be possible under the *given* exchange rate regime in most countries considered.

Although adjustment in shocks alone can be sufficient for making compliance with criteria more likely, it does not represent the least cost option in terms of stabilisation costs the policymaker faces. In case the current monetary regime remained in place, coordinated monetary and fiscal policies that are assisted by moderation in structural shocks (scenario V) yield lower stabilisation costs than mitigation of shocks alone. The further combination of these measures with adjustments in market structure would yield even better stabilisation results in some Visegrad States as well as Bulgaria and Romania (scenario VIII). In case monetary policy can be chosen optimally, again scenario V leads to lowest stabilisation costs in nearly all countries considered.[69] Given that the policymakers commit to the Maastricht criteria in their objective functions, scenario V still provides the lowest costs in case of Visegrad states and the Baltics. New entrants however do better when fiscal policy refrains from contributing to stabilisation efforts by monetary policy. Scenario III still turns out to be best.

[69] Romania provides the exception where scenario III turns out to be best.

Visegrad States	Adjustment in Markets	Shocks	Policies	Compliance with criterion: Inflation rate				Interest rate				Exchange rate				Stabilisation cost $\mathcal{E}[L_t]$ (in $\%^2 \times 10^{-3}$)			
Current monetary policy				CZ	HU	PL	SK	CZ	HU	PL	SK	CZ	HU	PL	SK	CZ	HU	PL	SK
I												✓	✓		✓				
II	×											✓							
III		×		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	0.194	0.171		0.145
IV			×			✓				✓		✓							
V		×	×	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	0.187	0.166		**0.141**
VI	×	×		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	0.195	0.171	**0.600**	0.145
VII	×		×			✓				✓		✓							
VIII	×	×	×	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	**0.186**	**0.164**		0.142
Optimal monetary policy				CZ	HU	PL	SK	CZ	HU	PL	SK	CZ	HU	PL	SK	CZ	HU	PL	SK
I						✓	✓					✓	✓						
II	×					✓	✓					✓	✓						
III		×		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	0.082	0.026	0.029	0.111
IV			×			✓	✓					✓	✓		✓				
V		×	×	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	**0.003**	**0.010**	**0.000**	**0.079**
VI	×	×		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	0.082	0.026	0.029	0.111
VII	×		×			✓	✓					✓	✓		✓				
VIII	×	×	×	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	0.003	0.010	0.000	0.079
Maastricht constrained monetary policy				CZ	HU	PL	SK	CZ	HU	PL	SK	CZ	HU	PL	SK	CZ	HU	PL	SK
I						✓						✓	✓						
II	×					✓	✓	✓	✓			✓	✓		✓				
III		×		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	0.129	0.107	0.065	0.111
IV			×			✓		✓	✓		✓	✓	✓		✓				
V		×	×	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	**0.104**	**0.089**	**0.041**	**0.079**
VI	×	×		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	0.129	0.107	0.065	0.111
VII	×		×		✓	✓		✓	✓		✓	✓	✓		✓				
VIII	×	×	×	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	0.104	0.089	0.041	0.079

Table 3.11: Adjustments for Maastricht compliance: Visegrad States. Bold: least cost combination of measures to attain a highly probable fulfilment under a certain monetary regime.

Baltic States	Adjustment in: Markets	Shocks	Policies	Compliance with criterion: Inflation rate EE	LT	LV	Interest rate EE	LT	LV	Exchange rate EE	LT	LV	Stabilisation cost $\mathcal{E}[L_t]$ (in $\%^2 \times 10^{-3}$) EE	LT	LV
Current monetary policy				EE	LT	LV	EE	LT	LV	EE	LT	LV	EE	LT	LV
I							✓	✓	✓	✓	✓	✓			
II	×							✓	✓		✓	✓			
III		×		✓	✓		✓	✓	✓	✓	✓	✓	0.126	0.125	
IV			×					✓			✓	✓			
V		×	×	✓	✓		✓	✓	✓	✓	✓	✓	**0.122**	**0.122**	
VI	×	×		✓	✓		✓	✓	✓	✓	✓	✓	0.126	0.125	
VII	×		×					✓	✓		✓	✓			
VIII	×	×	×	✓	✓		✓	✓	✓	✓	✓	✓	0.123	0.122	
Optimal monetary policy				EE	LT	LV	EE	LT	LV	EE	LT	LV	EE	LT	LV
I										✓	✓	✓			
II	×						✓		✓	✓	✓	✓			
III		×		✓	✓	✓	✓	✓	✓	✓	✓	✓	0.087	0.101	0.085
IV			×				✓	✓	✓	✓	✓	✓			
V		×	×	✓	✓	✓	✓	✓	✓	✓	✓	✓	**0.060**	**0.075**	**0.062**
VI	×	×		✓	✓	✓	✓	✓	✓	✓	✓	✓	0.087	0.101	0.085
VII	×		×				✓		✓	✓	✓	✓			
VIII	×	×	×	✓	✓	✓	✓	✓	✓	✓	✓	✓	0.060	0.075	0.062
Maastricht constrained monetary policy				EE	LT	LV	EE	LT	LV	EE	LT	LV	EE	LT	LV
I										✓	✓	✓			
II	×						✓	✓	✓	✓	✓	✓			
III		×		✓	✓	✓	✓	✓	✓	✓	✓	✓	0.092	0.110	0.091
IV			×				✓	✓	✓	✓	✓	✓			
V		×	×	✓	✓	✓	✓	✓	✓	✓	✓	✓	**0.064**	**0.077**	**0.065**
VI	×	×		✓	✓	✓	✓	✓	✓	✓	✓	✓	0.092	0.110	0.091
VII	×		×				✓	✓	✓	✓	✓	✓			
VIII	×	×	×	✓	✓	✓	✓	✓	✓	✓	✓	✓	0.064	0.077	0.065

Table 3.12: Adjustments for Maastricht compliance: Baltic States. Bold: least cost combination of measures to attain a highly probable fulfilment under a certain monetary regime.

2007	Adjustment in			Compliance with criterion						Stabilisation cost $\mathcal{E}[L_t]$ (in $\%^2 \times 10^{-3}$)	
Entrants	Markets	Shocks	Policies	Inflation rate		Interest rate		Exchange rate			
Current monetary policy				BG	RO	BG	RO	BG	RO	BG	RO
I								✓	✓		
II	×					✓		✓	✓		
III		×		✓	✓	✓	✓	✓	✓	0.135	0.189
IV			×			✓		✓	✓		
V		×	×	✓	✓	✓	✓	✓	✓	0.132	0.187
VI	×	×		✓	✓	✓	✓	✓	✓	0.135	0.191
VII	×		×			✓		✓	✓		
VIII	×	×	×	✓	✓	✓	✓	✓	✓	**0.130**	**0.185**
Optimal monetary policy				BG	RO	BG	RO	BG	RO	BG	RO
I								✓	✓		
II	×					✓		✓	✓		
III		×		✓	✓	✓	✓	✓	✓	0.108	**0.025**
IV			×			✓		✓	✓		
V		×	×	✓	✓	✓	✓	✓	✓	**0.016**	0.067
VI	×	×		✓	✓	✓	✓	✓	✓	0.108	0.025
VII	×		×			✓		✓	✓		
VIII	×	×	×	✓	✓	✓	✓	✓	✓	0.153	0.067
Maastricht constrained monetary policy				BG	RO	BG	RO	BG	RO	BG	RO
I					✓			✓	✓		
II	×				✓	✓		✓	✓		
III		×		✓	✓	✓	✓	✓	✓	**0.116**	**0.048**
IV			×		✓	✓		✓	✓		
V		×	×	✓	✓	✓	✓	✓	✓	0.153	0.083
VI	×	×		✓	✓	✓	✓	✓	✓	0.116	0.048
VII	×		×		✓	✓		✓	✓		
VIII	×	×	×	✓	✓	✓	✓	✓	✓	0.153	0.083

Table 3.13: Adjustments for Maastricht compliance: 2007 Entrants. Bold: least cost combination of measures to attain a highly probable fulfilment under a certain monetary regime.

3.9 Conclusions

Conditional on the economic situation as of 2007, most countries from Central and Eastern Europe that became members of the EU in the 2004 and 2007 accession wave did not meet the monetary Maastricht criteria jointly. We set up a small open economy framework estimated for each country based on national accounts data in order to explore conditions for fulfilment of the criteria. The two-sector model allows for sectoral real rigidities concerning markups in product markets and capital adjustment costs as well as nominal rigidities in the price formation process. In addition, various sources of economic shocks are incorporated that affect the economies in the short and longer run and improve the empirical plausibility of the framework. Taking into account real convergence, one obtains that meeting criteria remains uncertain in most countries. Upward inflation risks persist in the Baltic States as well as in 2007 Entrants and Poland. Visegrad States generally exhibit difficulties in meeting the interest rate criterion with high probability.

Recent convergence reports by the European Commission and the European Central Bank however reveal a preference for sustainable compliance with the imposed criteria. Our reading of this perception is that criteria should not only be met ‘by chance’ but with high probability throughout, also after the formal country examination. Consequently, we explored conditions for decreasing uncertainty in fulfilling the criteria for each country in turn. We found that loss-minimising monetary policy will not be sufficient but prove necessary. Furthermore, policies need to be accompanied by a conceivable degree of moderation in volatility of structural shocks in all countries. Against the backdrop of easing volatility in the economies, coordinated monetary and fiscal policies are especially helpful for fostering compliance with criteria.

Chapter 4

How will Bulgaria Cope with Shocks on its Way to Euro Adoption? A Microfounded Model for an Economy under the Currency Board

4.1 Introduction

Since 2007, Bulgaria is a member of the European Union and has intentions to adopt the euro within the next two to seven years.[1] Bulgaria has fared fairly well since installment of the currency board in 1997 which has helped to attain low inflation and macroeconomic stability in the first years following the regime change. The inflation rate has declined from over 1000% to single-digit figures by 2000 and bottomed out at some 2.3% in 2003. However, prices recorded a sustained upward trend subsequently, reaching an inflation rate of 7.5% on average in 2006 and 2007, as noted in the most recent convergence report, see European Commission (2008, p. 50). Inflation accelerated strongly since then further. Main drivers of rising inflation in recent years have been strong demand and wage growth, higher oil and food prices and increases in administered prices. Real GDP growth has been strong in recent years also and is above 6% since 2004. Growth was especially driven by robust domestic demand with a strongly negative contribution of net exports (European Commission, 2008, p. 51). The contribution of supply side productivity growth has instead been muted where growth fluctuated about 3.5% on average over the last years. Taken together, the inflation outlook appears to be tilted to the upside where continued strong demand and labour cost developments in a still tight labour market exceed factor productivity growth.

Against the backdrop of the current economic situation, there is an often voiced concern for

[1] An overview of recent developments in Bulgaria and in other new EU members from Central and Eastern Europe can be found in Geeroms (2007) and Schadler et al. (2005). See also Berger and Moutos (2004) and Sorsa (2002).

new EU members under fixed or heavily managed exchange rates that inflationary supply side pressures might prevent soon compliance with the inflation criterion and delay ERM II entry, as argued in the preceding chapter. Furthermore, the ongoing restructuring of the Bulgarian economy towards a market-based economy will still need expansionary fiscal policies, e.g. directed towards infrastructure investments. Accordingly, pressures on the headline inflation rate will also be triggered from the demand side. The framework proposed here rationalises these price pressures that arise from the supply and demand side in the short and long run within a calibrated model suitable for the Bulgarian economy. The small open economy produces traded (industry) and non-traded (service) goods as assumed throughout the dissertation. The empirically observed slow restructuring of firms is incorporated by allowing for sectoral hybrid inflation dynamics as well as physical capital adjustment costs. In order to match the institutional setup of Bulgaria, we incorporate the currency board mechanism and cover the balance of payments explicitly. We also investigate the consequences for external competitiveness when government spending falls on home-tradables instead on home services. As in preceding chapters, the law of one price holds when pricing tradable goods. As a result, foreign (euro area) firms and home firms cannot price-discriminate across the border (but keep their pricing power over their product in heterogenous goods markets).

The assumption of full pass-through from foreign tradables inflation to the domestic one needed for postulating the law of one price is heroic given the latest estimates for Bulgaria (see Dimitrova (2006), forthcoming). The exchange rate pass-through to overall inflation is found to be around 0.3. The second stage of the pass-through effect, defined as the elasticity of domestic prices to international prices, is 0.27. Full pass-through as assumed in the model can still be justified by the role of our model serving as a first contribution in order to understand developments in activity and prices in Bulgaria in dynamic equilibrium better. Also, pass-through can be assumed to increase, as trade and financial integration with other EU members advances.

Modelling the currency board mechanism in dynamic general equilibrium models has so far not found widespread attention. Models adapted to the Bulgarian case are rarely available. A general equilibrium model of the trade balance dynamics in Bulgaria is developed in Valev (2005) within a neoclassical growth model and thus abstracting from nominal rigidities. Desquilbet and Nenovsky (2003) discuss the stability of the currency board under the conditions of self-fulfilling exchange rate crises within a money in utility framework. Their model does not feature different production sectors and the authors acknowledge that inclusion of the latter would seriously affect the results on inflation and real exchange rate dynamics. We construct a model which is richer in structure than the above mentioned models and we follow again the New Open Economy Macroeconomics (NOEM) methodology, as in the preceding chapters. The basic structure of the model is closest to Natalucci and Ravenna (2005). Henriksson (2005), Benigno and Thoenissen (2003), as well as Bokil (2005) are other main influences. In Natalucci and Ravenna (2005), inflation dynamics are only of concern in the non-tradable sector. All firms in that sector act in a forward-looking way but are faced with Calvo (1983) type time-dependent price-setting restrictions. Production in the tradables sector is not differentiated and as the law of one price is assumed to hold, goods prices are determined in the world market.

In line with the arguments presented in chapters 2 and 3, we propose a richer structure regarding the setup of production as well as the assumptions about sectoral price setting. In consequence, we allow for hybrid inflation formation in both production sectors which generates 'humped-shaped' impulse-responses of inflation. Hybrid inflationary performance can be thought of as a proxy for the slow restructuring of firms within the Bulgarian economy. Consequently, it is necessary to drop the competitive market setting for tradables, contrary to Natalucci and Ravenna (2005). Accordingly, both tradables (industry goods) and non-tradables (services) markets are heterogenous which enables pricing power for firms over their product. Our model also takes into account that for a sufficient studying of business cycle dynamics in transition economies, investment and capital have to be incorporated.

In order to produce results that are reasonable for the institutional setting of Bulgaria, we integrate the currency board mechanism and model the balance of payments explicitly. Under the automatic currency board mechanism, the domestic base money is fully backed by the stock of official foreign reserve holdings which removes any discretionary monetary policy power from the central bank. The mechanism establishes a direct link between the balance of payments evolvements and the domestic money supply. As a result, a balance of payments deficit (of the private sector) causes automatic contraction in reserve money and therefore contracts domestic credit and accordingly for a surplus. We therefore obtain a direct link between the stock of official foreign reserves holdings and domestic base money supply for explaining monetary transmission. To highlight the character of this contribution as a first step of modelling the currency board within the NOEM, we refrain from any features that break the automation in the currency board mechanism. Accordingly, the central bank has no possibility of influencing the domestic base money by any means and there is no monetisation of government debt, as the fiscal stance has to be in balance every period.

Bulgaria's currency board actually features non-automatic elements that cut the direct link between the balance of payments and domestic money supply (see Miller (1999) for the empirical evidence). Namely, there exists the requirement of commercial banks to maintain reserves in the central bank. Further, Nenovsky and Hristov (2002) argue that the inclusion of government fiscal reserves in the liability side of a currency board (i.e. covering them with international reserves) creates a discretionary channel of monetary policy transmission in Bulgaria. Government revenue and expenditure policies therefore directly impact the reserve money and, hence, the money supply.

4.1.1 Main Results

Based on evaluating the economic dynamics in a two-sector framework calibrated for the Bulgarian economy, we obtain the following main findings:

- *Temporary supply or productivity shocks originating in the tradables sector are unlikely to cause severe inflationary pressures for the economy overall.*
- *Demand side shocks arising from a temporary increase in government expenditures however fuel home tradable inflation and improve the terms of trade. This result holds whether or not purchases fall on tradables or non-tradables.*
- *Government expenditures that favour non-traded goods over traded goods contribute to the worsening in external competitiveness.*
- *Real convergence, simulated as a permanent increase in total factor productivity in the tradable sector, is associated with a considerable appreciation of the external real exchange rate in the range of 2% to 5%.*

4.1.2 Organisation of the Chapter

We proceed as follows. Section 4.2 presents the macroeconomic framework we have already heavily drawn from in deriving results in chapter 3. Section 4.3 analyses the long-run equilibrium of the economy and section 4.4 describes the calibration based on Bulgarian data. Under 4.5, we explore the response of the economy following shocks that are crucial for Bulgaria, namely shocks in productivity and fiscal spending. Under 4.6, we evaluate long-run developments in the external real exchange rate following a trend-increase in overall factor productivity in the tradables sector. Conclusions follow.

4.2 The Model

The model built here also provides the main building blocks for the analysis conducted in the preceding chapter, with appropriate adjustments concerning the conduct of monetary policy. There, we were mainly concerned about fulfilment of monetary Maastricht criteria such that the derivation of main components of the model has been left out intentionally. In the following, we present the model in greater level of detail. Still, some recurrences in deriving expressions will be inevitable.

4.2.1 Households

Preferences and Decision Problem

The small open economy is inhabited by a continuum of households that reside on the interval $[0,1]$. In any period, a household j receives utility from consuming the consumption index C_s^j which is a composite of non-traded goods consumption $C_s^{N,j}$ and tradable goods consumption $C_s^{T,j}$. $C_s^{T,j}$ consists of a home produced tradable goods basket $C_{H,s}^j$ and consumption of foreign produced, imported, tradable goods $C_{F,s}^j$. The household derives utility from liquidity services provided by holding real money balances $\frac{M_s^j}{P_s}$. Utility from holding money balances is not covered by bond holdings which provide indirect utility through the income they generate. The household has disutility $-\mathcal{V}(L_s^j)$ by giving up leisure for supplying labour to firms in the tradable sector H and non-tradable sector N. Each household maximises the following life-time utility function

$$U_t^j = \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{(s-t)} \left\{ \mathcal{U}(C_s^j) + \mathcal{N}(\frac{M_s^j}{P_s}) - \mathcal{V}(L_s^j) \right\} \tag{4.1}$$

which is additively separable in the per-period functions $\mathcal{U}$, $\mathcal{N}$, and $\mathcal{V}$.[2] We assume the following functional forms which are concave in their arguments as in Natalucci and Ravenna (2005) and Bokil (2005)

$$\mathcal{U}(C_s^j) = \ln C_s^j \tag{4.2}$$

$$\mathcal{N}(\frac{M_s^j}{P_s}) = \chi \ln \frac{M_s}{P_s} \tag{4.3}$$

$$\mathcal{V}(L_s^j) = \vartheta \frac{(L_s^{S,j})^{\kappa}}{\kappa} \tag{4.4}$$

already used in chapter 3. (4.2) implies that the agent is risk-averse and that the intertemporal elasticity of substitution and the intratemporal relative risk aversion coincide and are equal to

[2] Additive separability ensures that marginal utility/disutility in one of the arguments does not depend on the respective other arguments.

unity. χ denotes the utility scale parameter for real money balances $\frac{M_s^j}{P_s}$. ϑ is the respective parameter for total labour supply of household j, given by $L_s^{S,j}$. $\kappa > 1$ denotes the inverse of the labour supply elasticity. Labour supply of household j is mobile within the country and hence across the sectors. It is perfectly substitutable between the industry sector H and the services sector N

$$L_s^{S,j} = L_{N,s}^j + L_{H,s}^j \tag{4.5}$$

$L_{N,s}^j$ denote hours worked in the N sector and $L_{H,s}^j$ are hours worked in the H sector.

Regarding consumption choices, we assume that the intratemporal elasticity of substitution between traded goods T and non-traded goods N is equal to one. We then obtain that preferences about total consumption are of the Cobb-Douglas form

$$C_s^j = \frac{(C_{T,s}^j)^\gamma (C_{N,s}^j)^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \tag{4.6}$$

where γ is the share of tradable consumption in overall consumption as already introduced in chapter 2. Tradable consumption $C_{T,s}^j$ is split between home produced tradable goods H and foreign tradable goods F

$$C_{T,s}^j = \frac{(C_{H,s}^j)^\nu (C_{F,s}^j)^{1-\upsilon}}{\nu^\nu (1-\nu)^{1-\nu}} \tag{4.7}$$

As a result, the intratemporal elasticity of substitution between home produced tradable goods H and foreign produced (imported) goods F equals one, as was the case between T and N consumption. ν is the share of home produced tradables in T consumption. Unitary elasticity of substitution between H and F goods represents a key ingredient such that the Law of One Price (LOP) will hold for tradable goods, as stressed in preceding chapters. The non-tradable consumption basket is defined as the aggregate of single items $c_{N,s}^j(z)$

$$C_{N,s}^j = [\int_0^1 (c_{N,s}^j(z))^{\frac{\rho-1}{\rho}} dz]^{\frac{\rho}{\rho-1}} \tag{4.8}$$

where $c_{N,s}^j(z)$ denotes the consumption of a non-tradable good by household j produced by firm z. The (harmonised) consumer price index P_s results from minimising total expenditures of household j for obtaining one unit of the consumption index C_s^j over the arguments $C_{N,s}^j$ and $C_{T,s}^j$. Therefore

$$P_s = (P_{T,s})^\gamma (P_{N,s})^{1-\gamma} \tag{4.9}$$

By the same procedure, we obtain the tradables price index

$$P_{T,s} = (P_{H,s})^{v}(P_{F,s})^{1-v} \tag{4.10}$$

The producer price index for non-tradables $P_{N,s}$ is given by

$$P_{N,s} = [\int_0^1 (p_{N,s}(z))^{1-\rho_N} dz]^{\frac{1}{1-\rho_N}} \tag{4.11}$$

Optimality Conditions

The budget constraint in period s for household j in nominal terms reads

$$B^j_{H,s-1} + S_t B^j_{F,s-1} + Q^j_s + M^j_{s-1} + \int_0^1 \Pi^j_{N,s}(z)dz + \int_0^1 \Pi^j_{H,s}(z)dz + W^j_{H,s}L^j_{H,s} + W^j_{N,s}L^j_{N,s} \tag{4.12}$$

$$+P_{N,s}R^{N,j}_s K^{N,j}_{s-1} + P_{H,s}R^{H,j}_s K^{H,j}_{s-1}$$

$$\geq P_s C^j_s + T^{\#}_s + M^j_s + P^{I,H}_s I^{H,j}_s + P^{I,N}_s I^{N,j}_s + \frac{B^j_{H,s}}{1+i_s} + \frac{S_s B^j_{F,s}}{1+i^*_s}$$

The household receives income from interest payments on home zero coupon bonds $B^j_{H,t-1}$ and foreign zero coupon bonds $S_s B^j_{F,s-1}$, lump-sum transfers from the government Q^j_s, from money holdings M^j_{t-1} and profits of firms. She receives factor income from supplying labour to firms in the H and N sector $W^j_{H,s}L^j_{H,s} + W^j_{N,s}L^j_{N,s}$ and renting out capital $R^{N,j}_s P_{N,s} K^{N,j}_{s-1} + R^{H,j}_s P_{H,s} K^{H,j}_{s-1}$. The revenues are used to finance total consumption and investment expenditures $P_s C^j_s + P^{I,H}_s I^{H,j}_s + P^{I,N}_s I^{N,j}_s$, to pay lump-sum taxes $T^{\#}_s$, to carry the money stock to the next period M^j_s, and to purchase domestic and foreign bonds for savings. As the private sector issues the bonds we can think of $B^j_{H,s}$ and $B^j_{F,s}$ as corporate bonds (commercial paper) issued by the household-owned firms. Note that if $B^j_{H,s} < 0$, household j is a net debtor/issuer of the home bond. As the government does not have an objective function and as the government balance is consolidated with the household balance, we can interpret the home corporate bond similarly as a bond issued by the government. As the intertemporal utility function U^j_t given by (4.1) has the usual property of local non-satiation as it is concave in its arguments and additively separable in per period utilities, efficiency requires that the budget constraint (4.12) will hold with equality.

i^*_s denotes the nominal world interest rate at which domestic agents can buy/sell foreign assets. i^*_s is comprised of the world interest rate $\tilde{i}^*_s$ adjusted for a risk premium that increases in net real foreign liabilities of the home economy. i_s denotes the domestic nominal short rate (the three month money market rate). Home households own the H and N firms and decide on investment spending and capital adjustment. Household j therefore has to take into account the law of accumulation of capital in the H and N sectors when deciding on the optimum capital stock K_s^J and period investment I_s^J, $J = H, N$. Capital accumulation therefore follows

$$K_s^N = \Phi(\frac{I_s^N}{K_{s-1}^N})K_{s-1}^N + (1-\delta)K_{s-1}^N \tag{4.13}$$

$$K_s^H = \Phi(\frac{I_s^H}{K_{s-1}^H})K_{s-1}^H + (1-\delta)K_{s-1}^H \tag{4.14}$$

Capital accumulation incurs concave adjustment costs $\Phi' > 0 > \Phi''$. The timing convention implies that the capital stock K_{s-1}^J is used in production in period s ('end of period concept'). This assumption will be of importance when the log-linearised model is solved, see Griffoli (2007) for a discussion.

We end up with the following optimality conditions. The *intratemporal* consumption decision between traded $C_{T,s}^j$ and non-traded goods $C_{N,s}^j$ consumption is guided by

$$\frac{C_{T,s}^j}{C_{N,s}^j} = \frac{\gamma}{1-\gamma}\frac{P_{N,s}}{P_{T,s}} = \frac{\gamma}{1-\gamma}Q_s^{-1} \tag{4.15}$$

where $Q_s = \frac{P_{T,s}}{P_{N,s}}$ denotes the internal real exchange rate, as in preceding chapters. Analogously, for the choice between the home produced tradables basket $C_{H,s}^j$ and the foreign produced (imported) tradables basket $C_{F,s}^j$

$$\frac{C_{H,s}^j}{C_{F,s}^j} = \frac{v}{1-v}T_s = \frac{v}{1-v}\frac{S_s}{P_{H,s}} \tag{4.16}$$

where T_s denotes the international relative price of tradables, i.e. the terms of trade. S_t denotes the nominal exchange rate, which is fixed under the currency board. We imposed that the law of one price holds for imports F, which is explained below. We observe from (4.16) that if foreign goods become relatively more expensive (an increase/deterioration in the terms of trade T_s), home households will substitute foreign goods consumption $C_{F,s}^j$ for home tradable goods consumption $C_{H,s}^j$. Accordingly, consumption of F goods rises and consumption of H goods falls. The magnitude of the effect depends on the share of H goods in the basket given by v. As the economy is small, it cannot affect world prices and therefore takes the foreign price level $P_{F,s}^*$ as given. We abstract from fluctuations in $P_{F,s}^*$ in this chapter and therefore normalised the latter to 1.[3] Without loss of generality, we assume that $s = t$ in the following.

[3] We could alternatively assume that the change in the foreign price level follows a stable exogenous stochastic process, as in chapter 3.

The *intertemporal* consumption/savings decision is guided by the Euler equation

$$\lambda_t^C = \beta \mathcal{E}_t[\lambda_{t+1}^C (1+i_t) \frac{P_t}{P_{t+1}}] \tag{4.17}$$

where $\lambda_t^C = (C_t^j)^{-1}$ is the marginal utility of total consumption for household j. Hence

$$\frac{1}{C_t^j} = \beta \mathcal{E}_t[\frac{1}{C_{t+1}^j}(1+i_t)\frac{P_t}{P_{t+1}}] \tag{4.18}$$

The assumed form for period utility derived from total consumption given by (4.2) implies that utility from consumption is additively separable in H, N, F consumption. We then obtain that the Euler equations for non-tradable and home-produced tradables consumption are

$$\frac{1}{C_{N,t}^j} = \beta \mathcal{E}_t[\frac{1}{C_{N,t+1}^j}\frac{P_t^N}{P_{t+1}^N}(1+i_t)] \tag{4.19}$$

$$\frac{1}{C_{H,t}^j} = \beta \mathcal{E}_t[\frac{1}{C_{H,t+1}^j}\frac{P_t^H}{P_{t+1}^H}(1+i_t)] \tag{4.20}$$

Further, the household decides on her total supply of labour according to the labour supply curve

$$\vartheta(L_{H,t}^j + L_{N,t}^j)^{\kappa-1} C_t^j = \frac{W_t}{P_t} \tag{4.21}$$

which states that the household provides labour up to the point where the marginal rate of substitution of consumption of goods and services for leisure equals the real wage in terms of total consumption units $\frac{W_t}{P_t}$. We have used the fact that due to mobility of labour between the sectors and the perfect substitutability of providing labour in the traded versus non-traded sector, nominal wages will equalise between sectors $W_t^H = W_t^N = W_t$. Note that this result also requires that taxation will not affect the optimum labour supply decision. As taxes $T_t^{\#}$ are set in a lump-sum fashion, taxation will not have first-order effects on disposable income. The trade-off between holding real money balances and total consumption is determined by

$$\chi(\frac{M_t^j}{P_t})^{-1} = \lambda_t^C - \beta \mathcal{E}_t[\lambda_{t+1}^C \frac{P_t}{P_{t+1}}]$$

By using the Euler equation (4.17), we obtain the money demand equation

$$\chi \frac{(\frac{M_t^j}{P_t})^{-1}}{\lambda_t^C} = \frac{i_t}{1+i_t} \tag{4.22}$$

The marginal rate of substitution of consumption for increasing money balances $\frac{M_t^j}{P_t}$ equals the discounted payoff on bond earnings which accrue in period $t+1$. Payoffs are discounted by using household's stochastic discount factor derived from (4.18).

In order to obtain an efficient investment and physical capital allocation, the household maximises utility with respect to the constraints (4.13) and (4.14) as well as the real household balance. Investment in the non-traded sector is therefore guided by

$$\begin{aligned} \lambda_t^C \frac{P_t^{I,N}}{P_t} Q_t^N &= \beta \mathcal{E}_t[\lambda_{t+1}^C \frac{P_{N,t+1}}{P_{t+1}} R_{s+1}^N] \\ &+\beta \mathcal{E}_t[\lambda_{t+1}^C \frac{P_{t+1}^{I,N}}{P_{t+1}} Q_{t+1}^N]\{[\Phi[\frac{I_{t+1}^N}{K_t^N}] - \Phi'[\frac{I_{t+1}^N}{K_t^N}]\frac{I_{t+1}^N}{K_t^N} + (1-\delta)\} \end{aligned} \tag{4.23}$$

where we have aggregated over all households. Q_t^N denotes Tobin's Q in the non-traded sector defined as the market value of capital over its replacement cost. Analogously, we obtain for the H sector

$$\begin{aligned} \lambda_t^C \frac{P_t^{I,H}}{P_t} Q_t^H &= \beta \mathcal{E}_t[\lambda_{t+1}^C \frac{P_{H,t+1}}{P_{t+1}} R_{t+1}^H] \\ &+\beta \mathcal{E}_t[\lambda_{t+1}^C \frac{P_{t+1}^{I,H}}{P_{t+1}} Q_{t+1}^H]\{[\Phi[\frac{I_{t+1}^H}{K_t^H}] - \Phi'[\frac{I_{t+1}^H}{K_t^H}]\frac{I_{t+1}^H}{K_t^H} + (1-\delta)\} \end{aligned} \tag{4.24}$$

Capital compared to labour is immobile between the sectors within the country, i.e. ex-post rental rates of equipment in the H and N sector, given by $R_{N,t}$ and $R_{H,t}$, can differ. In long-run equilibrium, however, rental prices will equalise as will become clear in section 4.3.

Private Sector Balance

The budget constraint for household j in nominal terms is given by (4.12). Aggregating over all home agents $j \in [0,1]$ we obtain the private sector balance

$$\begin{aligned} & B_{H,t-1} + S_t B_{F,t-1} + Q_t^{\#} + M_{t-1} + \Pi_{N,t} + \Pi_{H,t} + W_{H,t} L_{H,t} + W_{N,t} L_{N,t} \\ & + P_{H,t} R_t^H K_{t-1}^H + P_{N,t} R_t^N K_{t-1}^N \\ = \; & P_{T,t} C_{T,t} + P_{N,t} C_{N,t} + T_t^{\#} + M_t + P_t^{I,H} I_t^H + P_t^{I,N} I_t^N + \frac{B_{H,t}}{1+i_t} + \frac{S_t B_{F,t}}{1+i_t^*} \end{aligned} \tag{4.25}$$

We have used the fact that all households make same optimum decisions as the representative/average household, as they face the same set of constraints and preferences (elasticities). Hence $C_{T,t}^j = C_{T,t}$ and $\int_0^1 C_{T,t}^j dj = \int_0^1 C_{T,t} dj = C_{T,t}$, and so on for the other variables relevant for the household. Note that only in the closed economy we would have that $\int_0^1 B_{H,t}^j dj = 0$, as the domestic asset market would have to clear in the domestic economy and there would be no possibility of holding home bonds abroad. Here we have that $B_{H,t} + B_{H,t}^* = 0$ or $B_{H,t} = -B_{H,t}^*$, where $B^*_{H,t}$ denote home bonds held at foreign.

4.2.2 Firms in the Non-Traded and Traded Goods Sector

Domestic Production and Decision Problem

Both sectors are populated by a continuum of monopolistically competitive firms residing in the interval $[0,1]$. Imperfect substitutability of produced goods allows for price-setting power of firms over their product. Due to their negligible size, they cannot influence the overall aggregate price level of the respective sector and take it as given. In the setup considered here, we assume that domestic firms are wholly owned by home households and all profits are distributed to domestic residents in the form of dividends.[4]

Each firm $z \in [0,1]$ in the tradable sector H combines capital available at the beginning of period t, $K^H_{t-1}(z)$ and labour $L_{H,t}(z)$ according to the Cobb-Douglas production function

$$y_{H,t}(z) = A^H_t(z)(K^H_{t-1}(z))^{\alpha_H}(L_{H,t}(z))^{1-\alpha_H} \tag{4.26}$$

where $K^H_{t-1}(z) = \int_0^1 K^{H,j}_{t-1}(z)dj$ is capital rented out and $L_{H,t}(z) = \int_0^1 L^j_{H,t}(z)dj$ denotes labour supplied by all households to firm z. $A^H_t(z) = A^H_t$ is total factor productivity (the level of technology in production) in the tradable sector which is same for all firms. The level of technology is subject to persistent shocks and fluctuates about its steady state value. As an identity $A^H_t \equiv A^H \exp[a_{H,t}]$. $a_{H,t} \equiv \ln \frac{A^H_t}{A^H} \times 100\%$ is the percentage deviation of the level of technology over its steady state value. Therefore fluctuations derive from

$$a_{H,t} = \rho_{Y_H} a_{H,t-1} + \epsilon_{Y_H,t} \tag{4.27}$$

where $\epsilon_{Y_H,t}$ denotes an innovation that is n.i.d. with constant variance such that $\epsilon_{Y_H,t} \sim n.i.d.(0, \sigma^2_{Y_H})$.

Cost minimisation for the firm yields the standard factor demands

$$\frac{W_t}{P_{H,t}} = MC^H_t(z)(1-\alpha_H)\frac{y_{H,t}(z)}{L_{H,t}(z)} = MC^H_t(z)MPL^H_t(z) \tag{4.28}$$

$$R^H_t = MC^H_t(z)\alpha_H\frac{y_{H,t}(z)}{K_{H,t-1}(z)} = MC^H_t(z)MPK^H_t(z) \tag{4.29}$$

A positive productivity shock increases $y_{H,t}(z)$ and accordingly both the real returns on labour and physical capital. In the absence of nominal rigidities, such that prices are fully flexible, R^H_t

[4] We could relax this assumption by introducing foreign direct investment (FDI) which serves as an intermediate good in tradable goods production.

can be interpreted as the time-varying 'natural rate of interest' in the sense of Wicksell (1898). R_t^H will however in general not correspond to the natural rate associated with the long-run growth prospects of an economy (i.e. with the constant rate in the absence of real rigidities). The presence of the real marginal cost term $MC_t^H(z)$ in each equation instead makes clear that the degree of competition among firms in sector H will influence the costs of capital and labour besides the marginal product of labour and capital given by $MPL_t^H(z)$ and $MPK_t^H(z)$, respectively. Analogously in the non-traded sector

$$\frac{W_t}{P_{N,t}} = MC_t^N(z)(1-\alpha_N)\frac{y_{N,t}(z)}{L_t^N(z)} = MC_t^N(z)MPL_t^N(z) \tag{4.30}$$

$$R_t^N = MC_t^N(z)\alpha_N\frac{y_{N,t}(z)}{K_{N,t-1}(z)} = MC_t^N(z)MPK_t^N(z) \tag{4.31}$$

In both sectors, firms base their labour demand on sectoral factor costs for labour, i.e. on the real wage obtained by deflating the nominal wage by the sector-specific aggregate price levels $P_{H,t}$ and $P_{N,t}$, respectively. On the contrary, labour supply by households is determined from the labour supply curve (4.21), i.e. the consumption based real wage $\frac{W_t}{P_t}$. Finally, the labour market clears at all dates

$$L_t^S = \int_0^1 \{L_{H,t}^j + L_{N,t}^j\}dj = L_t^D = L_t$$

where for the sector-specific labour supplies $L_{H,t}^j = \int_0^1 L_{H,t}^j(z)dz$ and $L_{N,t}^j = \int_0^1 L_{N,t}^j(z)dz$.

Calvo-Pricing in the H and N sector

As a monopolistic competitor, each firm in the traded goods sector has to take into account the demand for its good when setting its price. Therefore, it can only sell more by lowering its price.[5] Demand for good z comprises consumption and investment demand by any household j given by $c_{H,t}^j(z)$ and $i_{H,t}^j(z)$, $j \in [0,1]$, government demand $g_{H,t}(z)$, and foreign demand $c_{H,t}^*(z)$ (exports). Therefore

$$c_{H,t}^j(z) = (\frac{p_{H,t}(z)}{P_{H,t}})^{-\rho_H}C_{H,t}^j, \qquad i_{H,t}^j(z) = (\frac{p_{H,t}(z)}{P_{H,t}})^{-\rho_H}I_{H,t}^j, \qquad g_{H,t}(z) = (\frac{p_{H,t}(z)}{P_{H,t}})^{-\rho_H}G_t$$

and

$$c_{H,t}^*(z) = (\frac{p_{H,t}^*(z)}{P_{H,t}^*})^{-\rho_H^*}C_{H,t}^* = (\frac{p_{H,t}(z)/s_t(z)}{P_{H,t}/S_t})^{-\rho_H}C_{H,t}^*$$

[5] We go through the reasoning for the tradable sector only. Results for the non-traded goods sector are derived analogously.

We have used that the law of one price holds for each product z in the tradable sector $p^*_{H,t}(z) = p_{H,t}(z)/s_t(z)$ which requires that the price elasticities of domestic and foreign demand concerning tradables are equal, $\rho^*_H = \rho_H$. Further $s_t(z) = S_t$, i.e. the nominal exchange rate is exogenous to any domestic producer and therefore in aggregate $P^*_{H,t} = P_{H,t}/S_t$. Finally, $c^*_{H,t}(z) = (\frac{p_{H,t}(z)}{P_{H,s}})^{-\rho_H} C^*_{H,t}$. Aggregating over all households and noting that they are alike concerning preferences and constraints, we obtain overall domestic private sector consumption and investment demand for good z

$$\begin{aligned} c_{H,t}(z) &= \int_0^1 c^j_{H,t}(z)dj = (\frac{p_{H,t}(z)}{P_{H,t}})^{-\rho_H} C_{H,t} \\ i_{H,t}(z) &= \int_0^1 i^j_{H,t}(z)dj = (\frac{p_{H,t}(z)}{P_{H,t}})^{-\rho_H} I_{H,t} \end{aligned}$$

Firm z sets $p_{H,t}(z)$ such that output $y_{H,t}(z)$ meets overall demand

$$y_{H,t}(z) = (\frac{p_{H,t}(z)}{P_{H,t}})^{-\rho_H}(C_{H,t} + C^*_{H,t} + G_{H,t} + I_{H,t}) = (\frac{p_{H,t}(z)}{P_{H,t}})^{-\rho_H} Y_{H,t} \tag{4.32}$$

The objective of the firm in each sector is to maximise the expected discounted flow of future profits. As firms are owned by households, future profits are discounted by the stochastic discount factor of households $\Delta_{s,t+s}$. We assume Calvo (1983) pricing in the sense that each firm faces an exogenous and fixed probability in the traded sector $1 - \theta_H$ and $1 - \theta_N$ in the non-traded sector of being able to reset prices in any period. When re-setting the price of the product, it has to take into account that the price has to remain optimal, given that there will be no resetting possibility till period $t + s$. The decision problem of the traded sector firm can then be stated as follows

$$\max_{\{p_{H,t}(z)\}} \mathcal{E}_t \sum_{s=0}^{\infty} (\theta_H)^s \Delta_{s,t+s} \{\frac{p_{H,t}(z)}{P_{t+s}} y_{H,t+s}(z) - MC^H_{t+s}(z) y_{H,t+s}(z)\}$$

$$\begin{aligned} \text{s.t. } y_{H,t+s}(z) &= (\frac{p_{H,t}(z)}{P_{H,t+s}})^{-\rho_H} Y_{H,t+s} \\ \text{s.t. } P^{(1-\rho_H)}_{H,t} &= (1-\theta_H)(p^o_{H,t})^{(1-\rho_H)} + \theta_H P^{(1-\rho_H)}_{H,t-1} \end{aligned}$$

The later condition states that at each point in time the home-traded price level is a weighted average of prices chosen by firms that re-set prices in t and those that could not. $\Delta_{s,t+s} = \mathcal{E}_t[\beta^s (\frac{C_{t+s}}{C_t})^{-1}]$ denotes the stochastic discount factor used for evaluating the expected future profit streams by the firm at date t. Optimal price setting by firm z then becomes

$$\frac{p^o_{H,t}(z)}{P_{H,t}} = \frac{\rho_H}{\rho_H - 1} \frac{\mathcal{E}_t \sum_{s=0}^{\infty}(\theta_H\beta)^s (C_{t+s})^{-1} MC^H_{t+s}(z)(\frac{P_{H,t+s}}{P_{H,t}})^{\rho_H} Y_{H,t+s}}{\mathcal{E}_t \sum_{s=0}^{\infty}(\theta_H\beta)^s (C_{t+s})^{-1}(\frac{P_{H,t+s}}{P_{H,t}})^{\rho_H - 1} Y_{H,t+s}} \tag{4.33}$$

4.2.3 Hybrid Inflation Dynamics

As in preceding chapters, inertia in the price level dynamics in each sector is introduced by assuming that there is a share ω_H of firms in the H sector and a share of ω_N firms in the N sector which are backward-looking as in Gali and Gertler (1999) and Amato and Laubach (2003). They are backward-looking in the sense that they have to use last period's optimally set prices $P^{\#}_{H,t-1}$ corrected for the change in the aggregate price level $\frac{P_{H,t-1}}{P_{H,t-2}}$ (last period's gross inflation rate) as informational sources when allowed to reset prices. For the H sector, the price set by backward-looking firms that can adjust prices in period t is therefore, in aggregate,

$$P^b_{H,t} = P^{\#}_{H,t-1} \frac{P_{H,t-1}}{P_{H,t-2}} = P^{\#}_{H,t-1}\,(1 + \pi_{H,t-1}) \tag{4.34}$$

The aggregate price level in the tradable sector then evolves according to

$$P_{H,t} = \left[(1-\theta_H) P^{\#,1-\rho_H}_{H,t} + \theta_H P^{1-\rho_H}_{H,t-1}\right]^{\frac{1}{1-\rho_H}} \tag{4.35}$$

In a first step, it is decided whether a firm can reset its price $p_{H,t}(z)$ at all, which happens with probability $1-\theta_H$. If a firm is allowed to reset prices, with probability $1-\omega_H$ the price is set in an optimising way according to (4.33) and in $\omega_H \times 100$ cases in a backward-looking fashion according to (4.34). Note that these probabilities are determined in an exogenous way, i.e. not depending on the state of the business cycle. They are therefore only time-dependent but not state-dependent. $P^{\#}_{H,t}$ describes the index of newly set prices by forward- and backward-looking firms in the H sector

$$P^{\#}_{H,t} = \left[(1-\omega_H) P^{f,1-\rho_H}_{H,t} + \omega_H P^{b,1-\rho_H}_{H,t}\right]^{\frac{1}{1-\rho_H}} \tag{4.36}$$

Log-linearising the above three indices about P_t yields

$$\begin{aligned}
p^b_{H,t} &= p^{\#}_{H,t-1} + \pi_{H,t-1} - \pi_{H,t} \\
p_{H,t} &= (1-\theta_H)\,p^{\#}_{H,t} + \theta_H p_{H,t-1} \\
p^{\#}_{H,t} &= (1-\omega_H)\,p^f_{H,t} + \omega_H p^b_{H,t}
\end{aligned}$$

Analogous relationships hold in the N sector. We eventually end up with the following sector-specific hybrid inflation dynamics

$$\pi_{H,t} = \lambda_H^b \pi_{H,t-1} + \lambda_H^{mc} mc_{H,t} + \lambda_H^f \mathcal{E}_t \pi_{H,t+1} \tag{4.37}$$

$$\pi_{N,t} = \lambda_N^b \pi_{N,t-1} + \lambda_N^{mc} mc_{N,t} + \lambda_N^f \mathcal{E}_t \pi_{N,t+1} \tag{4.38}$$

The deep parameters are $\lambda_{J,t}^b = \frac{\omega_J}{\theta_J + \omega_J(1-\theta_J(1-\beta))}$, $\lambda_{J,t}^{mc_J} = \frac{(1-\omega_J)(1-\theta_J)(1-\theta_J\beta)}{\omega_J(1-\theta_J+\theta_J\beta)+\theta_J}$, $\lambda_{J,t}^f = \frac{\beta\theta_J}{\omega_J}\lambda_{J,t}^b$ where $J = H, N$. For the share of backward-looking firms $\omega_J \to 0$, we obtain the forward-looking New Keynesian Phillips curves.[6] $mc_{J,t}$ denotes average real marginal cost in sector J.

4.2.4 Central Bank

The central bank receives income from seigniorage earnings $M_t - M_{t-1}$ when issuing base money M_t and interest earnings from one period foreign discount bonds $S_t B_{F,t-1}^C$ that mature at the beginning of period t. Revenues are used to purchase $B_{F,t}^C$ units of foreign pure discount bonds at a cost of $\frac{S_t B_{F,t}^C}{1+i_t^*}$ home currency units. The monetary authority is not allowed to purchase/sell bonds denominated in home currency. In other words, domestic credit has to be fully covered by the stock of foreign reserves. Therefore, the monetary base M_t as well as the short run domestic interest rate i_t are endogenous and cannot be influenced. The central bank thus respects the flow constraint

$$\underbrace{M_t - M_{t-1}}_{\text{change in liabilities}} = \underbrace{Z_t - Z_{t-1} + \frac{S_t B_{F,t}^C}{1+i_t^*} - S_t B_{F,t-1}^C + v_t}_{\text{change in assets}} \tag{4.39}$$

Changes in foreign reserves holdings $Z_t - Z_{t-1}$ arising from movements in the balance of payments are included on the asset side of the balance. $v_t > 0$ are nominal net transfers of the central bank to the government after bond purchases and money supply changes are accomplished. As v_t has no sign restriction, transfers can go either way and so this variable resembles the possibility that the government can deposit revenues in the central bank (which are liabilities to the central bank and have to be fully covered by foreign reserve holdings). Hence there is theoretically a way of financing government deficits by monetisation. As our model does not feature public debt and as the fiscal stance is to be balanced every period, this channel of discretionary monetary policy is closed.[7] $\frac{S_t B_{F,t}^C}{1+i_t^*} - S_t B_{F,t-1}^C$ is the change in net

[6] For a detailed derivation of the hybrid inflation dynamics see Holmsberg (2006) and the appendix to chapter 2.

[7] We see that one way of financing foreign bond purchases is by period t lump sum taxation by the government.

foreign bond holdings by the central bank. Further, the balance sheet of the central bank and the consolidated balance sheet of the monetary and financial institutions (MFIs) coincide such that $Z_t - Z_{t-1}$ comprises both official and private sector's holdings of foreign exchange. This derives from the fact that there is no explicit banking sector modelled that enables financial transactions between households and firms in reality.

Under the automatic currency board, the stock of domestic base money M_t at any point in time is fully backed by the stock of foreign reserve holdings $Z_t \geq M_t$ where efficiency requires that

$$Z_t = M_t \tag{4.40}$$

which also yields the (less restrictive) condition that at any point in time, the stock of currencies and coins in circulation evolves according to

$$M_t = M_{t-1} + Z_t - Z_{t-1} \tag{4.41}$$

(4.40) is the 'policy rule' of the central bank under the currency board and closes the model. Whereas monetary policy under the currency board was implemented by means of a policy rule in chapter 3 similar to a fixed exchange rate economy, the money stock is directly addressed here. The domestic base money supply adjusts mechanically to balance of payments deficits/surpluses and the central bank cannot sterilise this impact as it would be possible under a fixed exchange rate system (by changing home reserves in the opposite direction).We further see from (4.39) that the rule implies that $S_t B^C_{F,t-1} = \frac{S_t B^C_{F,t}}{1+i^*_t} + v_t$. Solving the latter forward we obtain

$$B^C_{F,t-1} = (\prod_{s=t}^{T} \frac{1}{1+i^*_s}) B^C_{F,s} + \sum_{s=t}^{T} (\prod_{j=t}^{s-1} \frac{1}{1+i^*_j}) \frac{v_s}{S_s}$$

with $\prod_{j=t}^{t-1} \frac{1}{1+i^*_j} \equiv 1$. Ruling out Ponzi-schemes, i.e. imposing the condition that the initial central bank assets have to be redeemed sooner or later requires $\lim_{T\to\infty}(\prod_{s=t}^{T} \frac{1}{1+i^*_s}) B^C_{F,s} = 0$. The initial asset position is therefore (ex post[8]) sustainable if

$$B^C_{F,t-1} = \sum_{s=t}^{\infty} (\prod_{j=t}^{s-1} \frac{1}{1+i^*_j}) \frac{v_s}{S_s}$$

A positive present discounted value of transfers to the government has to be covered by an initial net asset position in surplus.

[8] I.e. after all values have realised and expectation operators can be omitted.

4.2.5 Government

In our benchmark model, government consumption 'goes into the ocean' in the sense that it does not provide any utility to households.[9] Under this assumption, the government balance is given by

$$T_t^{\#} + v_t = P_{H,t}G_{H,t} + P_{N,t}G_{N,t} + Q_t^{\#} \tag{4.42}$$

Expenditures cover government purchases on home produced tradables $P_{H,t}G_{H,t}$ and non-tradables $P_{N,t}G_{N,t}$ as well as lump-sum transfers $Q_t^{\#} = \int_0^1 Q_t^{,j} dj$ to households (social security spending). Further we have assumed that government expenditures on the tradable good fall on home produce only. Lump-sum taxation $T_t^{\#} = \int_0^1 T_t^{\#,j} dj$ and transfers from the central bank captured by v_t serve as source of financing. We assume that the government follows simples rules when deciding about sectoral spending on H and N goods that are not linked to the state of the economy. As a result, the government chooses spending in a non-activist way

$$G_{J,t} = G_J \exp\left[g_{J,t}\right] = G_J \exp[\rho_{G_J} g_{J,t-1} + \epsilon_{G_J,t}] \tag{4.43}$$

where $g_{J,t} \equiv \ln G_{J,t} - \ln G_J$ denotes the log-deviation of $G_{J,t}$ about its steady state value G_J and $0 \leq \rho_{G_J} \leq 1$ describes persistency of spending through time. If persistency is large, current spending is mainly explainable by spending behaviour in the recent past. According to (4.43) in connection with (4.42), the government has no measures of influencing the business cycle, unlike it was considered in preceding chapters. There is no other objective than to immediately spend what is earned from lump sum taxation $T_t^{\#}$ and interest income on foreign-currency denominated bonds made available by $v_t > 0$.

4.2.6 Foreign Sector

Uncovered Interest Parity

Maximising expected utility (4.1) with respect to $\{B_{H,t}, B_{F,t}\}$ in the private sector balance (4.25) yields the uncovered interest parity condition

$$\mathcal{E}_t[\lambda_{t+1}^C \frac{P_t}{P_{t+1}} \left\{(1+i_t) - (1+i_t^*)\frac{S_{t+1}}{S_t}\right\}] = 0$$

[9] Further, revenues are not used to cure inefficiencies in the economy, i.e. there is no offsetting of inefficiencies in equilibrium output caused by pricing power of monopolistically competitive firms.

From the Euler equation (4.18) we obtain $\mathcal{E}_t[\lambda^C_{t+1}\frac{P_t}{P_{t+1}}] = \frac{\lambda^C_t}{\beta(1+i_t)}$ and by simplifying

$$\begin{aligned} 0 &= \mathcal{E}_t[\frac{\lambda^C_t}{\beta(1+i_t)}\{(1+i_t)-(1+i^*_t)\frac{S_{t+1}}{S_t}\}] \\ (1+i_t) &= (1+i^*_t)\mathcal{E}_t\frac{S_{t+1}}{S_t} \end{aligned} \quad (4.44)$$

In our model, UIP holds up to a risk premium. The interest rate at which home households can borrow/lend internationally i^*_t (for which the UIP holds) and the exogenous world interest rate $\tilde{i}^*_t$ are linked by

$$(1+i^*_t) = (1+\tilde{i}^*_t)\phi[-\frac{F_t}{P_t}] \quad (4.45)$$

where $\frac{F_t}{P_t}$ denote net real foreign assets ($\frac{F_t}{P_t} > 0$) or net foreign real liabilities ($\frac{F_t}{P_t} < 0$), respectively. Equation (4.45) ensures the stationarity of the economy when exposed to temporary shocks, e.g. a world interest rate shock.[10] As $\phi' > 0$, net foreign liabilities will imply a positive spread in borrowing costs, $i^*_t > \tilde{i}^*_t$ such that it becomes increasingly expensive to hold foreign bonds as a domestic household. We further assume that interest rates converge in steady state which requires that $\phi[-\frac{F}{P}] = 1$ such that all short run nominal interest rates in the model equalise, $i = i^* = \tilde{i}^*$.

Terms of Trade and the Internal Real Exchange Rate

Allowing for a two-sector production structure leads to the introduction of two relative prices that guide the overall consumption decisions of the household. As in preceding chapters, the consumption based internal real exchange rate in levels is given by

$$Q_t = \frac{P_{T,t}}{P_{N,t}} \quad (4.46)$$

and the consumption based external real exchange rate for tradables is given by the relative price of imports in terms of home produced tradables

$$T_t = \frac{P_{F,t}}{P_{H,t}} \quad (4.47)$$

Movements in the terms of trade T_t indicate movements in international relative prices abstracting from movements in non-tradables price developments (that would be included in the external real exchange rate $S_tP^*_t/P_t$ as argued below). We assume that the law of one price holds in imported goods such that the market for imported goods is fully competitive. Hence

[10] See also Schmitt-Grohe and Uribe (2003) for this point.

we can write $P_{F,t} = S_t P^*_{F,t}$. Movements in the nominal exchange rate S_t lead thus in general one-for-one to changes in the domestic price of imports $P_{F,t}$ such that there is full pass-through. As we normalised $P^*_{F,t} = 1$, we eventually get that $T_t = S_t / P_{H,t}$. Further, in the currency board country, the exchange rate is irrevocably pegged to the anchor currency, and therefore $S_t = S$. Full pass-through then implies that any movements in the foreign price of tradable goods should cause one-for-one movements in the domestic price of imports. The consumer price index (4.9) can be decomposed in contributions by the nominal exchange rate, the internal real exchange rate, and the terms of trade such that

$$P_t = S Q_t^{\gamma-1} T_t^{-v} \tag{4.48}$$

A depreciation of the internal real exchange rate (a rise in Q_t such that non-tradable goods become relatively less expensive) or a deterioration in the terms of trade (a rise in T_t such that exports become relatively less expensive) will lead to a decrease in the price level and vice versa.

Relative prices Q_t and T_t can also be used to show how changes in the external competitiveness affect domestic real marginal cost in each sector. From (4.28) and (4.30) we directly obtain

$$\frac{P_{N,t}}{P_{H,t}} = \frac{(1-\alpha_H)}{(1-\alpha_N)} \frac{MC_s^H}{MC_s^N} \frac{Y_{H,t}}{Y_{N,t}} \frac{L_s^N}{L_s^H} \tag{4.49}$$

or

$$\frac{T_t^{1-v}}{Q_t} = \frac{(1-\alpha_H)}{(1-\alpha_N)} \frac{MC_t^H}{MC_t^N} \frac{Y_{H,t}}{L_t^H} \frac{L_t^N}{Y_{N,t}} = \frac{MPL_{H,t}}{MPL_{N,t}} \frac{MC_t^H}{MC_t^N} \tag{4.50}$$

A temporary/permanent real depreciation (an increase in Q_t) - ceteris paribus - decreases real marginal cost in the tradable sector MC_t^H. The same effect can be attained by an improvement (a decrease) in the terms of trade T_t, as the output price $P_{H,t}$ increases. Which effect dominates, remains an open question. Furthermore, an increase in the productivity of labour in the traded sector relative to the non-traded sector $\frac{MPL_{H,t}}{MPL_{N,t}} > 1$ (that can come about an increase in the level of technology $A_{H,t}$ that affects $Y_{H,t}$) causes a real appreciation in Q_t and/or a deterioration (an increase) in the terms of trade. Higher factor productivity in the H sector over the N sector, a situation typical for transition economies, and Bulgaria in particular, can be further inspected. Log-linearising (4.49) about the zero inflation steady state (the steady that exhibits a constant price level), one obtains

$$\underbrace{y_{H,t} - l_t^H}_{\text{labour productivity } H} + \underbrace{mc_t^H + p_{H,t}}_{\text{nominal marginal cost } H} = \underbrace{y_{N,t} - l_t^N}_{\text{labour productivity } N} + \underbrace{mc_t^N + p_{N,t}}_{\text{nominal marginal cost } N} \tag{4.51}$$

When prices are very sticky such that $p_{H,t} = p_{N,t} = 0$, a (temporary/permanent) increase in the (log-linear) labour productivity in the traded-goods sector $y_{H,t} - l_t^H$ will lead to a (temporary/permanent) increase in real marginal cost in the non-traded sector (a temporary/permanent deviation above the steady state) as an analogous increase is absent in the non-traded sector $y_{N,t} - l_t^N = 0$. If the firms in the N sector can reset prices, they can offset the increase in real marginal cost by an equal (percentage) increase in output prices of N goods. Then, the increase in productivity in the H sector results in higher prices in the non-traded goods sector and feeds into sectoral inflation dynamics, which can be seen from the hybrid Phillips-curve given by (4.38). It therefore also feeds into the headline inflation rate π_t which follows from (4.9).

Current Account

In the open economy, the difference between total income and domestic consumption is defined as the current account. To obtain total income, we write the resource constraints of the home economy in nominal terms

$$\begin{aligned} P_{H,t}Y_{H,t} &= P_{H,t}C_{H,t} + P_{H,t}I_{H,t} + P_{H,t}G_{H,t} + P_{H,t}C^*_{H,t} \\ P_{N,t}Y_{N,t} &= P_{N,t}C_{N,t} + P_{N,t}I_{N,t} + P_{N,t}G_{N,t} \end{aligned}$$

In our framework, investment goods and final output goods are assumed to exhibit the same price elasticities of demand and hence have the same equilibrium price $P_t^{I,N} I_t^N = P_t^N I_t^N$ and $P_t^{I,H} I_t^H = P_t^H I_t^H$.[11] Further note that $P_{T,t}C_{T,t} = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$. To obtain the current account, we sum (4.25), (4.42), (4.39) and take into account aggregate equilibrium profits in both sectors to obtain

$$B_{H,t-1} + S_t B_{F,t-1} + S_t B^C_{F,t-1} + P_{H,t}C^*_{H,t} = P_{F,t}C_{F,t} + \frac{B_{H,t}}{1+i_t} + \frac{S_t B_{F,t}}{1+i_t^*} + \frac{S_t B^C_{F,t}}{1+i_t^*} + Z_t - Z_{t-1}$$

The nominal current account (the current account at market prices) is equal to the left hand side of the following equation

$$B_{H,t-1} + S_t B_{F,t-1} + S_t B^C_{F,t-1} + NX_t = \frac{B_{H,t}}{1+i_t} + \frac{S_t(B_{F,t} + B^C_{F,t})}{1+i_t^*} + Z_t - Z_{t-1} \tag{4.52}$$

where $NX_t = P_{H,t}C^*_{H,t} - P_{F,t}C_{F,t}$ denotes the nominal trade balance expressed in home currency at current prices and other terms denote factor income on bonds bought at the end of period

[11] One could relax this assumption in order to study FDI as intermediate good of production, see Natalucci and Ravenna (2005) for this point.

$t-1$ and paid out at the beginning of t.

Balance of Payments

The Balance of Payments (BOP) is defined as

$$BOP_t \equiv CA_t + FA_t - ZZ_t = 0$$

where CA_t denotes the current account, FA_t denotes the financial account and ZZ_t denotes the foreign reserves/exchange balance.[12] $ZZ_t - FA_t$ equals the change in the net asset position of the economy which is equal to the current account. In a world of a purely floating rate, the exchange rate adjusts such that $CA_t + FA_t = 0$. Under the currency board, an increase in nominal foreign reserves Z_t translates into an increase in domestic base money supply M_t one by one which directly derives from (4.41). After rearranging (4.52) we can write the BOP as

$$BOP_t \equiv \underbrace{B_{H,t-1}+S_t B_{F,t-1}+S_t B^C_{F,t-1}+NX_t}_{CA_t} + \underbrace{-\frac{B_{H,t}}{1+i_t}-\frac{S_t(B_{F,t}+B^C_{F,t})}{1+i^*_t}}_{FA_t} - \underbrace{(Z_t - Z_{t-1})}_{ZZ_t} = 0 \tag{4.53}$$

A BOP surplus of the private sector (i.e. the case where $CA_t + FA_t > 0$) induced by a current account surplus (an increase in net exports/net factor income) that is accompanied by a financial account deficit (a net capital export) implies that there is a positive change in net assets. As the exchange rate is fixed, the increased demand for domestic currency cannot be matched by an increase in price of domestic currency, such that there must occur an offsetting net increase in the foreign reserves balance, $ZZ_t > 0$ in order to balance the BOP.

Capital imports from FDI flows (purchases of home corporate bonds by foreigners, such that $B_{H,t} < 0$) however have no direct effect on the money supply. The increase in liabilities to foreigners of the home private sector is matched by an equal increase in foreign exchange handed over to domestic firms in exchange for the bonds purchased. There is nevertheless an indirect effect that accrues in future periods via the future factor income on bond holdings transferred to abroad that enters the current account.

We can denote net nominal foreign assets, i.e. assets acquired at the end of period t which mature at beginning of period $t+1$ by $F_t \equiv \frac{B_{H,t}}{1+i_t} + \frac{S_t(B_{F,t}+B^C_{F,t})}{1+i^*_t}$. In our framework, net nominal foreign assets are equal to the negative financial account $F_t = -FA_t$, as the full principal of last period's bond purchases is paid out as interest income and therefore only enters the current,

[12] The BOP features double book-keeping in the sense that each cross-border transaction has two sides, the 'credit' and the 'debit' part. Accordingly, each goods export/import triggers both a goods as well as a financial flow. In case of financial asset export/import, both transactions take place in the financial account.

but not the financial account. Using the UIP given by (4.44) one obtains

$$\begin{aligned} F_t &= \frac{B_{H,t} S_t}{(1+i_t^*) E_t S_{t+1}} + \frac{S_t (B_{F,t} + B_{F,t}^C)}{(1+i_t^*)} \frac{\mathcal{E}_t S_{t+1}}{\mathcal{E}_t S_{t+1}} \\ F_{t-1}(1+i_{t-1}^*) \frac{\mathcal{E}_{t-1} S_t}{S_{t-1}} &= B_{H,t-1} + \mathcal{E}_{t-1} S_t B_{F,t-1} + \mathcal{E}_{t-1} S_t B_{F,t-1}^C \end{aligned}$$

As returns on period $t-1$ bond holdings are paid out at beginning of period t when S_t has realised, we can drop expectations[13]

$$F_{t-1}(1+i_{t-1}^*) \frac{S_t}{S_{t-1}} = B_{H,t-1} + S_t B_{F,t-1} + S_t B_{F,t-1}^C$$

Rewriting the balance of payments (4.53), we obtain the evolvement of nominal net foreign assets

$$F_t = (1+i_{t-1}) F_{t-1} + NX_t - (Z_t - Z_{t-1}) \tag{4.54}$$

If $F_t > 0$, the domestic economy has a positive net foreign asset position to abroad and vice versa. We can rewrite (4.54) to obtain a condition for the intertemporal solvency of the home country. We deflate by the CPI price index to obtain the real asset position in period t

$$F_t^r = \frac{1+i_{t-1}}{1+\pi_t} F_{t-1}^r + NX_t^r - (Z_t^r - \frac{Z_{t-1}^r}{1+\pi_t}) \tag{4.55}$$

where $F_t^r = \frac{F_t}{P_t}$. Solving forward yields the condition

$$F_{t-1}^r = \mathcal{E}_t \sum_{s=t}^{\infty} (\prod_{j=t}^{s} (\frac{1+i_{j-1}}{1+\pi_j})^{-1}) \left\{ Z_s^r - \frac{Z_{s-1}^r}{1+\pi_s} - NX_s^r \right\} \tag{4.56}$$

where we have ruled out Ponzi-schemes.[14] The intertemporal net asset position gives guidance for the intertemporal sustainability of current account deficits under the currency board. If the economy is initially (at date $t-1$) a net borrower from abroad, i.e. $F_{t-1}^r < 0$ it needs to sell real foreign reserves and/or attain net real trade surpluses. This condition accounts for the fact that the economy - as typical under the economic transition process - maintains a current account deficit for some time as (4.56) does not need to hold any period. The condition only requires that the expected present discounted income flow from selling reserves/increases in net-trade has in sum to be positive and large enough to pay off initial net liabilities to abroad. Analogous conditions for intertemporal solvency were imposed in chapter 2 under (2.52) and

[13] We have not fixed the nominal exchange rate S_t in this section such that results can be compared to the case of operating under a flexible exchange rate regime.

[14] Therefore, the date $t-1$ present discounted value of real assets in the distinct future is zero.

chapter 3 under (3.18).

4.3 The Steady State

The steady state describes the deterministic equilibrium of the economy where all prices are flexible and optimum decisions of households and firms are guided by relative prices only. In order to show that the steady state is stationary and unique (in the absence of steady state growth), we explain all (ratios of) endogenous and state variables by exogenous parameters only.

4.3.1 Pricing Decision of Firms and Steady State Price Levels

In steady state, all prices are flexible and therefore the price-resetting probability for forward-looking firms is $1-\theta_H = 1-\theta_N = 1$. As we abstract from deterministic (or even stochastic) steady state growth, we obtain steady state values by omitting the time subscript from the variables. From (A.82) - (A.84) directly follows that

$$P_J^b = P_J^{\#}, \qquad P_J = P_J^{\#}, \qquad P_J^b = P_J^f$$

With monopolistically competitive firms, the steady state does however not yield the first best outcome. Market power causes equilibrium prices to be above those of the competitive outcome where all firms are price takers which can be seen from

$$\frac{p_H^o(z)}{P_H} = 1 = \frac{\rho_H}{\rho_H - 1} MC^H > MC^H \tag{4.57}$$

$$\frac{p_N^o(z)}{P_N} = 1 = \frac{\rho_N}{\rho_N - 1} MC^N > MC^N \tag{4.58}$$

As the price elasticities of demand $1 < \rho_H, \rho_N < \infty$, firms receive a markup over real marginal costs when selling at price $\frac{p_H^o(z)}{P_H}$ and $\frac{p_N^o(z)}{P_N}$, respectively.[15] For $\rho_H, \rho_N \to \infty$ we would obtain the competitive 'first-best' result in both sectors where output prices reflect real marginal cost and the monopoly power goes to zero. We further assume that $P_H = P_N = S = W = 1$.[16] As earlier $P_{F,t}^* = 1$ and by the LOP $P_{F,t}^* = \frac{P_{F,t}}{S_t}$, it also holds that $P_F^* = P_F = 1$. Further, $p_H^o(z) = p_N^o(z) = 1$ for all firms $z \in [0,1]$. Steady state price indices for the tradables price

[15] $\frac{\rho_H}{\rho_H-1}$ and $\frac{\rho_N}{\rho_N-1}$ also have an interpretation as sector-specific Lerner-indices of monopoly power.
[16] Note that fixing prices at any other arbitrary level would be in line with a zero inflation rate steady state.

index P_T and the CPI price index P are then given by

$$P_T = (P_H)^v (P_F)^{1-v} = 1, \qquad P = (P_T)^\gamma (P_N)^{1-\gamma} = 1$$

4.3.2 The Real Economy

We obtain from the Euler equation given by (4.17) that subjective discounting (based on the rate of time preference) and market discounting (based on the three month return on a nominal asset) are linked by

$$\frac{1-\beta}{\beta} = i \tag{4.59}$$

We further find that capital accumulation in steady state is

$$K^N = \Phi(\frac{I^N}{K^N})K^N + (1-\delta)K^N, \qquad K^H = \Phi(\frac{I^H}{K^H})K^H + (1-\delta)K^H$$

with

$$\delta = \Phi(\frac{I^N}{K^N}) = \Phi(\frac{I^H}{K^H})$$

For the functional form for capital adjustment costs Φ we assume that $\Phi(\delta) = \delta$, as in Gali et al. (2004). It follows that $\Phi'(\delta) = 1$ and that $\delta = \frac{I^H}{K^H} = \frac{I^N}{K^N}$. From the latter we see that in long-run equilibrium the share of new investment to physical capital just equals the depreciation rate of existing capital in order to leave the capital stock of the economy constant. Nominal and real rates of returns are linked by

$$1 + i = \frac{1}{1-\beta(1-\delta)} R^H = \frac{1}{1-\beta(1-\delta)} R^N \tag{4.60}$$

such that in each sector J the return on physical capital equals the return on domestic bonds, once depreciation is taken into account

$$R^J - \delta = i \tag{4.61}$$

As both sectors face the same depreciation rate δ of real physical capital, real returns on sectoral physical capital utilisation have to equalise in steady state. Note that as inflation is zero in steady state, there is no inflation premium present that bond holders would call for in a risky environment.

From the resource constraint in the tradable sector

$$1 = \frac{C_H}{Y_H} + \delta\frac{K^H}{Y_H} + \frac{G_H}{Y_H} + \frac{C_H^*}{Y_H}$$

By the factor demands for physical capital (4.29) and (4.31), we obtain the capital-output ratios

$$\frac{K^H}{Y_H} = (\frac{1}{\alpha_H}\frac{1-\beta(1-\delta)}{\beta}\frac{\rho_H}{\rho_H - 1})^{-1}, \qquad \frac{K^N}{Y_N} = (\frac{1}{\alpha_N}\frac{1-\beta(1-\delta)}{\beta}\frac{\rho_N}{\rho_N - 1})^{-1} \tag{4.62}$$

where we used that firms' real marginal costs in steady state is $\frac{\rho_H - 1}{\rho_H}$ and $\frac{\rho_N - 1}{\rho_N}$ respectively. Hence steady state investment-capital ratios are the same in both sectors. From the labour demands given by (4.28) and (4.30), we recover the labour shares $\frac{L^J}{Y_J}$

$$\frac{L^H}{Y_H} = (\frac{1}{1-\alpha_H}\frac{\rho_H}{\rho_H - 1})^{-1}, \qquad \frac{L^N}{Y_N} = (\frac{1}{1-\alpha_N}\frac{\rho_N}{\rho_N - 1})^{-1}$$

We observe that due to monopolistic competition, labour shares are lower than under the first best, i.e. in the absence of real rigidities. From aggregate supply in the economy, we obtain accordingly the capital-labour ratios in both sectors $\frac{K^J}{L^J}$

$$\frac{K^H}{L^H} = (\frac{Y_H}{L^H})^{\frac{1}{\alpha_H}} = (\frac{1}{1-\alpha_H}\frac{\rho_H}{\rho_H - 1})^{\frac{1}{\alpha_H}}, \qquad \frac{K^N}{L^N} = (\frac{Y_N}{L^N})^{\frac{1}{\alpha_N}} = (\frac{1}{1-\alpha_N}\frac{\rho_N}{\rho_N - 1})^{\frac{1}{\alpha_N}}$$

where we have aggregated over the average firm's production and used that by assumption $A^H = A^N = 1$. The steady state consumption shares $\frac{C_H}{Y_H}$ and $\frac{C_N}{Y_N}$ are uniquely determined by exogenous variables and parameters which follows from

$$\begin{aligned} \frac{C_H}{Y_H} &= 1 - \delta(\frac{1}{\alpha_H}\frac{1-\beta(1-\delta)}{\beta}\frac{\rho_H}{\rho_H - 1})^{-1} - \frac{G_H}{Y_H} - \frac{C_H^*}{Y_H} \\ \frac{C_N}{Y_N} &= 1 - \delta(\frac{1}{\alpha_N}\frac{1-\beta(1-\delta)}{\beta}\frac{\rho_N}{\rho_N - 1})^{-1} - \frac{G_N}{Y_N} \end{aligned}$$

where the government shares $\frac{G_H}{Y_H}$, $\frac{G_N}{Y_N}$ and the export quota $\frac{C_H^*}{Y_H}$ are set exogenously.

4.3.3 Current Account and Balance of Payments

In (4.53) we obtained the balance of payments and that in steady state

$$B_H + SB_F + SB_F^C + NX + (-\frac{B_H}{1+i} - \frac{S(B_F + B_F^C)}{1+i^*}) - (Z - Z) \equiv 0 \tag{4.63}$$

$$\frac{i}{1+i}B_H + \frac{i^*}{1+i^*}(B_F + B_F^C) = -NX \tag{4.64}$$

From the risk premium equation (4.45) follows that $\phi[F] = 1$. Hence in steady state, home households can borrow at the world interest rate

$$i^* = \tilde{\imath}^* \tag{4.65}$$

and all nominal interest rates equalise, $i = i^* = \tilde{\imath}^*$ as noted earlier. Then steady state net financial assets can be written as

$$\frac{\tilde{\imath}^*}{1+\tilde{\imath}^*}(B_H + B_F + B_F^C) = -NX$$

We can express this steady state relationship in terms of units of the home produced tradable output

$$\frac{\tilde{\imath}^*}{1+\tilde{\imath}^*}(\frac{B_H + B_F + B_F^C}{Y_H}) = -(\frac{C_H^* - C_F}{Y_H})$$

Real net financial assets equal nominal net financial assets in steady state and can be obtained by omitting all time subscripts from (4.55)

$$\frac{NX}{F} = \frac{\beta - 1}{\beta} < 0 \tag{4.66}$$

The latter equation implies that if the steady state financial assets F are negative, they have to be offset by a positive net trade balance NX in order to yield a balanced net financial asset account in long-run equilibrium, and vice versa.[17] Then the steady state reserves to asset ratio $\frac{Z}{F}$ has to be negative as well. In other words, in steady state the current account has to be balanced. Otherwise, indeterminacy of the long-run equilibrium would result.

4.4 Calibration

We calibrate our model for quarterly data.[18] Regarding preferences, household's discount factor is set to $\beta = 0.99$ implying a quarterly zero inflation steady state interest rate of $\frac{1}{\beta} - 1 = 1.01\%$ and an annualised interest rate of $(\frac{1}{\beta})^4 - 1 = 4.1\%$. We set the Frisch elasticity of labour supply $\frac{1}{\kappa}$ to $\frac{1}{2}$ as in Natalucci and Ravenna (2005). Following Valev (2005), who relies on data from the 1997 Bulgarian input-output matrix, we assume that preferences are tilted towards tradable goods consumption leading to a share of non-traded consumption of $1 - \gamma = 0.4085$. For the share of home produced goods in the tradables basket, we set $v = 0.52$ as in Natalucci and Ravenna (2005) who estimated the latter for the Czech economy. For the labour supplies

[17] We could have also imposed long-run equilibrium on (4.56) to obtain the same steady state relationship.
[18] The detailed set of calibrated parameters is provided in appendix C.1 on page 302.

in each sector we assume that $\frac{L_N}{L} = \frac{L_H}{L} = 0.5$.

As becomes visible of the log-linearised model shown in the appendix in section C.2 on page 304, dynamics depend on the initial asset position of the economy. From the External Sectors Indicators of the Balance of Payments statistics of the BNB we obtain the BNB foreign reserves to asset ratio.[19] We average the ratio for the time span 1999 – 2003 which yields $\frac{Z}{F} = -0.9$ as the steady state value. Further, net external debt to GDP averaged over the same time span is 33.1%. In the absence of specific data, we assume that home produced tradable gross value added amounts to half of total GDP from which we can proxy that $\frac{F}{Y_H}$ is equal to -0.6. We set exports as share of net trade $\frac{C_H^*}{NX}$ to 3/2, thereby departing largely from the empirical average Bulgarian value given by -3.4 for the considered time span. Employing the value taken from the data would induce non-stationarity in the model, i.e. there would be no return to the initial steady state after temporary shocks.[20] We set the time-invariant risk premium given by ξ equal to 5 percent.

Concerning domestic production, the quarterly depreciation rate of physical capital is set to the standard value of $\delta = 0.025$. As in Natalucci and Ravenna (2005), the elasticity of Tobin's Q with respect to the investment-capital ratio is 0.5 in both production sectors. We assume that home tradable production technology is twice as capital intensive as in the non-traded sector, i.e. $\alpha_H = 0.67$ and $\alpha_N = 0.33$.

The price resetting-probabilities $1-\theta_N$, $1-\theta_H$ and the share of backward-looking firms ϖ_N, ϖ_H determine the characteristics of the inflation dynamics in the respective sector. Estimates of sector-specific hybrid inflation dynamics for Bulgaria are absent in the literature.[21] We use - not-sector specific - estimates from Lendvai (2005) for the case of Hungary which we treat as a proxy for inflation dynamics in transition economies. The author finds the share of backward-looking firms to be in the interval of $[0.3, 0.55]$, hence we set $\varpi_N = \varpi_H = 0.4$. Therefore, the share of forward-looking firms dominates in both sectors. This calibration is in line with the point made earlier that small open economies tend to have rather forward-looking price dynamics, see the discussion in the introduction to chapter 2. The estimated probability in any period that a firm cannot reset its price is in the interval between 0.45 to 0.6, hence we assume $\theta_N = \theta_H = 0.55$ to be reasonable values. The setting implies that the average duration of forward-looking price contracts - which is the Benigno and López-Salido (2002) measure of

[19] BNB reserve assets (in millions of euros) / net external debt (in millions of euros), see Bulgarian National Bank (2006).

[20] These assumptions have some implications for other shares in the external sector in long-run equilibrium. From (4.66) follows that the current account $NX + iF$ has to be balanced in steady state which requires $\frac{NX}{Y_H} = \frac{\beta-1}{\beta}\frac{F}{Y_H} = 0.0061$. For $\frac{C_H^*}{NX} = 3/2$ we obtain that $\frac{C_H^*}{Y_H} = \frac{C_H^*}{NX}\frac{NX}{Y_H} = \frac{3}{2}0.0061 = 0.0092$.

[21] See however the results from the Bayesian estimation presented in chapter 3.

nominal rigidity - is $D^J = \frac{1}{1-\varpi_J}\frac{1}{1-\theta_J} = 3.7$ quarters.[22] It therefore takes about one year until optimally set price contracts are reset.

Estimates of equilibrium markups in Bulgarian manufacturing sectors for the time-span 1995 – 2001 are provided in Dobrinsky et al. (2004). We extract an average value around 1.2 and assume that the markup is same for both sectors. In other words, firms charge in equilibrium prices which are 20% above real marginal cost. From (4.57) and (4.58) we then obtain that price elasticities of demand are given by $\rho_H = \rho_N = 6$. Finally, government consumption is set to 10 percent of sector-specific output.

4.5 Business Cycle Dynamics under the Currency Board

4.5.1 Fluctuations in Productivity

To explore the effects of a temporary increase in the level of tradables technology, we assume that factor productivity in the tradable sector $A_{H,t}$ is higher than in the non-tradable sector $A_{N,t}$ for some time until it again reaches its steady state value $A_H = 1$. Non-traded sector productivity remains at its steady state value $A_N = 1$ throughout. For an autocorrelation coefficient in the total factor productivity process of $\rho_{Y_H} = 0.85$, we attain that after approximately 24 quarters, or 6 years A_H will have returned to its steady state value.

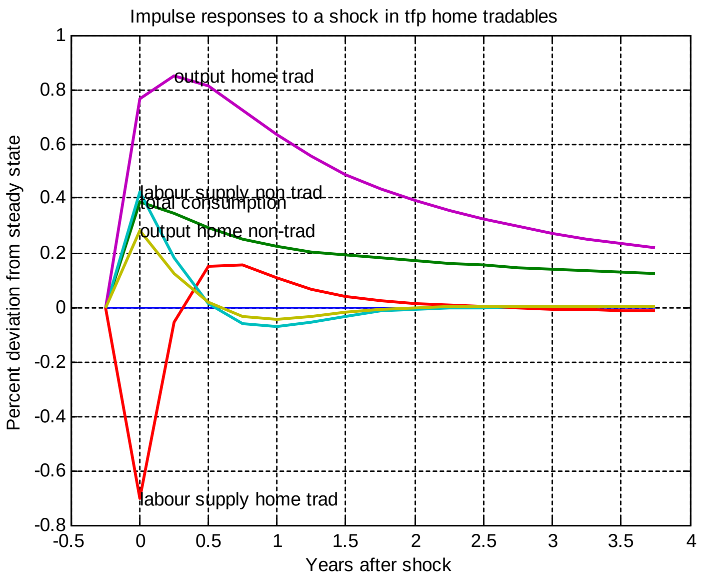

Figure 4.1: Real sector after a temporary shock to tradables technology.

Figures 4.1 to 4.3 illustrate developments in the real, the financial, and the external sector

[22] The average duration of price contracts in quarters (including both prices set by forward-looking firms as well as backward-looking firms) is instead given by $\frac{1}{1-\theta_J}$.

following the technology shock. From 4.1 we see that there is persistent higher growth of real output in the tradable sector.[23] The humped-shaped response can be attributed to the presence of costs in changing the initial stock of capital. The improved efficiency of firms in the traded-goods sector coupled with fixed output demand due to the price rigidity results in initially less labour demand in the tradable sector. The higher productivity reduces real marginal cost in the tradable sector and it is profitable for firms which are able to reset prices to lower prices in order to sell more, causing a drop in tradable inflation. Higher productivity allows for higher real wages in the H sector and due to labour mobility across the sectors, real wages in the N sector will increase as well. The latter can only be accomplished by higher final goods prices in the non-tradable sector generating the initial surge in non-tradable inflation. This effect becomes visible from figure 4.2 where the response in inflation is humped-shaped due to the presence of backward-looking firms in the N sector that cannot adjust price-setting immediately.

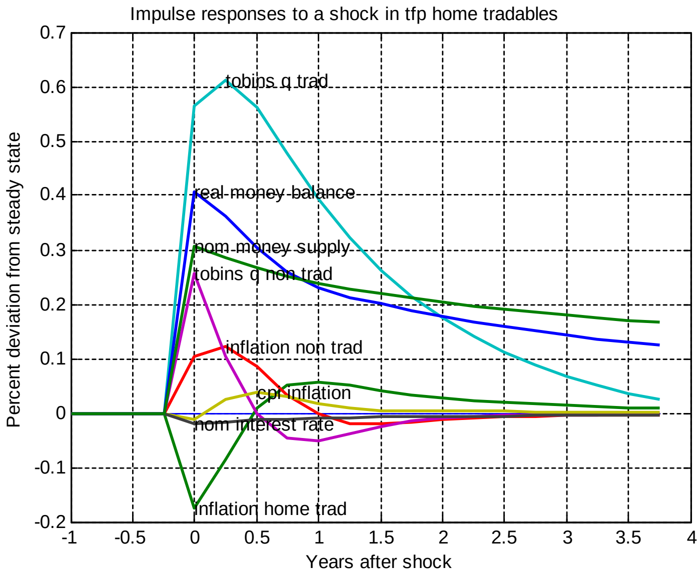

Figure 4.2: Financial sector after a temporary shock to tradables technology.

As increases in non-tradable inflation outweigh initial decreases in home-tradable inflation, there is a net increase in CPI inflation. The impact of the technology shock on CPI inflation is not that pronounced and starts dying out after 2 years as movements in H and N inflation rates start compensating for each other. The effect of the productivity gain in the H sector on

[23]Note that when abstracting from steady state growth, as assumed here, the change in log-deviations equals the growth rate in the variable:

$$\ln X_t - \ln X - (\ln X_{t-1} - \ln X) = \ln X_t - \ln X_{t-1}$$

the internal real exchange rate is more pronounced leading to a large persistent appreciation illustrated in figure 4.3. One observes that the effect on Q_t is more persistent than the effect on domestic producer price inflation rates π_J which can be explained by the increase in tradable consumption that derives from higher tradable output.[24]

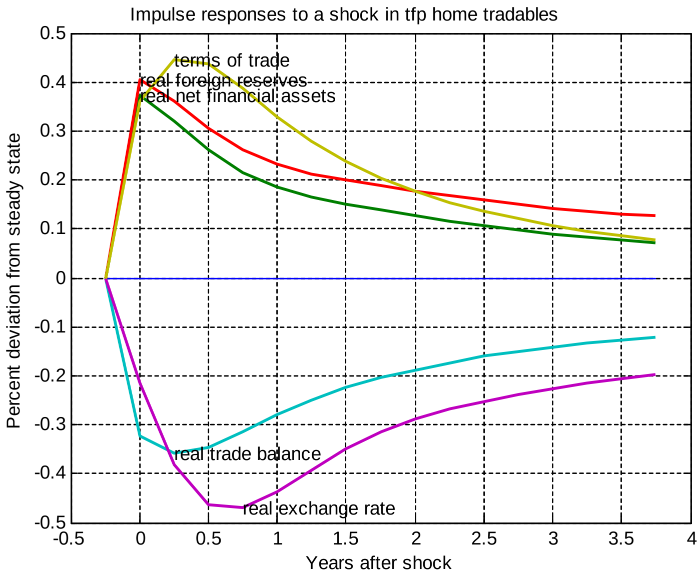

Figure 4.3: External sector after a temporary shock to tradables technology.

The productivity shock improves the financial sector sentiment given by Tobin's Q in the respective sectors which becomes clear from figure 4.2. The improvement in the domestic financial market climate by the surge in the market value of capital over its replacement costs in the H sector improves the net foreign asset position of the economy. The balance of payment turns temporarily into surplus by the respective increase in the net financial wealth as can be seen from figure 4.3. The accumulation of real foreign reserves causes a domestic monetary expansion which increases holdings of real cash balances and decreases the short-run interest rate, further contributing to the increase in total consumption. As there is a positive change in net foreign reserves, the current account is in surplus and wealth effects from factor income dominate the visible decrease in the real trade balance. Concerning compliance with the Maastricht inflation criterion, we obtain that for the given calibration a temporary technology shock (and therefore cyclical fluctuations in tradables technology) would not push inflation and the nominal interest rate above the Maastricht limit. However this question can only be answered sufficiently by simulating the model and investigating the volatility in Maastricht variables (see chapter 3 for this approach).

[24] Note that the internal real exchange rate can in log-linear terms be written as $q_t = c_{N,t} - c_{T,t}$. Following the productivity-shock, there is a persistent surge in tradable consumption.

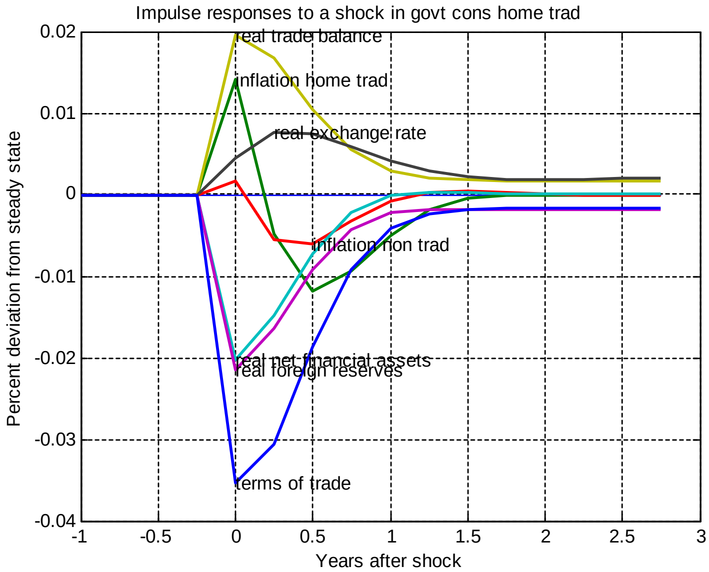

Figure 4.4: External sector after a fiscal shock to tradables consumption.

4.5.2 Fiscal Policies and the Trade Balance

Turning to shocks that originate from the demand side, we investigate the impact of a transitory increase in government spending in the N and H sectors, respectively. The standard flexible-price Real Business Cycle closed-economy framework predicts a decline in total consumption in response to a rise in government spending, see Wickens (2008). Ricardian consumers consume out of life-time wealth and they smooth away income fluctuations caused by the initial surge in government spending. This result is also visible in our sticky-price open economy model as becomes clear from figures 4.4 and 4.5.[25]

Turning to figure 4.4, we see that a temporary fiscal policy shock increases home tradable inflation and appreciates the terms of trade. At the same time, the internal real exchange rate depreciates as non-tradables have become relatively cheaper. The higher inflation triggered by demand side pressures contracts the real domestic base money supply which under the currency board mechanism leads to a one-for-one reduction in the stock of real foreign reserve holdings $\frac{Z_t}{P_t}$. De-accumulation in reserves results in a deterioration of the intertemporal foreign asset position. Households foresee their loss in life-time wealth and smooth the effect away by initially decreasing total consumption. Following the shock, home tradable output increases and as domestic absorbtion by households decreases, a positive net trade balance builds up.[26]

[25] We further can extract the impact of government spending on the real net foreign asset position of the economy.

[26] The impulse response of sector-specific output and total consumption is omitted from both graphs to enhance clarity of the exposition.

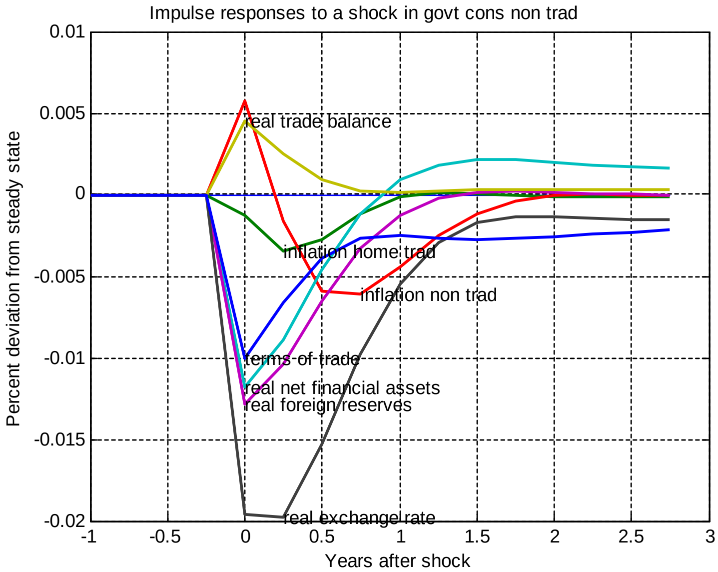

Figure 4.5: External sector after a fiscal shock to non-tradables consumption.

As there is an overall loss in reserves, the intertemporal loss in net real wealth of the economy dominates.

Turning to figure 4.5, we see that mainly the same qualitative conclusions arise if government expenditures fall on services, i.e. N consumption. The main difference concerns the development of the internal real exchange rate. Whereas the terms of trade and the internal real exchange rate moved in opposite directions in case of tradables, now both measures decrease. The initial surge in spending causes an increase in non-tradable inflation and hence leads to a rise in the relative price of non-tradables. We see that the internal real exchange rate appreciation is less persistent than the development in the home non-tradable inflation.

4.6 Developments of the Real Exchange Rate in the Long-Run

Real convergence in GDP is expected to be driven by higher productivity growth in the sector open to trade brought about permanently better technology, physical capital productivity, as well as improvements in management skills. According to the Balassa-Samuelson hypothesis, higher productivity growth in the traded goods sector $A_{H,s+j}/A_{H,s} - 1$ will however also lead to a permanent increase in home consumer prices, see Balassa (1964) and Samuelson (1964). The original proposition assumes homogeneity in tradable goods across countries. As the home economy is small, the price of exported goods will then be determined in the world market. Higher productivity in the industry sector H generates higher output per unit of capital and labour and resulting higher income can be used to increase sectoral wages W_s^H. As labour is mobile between sectors (and as the assumed lump sum tax $T_t^{\#}$ will not distort the first order condition of labour supply), growth in productivity in the H sector will lead to a permanent increase in the overall wage level in the economy, such that $W_s^H = W_s^N = W_s$. In the N sector, firms will be willing to produce the same level of output as before only, if output prices increase since factor costs have risen and there are no spillovers from the productivity gain in the H sector. Hence, non-tradables prices $P_{N,s}$ will permanently increase compared to tradable prices $P_{T,s}$ and the real exchange rate Q_s will appreciate permanently. The consumer price level P_s will increase according to (4.48) until productivity has converged to the level of the world economy.

Our model does however not meet all the required assumptions for the effect to be present: Albeit labour is mobile between sectors and the world interest rate is exogenous to home households and firms, the traded goods market is not homogenous. As outlined above, home and foreign tradable firms specialise in producing a bundle of tradable goods. Heterogeneity in goods within a sector and across countries allows industrial firms in each region to have pricing power over their product at home and abroad. Pricing power of firms in the tradable sector allows (some) firms to decrease prices following higher productivity growth such that the overall price level $P_{H,s}$ will decrease relative to the imported goods price index $P_{F,s}$ and the terms of trade worsen (T_s increases).[27] Therefore, movements in the terms of trade might absorb some of the pressure on the overall domestic wage level and on non-traded goods prices $P_{N,s}$ and eventually on the real exchange rate Q_s. In order to obtain the net effect on external competitiveness, the external real exchange rate is introduced, similar to (2.13) in chapter 2 on page 37. The external real exchange rate E_t can be decomposed into contributions by the terms of trade and the internal real exchange rate as follows

$$E_t \equiv S\frac{P_t^*}{P_t} = S\frac{P_{T,t}^*}{P_t} = \frac{(P_H^*)^{v^*}(P_F^*)^{1-v^*}}{Q_t^{\gamma-1}T_t^{-v}} \overset{l.o.p.}{=} \frac{T_t^{-v^*}}{Q_t^{\gamma-1}T_t^{-v}} = \frac{T_t^{v-v^*}}{Q_t^{\gamma-1}} = Q_t^{1-\gamma} \tag{4.67}$$

where the nominal exchange rate S is fixed under the currency board and normalised to 1 as explained earlier.[28] We observe that the law of one price (no home bias in consumption, $v = v^*$) insulates developments in E_t from movements in the terms of trade. The appreciation in E_t (a decrease) will therefore be only determined by the internal real exchange rate. When

[27] Note that in case prices would be flexible, all firms would reset prices such that the aggregate index would not move.

[28] Further, the foreign producer price level is normalised to one, $P_F^* = 1$ as throughout the exposition in this chapter. We also made use of the law of one price, such that $P_F^* = P_F$ as well as $P_H^* = P_H$. The home CPI price level is given by (4.48).

Long-run real convergence	$A_{H,t+40}/A_{H,t}^{ss} - 1$				
	10%	*15%*	*20%*	*25%*	*30%*
Gross value added H sector Y_H	15.9	22.7	29.7	36.6	43.7
Gross value added N sector Y_N	-0.8	-1.0	-1.2	-1.3	-1.5
Real net financial assets F/P	3.4	4.9	6.3	7.7	9.0
Consumption C	2.3	3.6	4.8	6.0	7.1
Real wage W/P	3.2	4.7	6.2	7.7	9.1
Internal real exchange rate Q	-4.9	-7.2	-9.3	-11.2	-13.1
Terms of trade T	15.8	22.1	28.4	34.7	41.0
External real exchange rate E	-2.0	-2.9	-3.8	-4.6	-5.3

Table 4.1: Long-run consequences of a trend-productivity increase in the tradables sector. Values denote the permanent deviation from the initial steady state in %.

log-linearising about the deterministic steady state, one obtains

$$\hat{E}_t = (1 - \gamma)\,\hat{Q}_t \tag{4.68}$$

An improvement in the internal real exchange rate Q_t (a decrease, such that $\hat{Q}_t < 0$) directly leads to an appreciation of the external real exchange rate (in favour of the Balassa-Samuelson hypothesis). Thus, homogeneity between traded goods is crucial for the Balassa-Samuelson effect to come through fully, see also Benigno and Thoenissen (2003) and Altissimo et al. (2005).

In order to investigate the spillovers from the internal real exchange rate on the external real exchange rate in the long run, we assume that tradable productivity $A_{H,t+j}$ grows over a prolonged period of time. An innovation to the level of technology in the tradable sector H given by $\epsilon_{H,t}$ then has lasting consequences on output Y_H and productivity and will cause output and other key variables to remain permanently away from the initial steady state. Accordingly, we assume that $A_{H,t+j}$ grows over a ten year period in the range of 10% to 30%.[29] From table 4.1 we observe that in the long-run, the internal real exchange rate appreciation is pronounced and can be expected to range from 4.9% up to 13.1%. The appreciation is accompanied by an improvement in the net real financial asset position and a worsening in the terms of trade in the range of 15.8% to 41.0%. Households increase their overall consumption in the range of 2.3% up to 7.1%. As the terms of trade effect does not counteract the internal real appreciation, the external exchange rate will appreciate considerably, in the range of 2.0% to 5.3%.

[29] We determine $\rho_{Y_H} > 1$ in (4.27) such that $A_{H,T}/A_{H,t}^{ss} - 1$ is 10%, 15%, .., 30% with $T = 40$ quarters under constant average growth. We again make use of (3.46) in order to account for the fact that the log-linear deviations generated in the simulations no longer provide reasonably well approximations of the overall growth rates.

4.7 Conclusions

This chapter investigated the transmission of productivity and fiscal shocks under the currency board mechanism within a new open economy framework. The model presented here also served as foundation for the analysis carried out in the preceding chapter. Instead of estimating the model as in the comparative study in chapter 3, the model was calibrated. Further, the focus of monetary transmission was on the money stock directly such that the interest rate is endogenously defined. Earlier, policy under the currency board was implemented similarly as in a fixed exchange rate economy. We found that inflationary pressures resulting from temporary higher capacity utilisation in the tradable sector (triggered by a temporary technology shock) will not have a persistent impact on overall CPI inflation along the business cycle. A permanent technology shock instead might cause a persistent appreciation of the external real exchange rate in the long run.

Demand side shocks can considerably influence the path of development of external competitiveness. Fiscal spending that falls on either tradable or non-tradable goods impacts positively on the trade balance but still leads to current account deficits due to the long-run worsening of the net foreign asset position. When fiscal spending falls on traded (non-traded) goods, a lasting real depreciation (appreciation) of the internal real exchange rate is present. In the absence of home bias, an improvement in the internal real exchange rate directly causes an improvement in the external real exchange rate thereby worsening external competitiveness. Whether supply or demand side inflationary pressures will dominate in the future, remains to be seen.

Appendix A

Appendix to Chapter 2

A.1 Additional Tables

Productivity shocks	Volatility, in percentage points				Persistence			
	$\sigma_{\epsilon_{S_H}}$	$\sigma_{\epsilon_{S_N}}$	$\sigma_{\epsilon_{S_F}}$	$\sigma_{\epsilon_{S_{N*}}}$	$\rho_{\epsilon_{S_H}}$	$\rho_{\epsilon_{S_N}}$	$\rho_{\epsilon_{S_F}}$	$\rho_{\epsilon_{S_{N*}}}$
Baseline	$2\sigma_{\epsilon_{S_N}}$	0.598	$2\sigma_{\epsilon_{S_{N*}}}$	0.598	0.823	0.823	0.823	0.823
Current Euro Area	$2\sigma_{\epsilon_{S_N}}$	0.598	$2\sigma_{\epsilon_{S_{N*}}}$	0.598	0.823	0.823	0.823	0.823
Large Member Area	$2\sigma_{\epsilon_{S_N}}$	0.598	$2\sigma_{\epsilon_{S_{N*}}}$	0.598	0.823	0.823	0.823	0.823
Enlarged Area	$2\sigma_{\epsilon_{S_N}}$	1.600	$2\sigma_{\epsilon_{S_{N*}}}$	0.598	0.850	0.850	0.823	0.823
Fiscal spending shocks	Volatility, in percentage points				Persistence			
	$\sigma_{\epsilon_{G_H}}$	$\sigma_{\epsilon_{G_N}}$	$\sigma_{\epsilon_{G_F}}$	$\sigma_{\epsilon_{G_{N*}}}$	$\rho_{\epsilon_{G_H}}$	$\rho_{\epsilon_{G_N}}$	$\rho_{\epsilon_{G_F}}$	$\rho_{\epsilon_{G_{N*}}}$
Baseline	0.325	0.325	0.325	0.325	0.949	0.949	0.949	0.949
Current Euro Area	0.325	0.325	0.325	0.325	0.949	0.949	0.949	0.949
Large Member Area	0.325	0.325	0.325	0.325	0.949	0.949	0.949	0.949
Enlarged Area	0.325	0.325	0.325	0.325	0.949	0.949	0.949	0.949
Monetary policy shock	Volatility, in percentage points				Persistence			
	σ_η				ρ_η			
Baseline	0.162				0			
Current Euro Area	0.162				0			
Large Member Area	0.162				0			
Enlarged Area	0.162				0			

Table A.1: Calibration of scenarios: shocks.

Comparing moments	Current Euro Area historical	model	Large Member Area historical	model	Enlarged Euro Area historical	model
Volatility, pp						
Stdev($\hat{C}_t$)	NO	0.79	0.42	0.91	0.42	1.14
Stdev($\hat{C}_t^*$)	NO	0.78	0.87	0.84	0.86	1.07
Stdev($\hat{C}_t^U$)	0.85	0.79	0.64	0.85	NO	1.07
Stdev($\hat{Y}_t$)	NO	1.03	0.69	1.02	0.74	1.23
Stdev($\hat{Y}_t^*$)	NO	1.08	1.04	1.12	0.85	0.86
Stdev($\hat{Y}_t^U$)	0.86	$0.90^{(ii)}$	0.85	$0.91^{(ii)}$	0.82	$0.86^{(ii)}$
Stdev($\hat{\pi}_t$)	NO	0.28	0.36	0.27	$0.61^{(iii)}$	0.35
Stdev($\hat{\pi}_t^*$)	NO	0.28	0.40	0.25	0.43	0.30
Stdev($\hat{\pi}_t^U$)	0.43	0.28	$0.38^{(i)}$	$0.26^{(ii)}$	$0.44^{(i)}$	$0.30^{(ii)}$
Correlations						
Corr($\hat{C}_t$, $\hat{C}_t^U$)	NO	1.00	0.74	0.96	-0.299	0.93
Corr($\hat{C}_t^*$, $\hat{C}_t^U$)	NO	1.00	0.98	0.98	0.99	1.00
Corr($\hat{Y}_t$, $\hat{Y}_t^U$)	NO	$0.94^{(ii)}$	0.86	$0.62^{(ii)}$	0.43	$0.35^{(ii)}$
Corr($\hat{Y}_t^*$, $\hat{Y}_t^U$)	NO	$0.69^{(ii)}$	0.98	$0.87^{(ii)}$	0.99	$1.00^{(ii)}$
Corr($\hat{\pi}_t$, $\hat{\pi}_t^*$)	NO	1.00	0.54	0.98	0.48	0.90

Table A.2: Historical and model based business cycle characteristics for the three scenarios: output, consumption, and inflation rates (cyclical components). Model based data for the current euro area is based on a decomposition of the union in Germany and other main euro area members. NO: not observable. (i) weighted average of country components. (ii) moment calculated from simulating time series. For empirical variables: Cyclical component extracted by the Hodrick-Prescott (1997) filter. (iii) Standard deviation of the weighted quarter-on-quarter CPI inflation rates of the EU9, range: 2001q1 - 2007q3. Weights derived from country weights in the EU25 (European Union). Own calculations.

Shock decomposition			Supply$^{(i)}$				Demand$^{(ii)}$			Monetary
Current Euro Area	S_H	S_N	S_F	S_{N^*}	S_T	g_H	g_N	g_F	g_{N^*}	η
$\hat{C}_t$	7.5	0.5	0.8	0.1	68.0	1.3	5.0	0.3	0.7	15.9
$\hat{C}_t^*$	7.6	0.4	0.8	0.2	68.8	1.4	3.1	0.3	1.9	15.7
$\hat{C}_t^U$	7.5	0.5	0.8	0.1	68.4	1.3	4.3	0.3	0.9	15.9
$\hat{Y}_t$	3.6	0.5	1.5	0.1	31.0	2.8	46.5	0.3	0.2	13.5
$\hat{Y}_t^*$	5.6	0.3	3.6	0.2	28.5	1.4	0.7	1.3	46.5	11.9
$\hat{Y}_t^U$	*	*	*	*	*	*	*	*	*	*
$\hat{Y}_t^U$ factor model			12.0		34.0		40.0			14.0
π_t	18.8	0.1	2.0	0.0	74.4	0.9	0.0	0.2	0.0	3.6
π_t^*	18.8	0.1	2.0	0.0	74.4	0.9	0.0	0.2	0.0	3.6
π_t^U	18.8	0.1	2.0	0.0	74.4	0.9	0.0	0.2	0.0	3.6
π_t^U factor model			10.0		53.0		27.0			10.0
Large Member Area	S_H	S_N	S_F	S_{N^*}	S_T	g_H	g_N	g_F	g_{N^*}	η
$\hat{C}_t$	4.3	0.7	3.0	0.2	72.7	0.5	5.2	1.3	1.5	10.6
$\hat{C}_t^*$	4.8	0.2	3.5	0.4	71.5	0.4	0.9	1.1	3.9	13.6
$\hat{C}_t^U$	4.6	0.3	3.3	0.3	72.4	0.4	1.9	1.2	2.8	12.8
$\hat{Y}_t$	3.3	1.6	4.7	0.2	37.8	0.7	39.5	0.8	0.5	10.9
$\hat{Y}_t$ factor model			15.0		27.0		46.0			11.1
$\hat{Y}_t^*$	3.3	0.2	3.1	0.4	40.0	0.5	0.8	2.4	38.4	11.1
$\hat{Y}_t^*$ factor model			10.0		40.0		31.0			19.0
$\hat{Y}_t^U$	*	*	*	*	*	*	*	*	*	*
π_t	9.4	0.2	6.7	0.1	77.8	0.3	0.7	0.8	0.1	3.9
π_t factor model			10.0		54.0		26.0			10.0
π_t^*	10.4	0.1	7.1	0.0	78.3	0.3	0.1	0.9	0.1	2.8
π_t^* factor model			10.0		53.0		27.0			10.0
π_t^U	*	*	*	*	*	*	*	*	*	*
Enlarged Euro Area	S_H	S_N	S_F	S_{N^*}	S_T	g_H	g_N	g_F	g_{N^*}	η
$\hat{C}_t$	2.3	8.3	5.2	0.3	71.9	0.0	2.8	2.2	2.3	4.8
$\hat{C}_t^*$	2.2	0.0	5.0	1.1	74.2	0.0	0.0	2.3	8.7	6.5
$\hat{C}_t^U$	2.2	0.0	5.1	1.1	74.5	0.0	0.0	2.3	8.3	6.5
$\hat{Y}_t$	15.	28.6	3.1	1.3	28.4	0.6	18.0	0.6	0.8	3.9
$\hat{Y}_t^*$	1.0	0.0	2.3	0.3	31.7	0.0	0.0	2.8	44.8	13.8
$\hat{Y}_t^U$	*	*	*	*	*	*	*	*	*	*
π_t	5.4	13.5	12.5	0.8	58.8	0.0	1.4	1.1	1.2	5.3
π_t^*	6.4	0.0	14.8	0.4	71.6	0.0	0.0	1.5	1.2	4.1
π_t^U	*	*	*	*	*	*	*	*	*	*

Table A.3: Variance decomposition of real GDP, consumption, and CPI inflation rates for the three scenarios. Values obtained from the asymptotic covariance matrix. Due to contemporaneous correlation, Cholesky decomposition is assumed for country-specific shocks. S_T represents a composite productivity and labour supply shock in the factor model, whereas the model demand shocks comprise the common demand shock and US shocks in the empirical model. For factor model results, see Eickmeier (2006), tables 7 and 8. (i) Cholesky ordering in case of country-specific industry shocks $e_{S_H} \rightarrow e_{S_F}$. (ii) Cholesky ordering for fiscal shocks within each country: $e_{G_N} \rightarrow e_{G_H}$ and $e_{G_{N^*}} \rightarrow e_{G_F}$. When reversing the role, the contributions of correlated shocks within each period would be reversed. Model based data for the current euro area is based on a decomposition of the union in Germany and other main euro area members.

A.2 Additional Impulse Responses in the Baseline Scenario

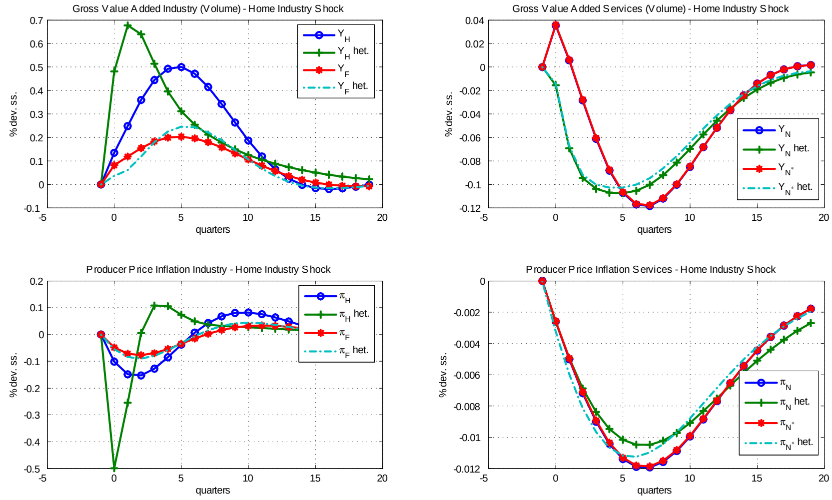

Figure A.1: Innovation in industry productivity S_H - response of gross value added and producer price indices: higher flexibility only in the industry sector.

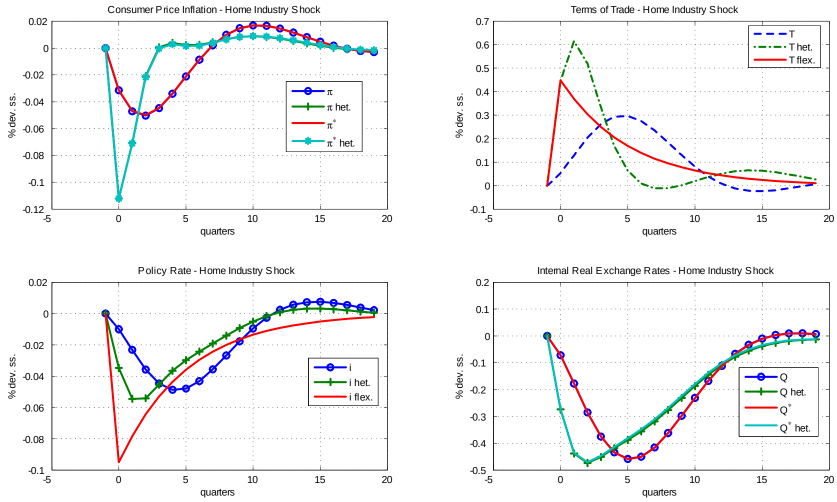

Figure A.2: Innovation in industry productivity S_H - response of the CPI inflation rate, the policy rate, the internal real exchange rates, and the terms of trade: higher flexibility only in the industry sector.

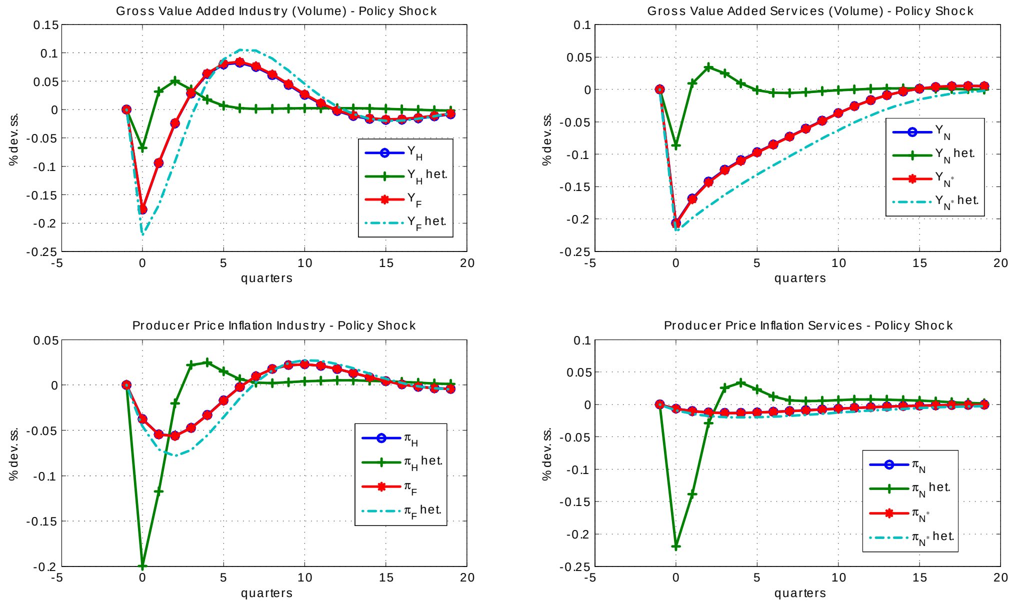

Figure A.3: Innovation in monetary policy - response of gross value added and producer price inflation rates.

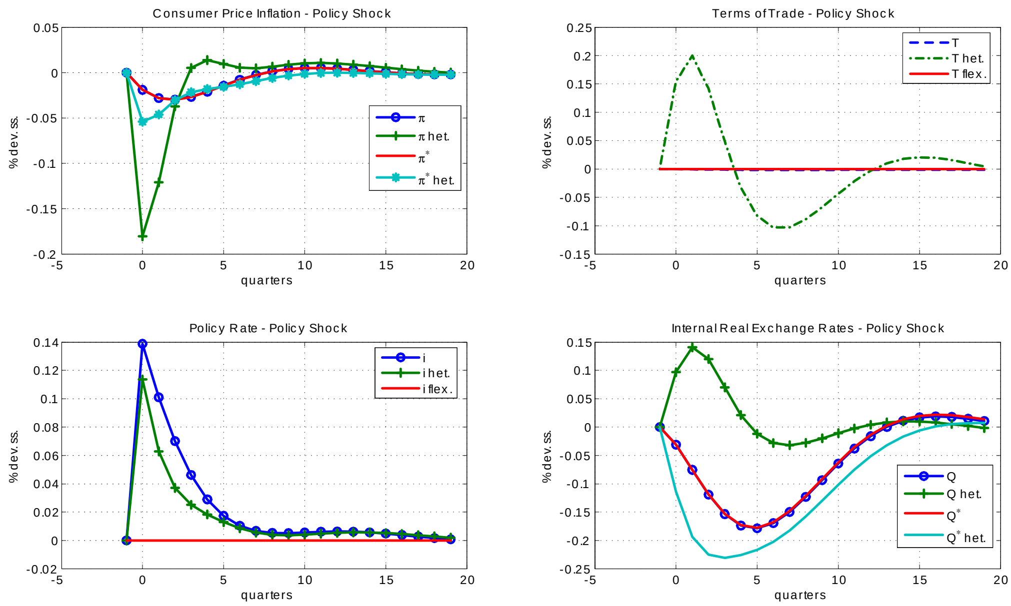

Figure A.4: Innovation in monetary policy - response of the CPI inflation rate, the policy rate, the internal real exchange rates, and the terms of trade.

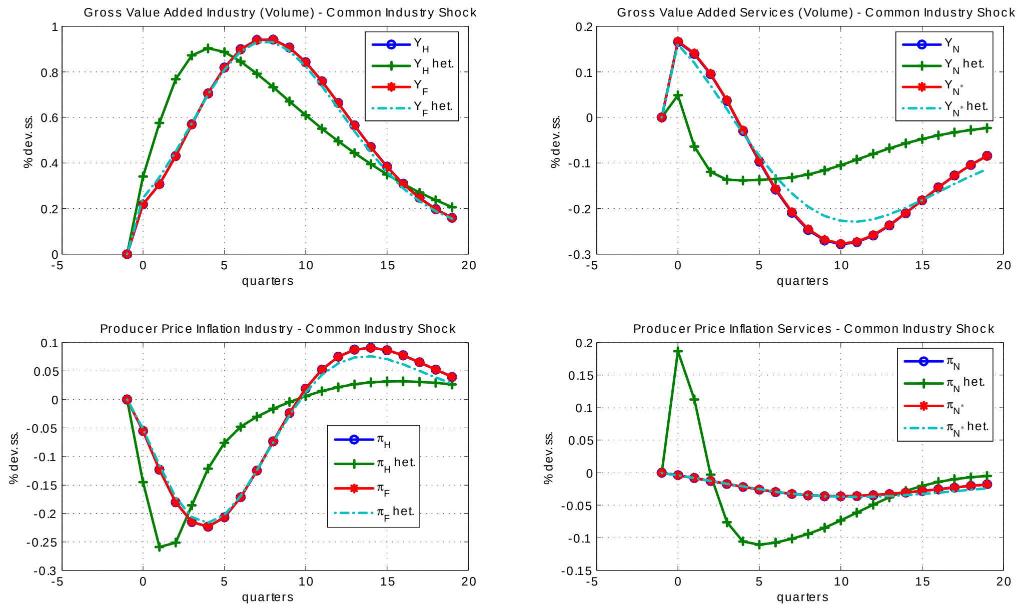

Figure A.5: Innovation in common industry productivity S_T - response of gross value added and producer price indices.

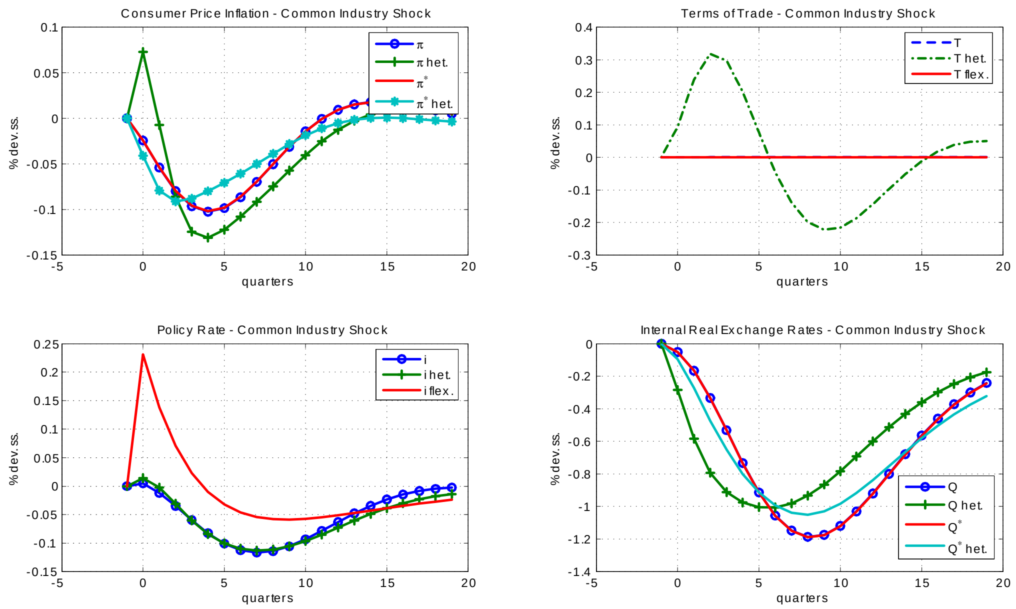

Figure A.6: Innovation in common industry productivity S_T - response of the CPI inflation rate, the policy rate, the internal real exchange rates, and the terms of trade.

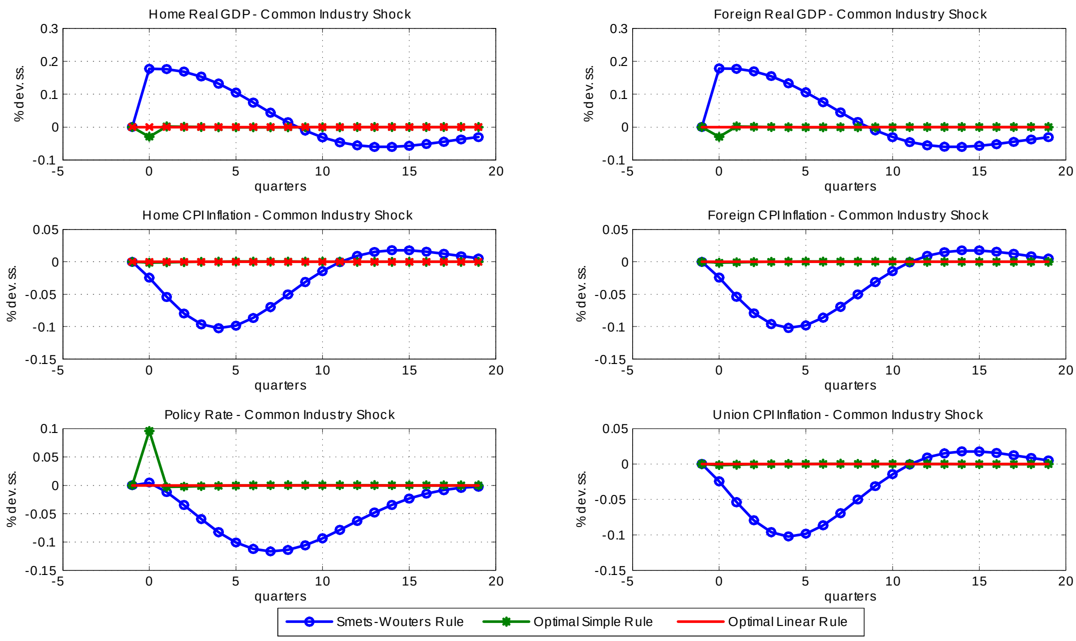

Figure A.7: Optimal stabilisation of a common productivity shock S_T.

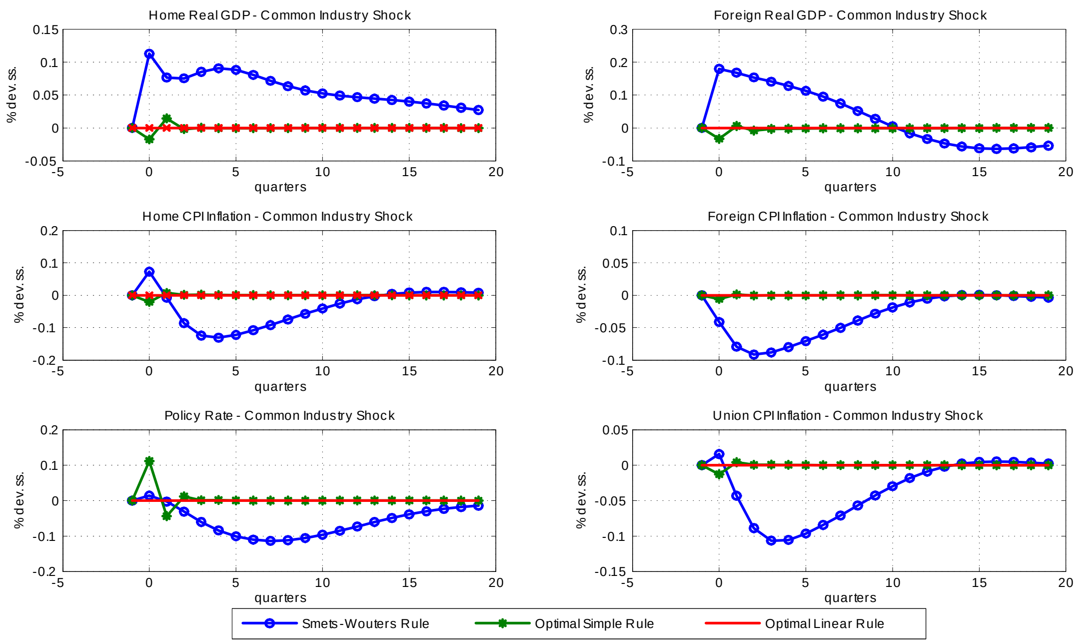

Figure A.8: Optimal stabilisation of a common productivity shock S_T: home flexible.

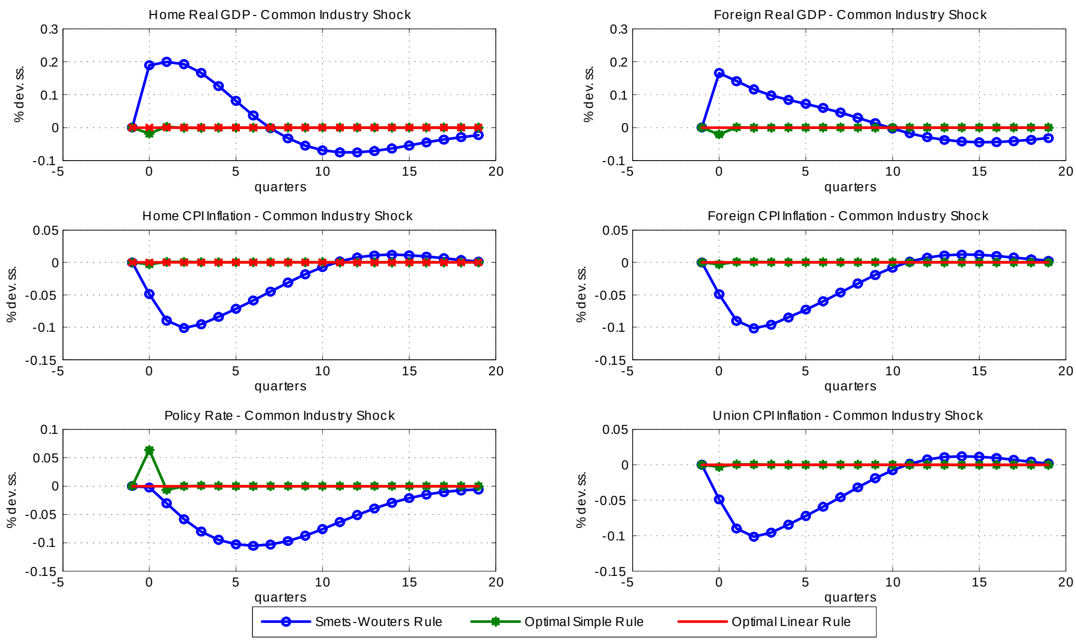

Figure A.9: Optimal stabilisation a common supply shock S_T under sectoral heterogeneity.

A.3 The Steady State

The steady state is the static, stationary and deterministic long-run equilibrium in which all dynamic adjustments have vanished. All stochastic structural shocks are zero and all variables in levels take their expected values. Especially, prices are fully flexible and all nominal rigidities are absent. Due to real rigidities that derive from monopolistic competition, the steady state is generally inefficient (the natural rate of output is below potential output $Y^{ss} < Y^{ss,eff}$). However, we will later on assume that an appropriate subsidy to labour can replicate the efficient long-run equilibrium outcome $Y^{ss,eff}$ such that the steady state can be made efficient. Therefore, a single expansion point for log-linear deviations about the long-run equilibrium can be provided.

A.3.1 Consumption and Output

The steady state assumption of a constant price level implies a constant nominal discount factor

$$V \equiv \beta \frac{\mathcal{U}_C(C)}{\mathcal{U}_C(C)} \frac{P}{P} = \beta \tag{A.1}$$

and by using the no-arbitrage condition (2.12) one obtains that market and subjective discounting of future consumption is linked by

$$1 + i = \frac{1}{\beta} \tag{A.2}$$

As inflation is absent in steady state (the price level is not growing in the long-run equilibrium), it also holds that the nominal interest rate equals the natural or real interest rate $i = r$ which is also respected by the monetary policy rule (2.46) in long-run equilibrium. As $\beta \equiv \frac{1}{1+\varsigma}$ where ς denotes the subjective rate of time preference by households, one obtains that the real interest rate (the Euler interest rate used for equilibrating intertemporal consumption) is determined by household's valuation of future consumption streams, such that $i = r = \varsigma$.[1] If ς is increasing, there is a higher discounting of future consumption streams which implies that the discount factor decreases and future consumption adds less to lifetime utility (2.1). As a consequence, consumption is tilted towards streams that are not too far in the future.

The stochastic shocks characterising production[2] captured by A_J, the preference shock in money demand ξ^i as well as shocks in government spending G_J are all zero[3]

$$A_J = 0, \qquad \xi = \xi^* = 0, \qquad G_J = 0$$

for $J = H, F, N, F^*$. The steady state perfect risk sharing condition in nominal terms is

$$\frac{\mathcal{U}_C(C)}{\mathcal{U}_{C^*}(C^*)} E = \kappa \tag{A.3}$$

Note that as our steady state coincides with the initial equilibrium $\frac{\mathcal{U}_C(C)}{\mathcal{U}_{C^*}(C^*)} E = \frac{\mathcal{U}_C(C_0)}{\mathcal{U}_{C^*}(C_0^*)} E_0 = 1$, the steady state real exchange rate is $E = E_0$. It therefore depends on the initial distribution of wealth. We follow Matsumoto (2007) and normalise $\kappa = 1$ which implies that $C_0 = C_0^*$.

[1] A high discount factor β implies a low rate of time preference ς. Therefore, the present discounted value of future consumption has a great impact on lifetime utility (the agent is patient in consumption choices).

[2] A_J denotes a preference shock to labour supply, which can also be interpreted as a shock to the level of technology in our framework, not technology (the technological state of production) itself. Technology therefore is constant in steady state.

[3] Note that the shocks are already linear such that no linearisation is necessary.

From sectoral market clearing, we obtain that sectoral gross value added Y_J needs to equal

$$\begin{array}{ll} Y_H = nC_H + (1-n)C_H^* & Y_F = (1-n)C_F^* + nC_F \\ Y_N = C_N & Y_{N^*} = C_{N^*}^* \end{array}$$

as there cannot be net savings of sectoral produce on the aggregate level when all claims are redeemed. From the aggregate demand Y^J in the tradable sectors (2.55) and (2.57) we obtain that

$$Y^H = T^{1-n} C_T^U, \qquad Y^F = T^{-n} C_T^U$$

and therefore $Y^H = Y^F T$ and $Y_H = Y_F T$. We further can write consumption at home as $C = \frac{1}{\gamma} C_T Q^{1-\gamma}$ and $C^* = \frac{1}{\gamma^*} C_T^* Q^{*(1-\gamma^*)}$. If the share of tradables $\gamma = \gamma^* = 1$, we obtain that home total consumption C equals home tradables consumption C_T as in the Benigno (2004) case.

A.3.2 Labour Markets

We define

$$(1-\Phi^J) \equiv (1-\tau^J)\frac{\rho_J - 1}{\rho_J}$$

where $0 \leq \Phi^H < 1$ indicates the efficiency of the fiscal policy to offset the equilibrium real rigidity by means of setting a tax τ^H, see also the appendix to Benigno (2004, page xi). We assume that this tax τ^J is sector specific and available in all sectors, such that

$$\tau^J = 1 - (1-\Phi^J)\frac{\rho_J}{\rho_J - 1}, \qquad J = \{H, F, N, N^*\} \tag{A.4}$$

Using the expression for τ^J, labour market equilibria defined in paragraph 2.2.2 can be rewritten in steady state as[4]

$$\begin{array}{ll} (1-\Phi^H)\mathcal{U}_C(C) = \frac{T^{1-n}}{Q^{1-\gamma}}\mathcal{V}_{y_H}(y_H(h), A^H), & \left(1-\Phi^N\right)\mathcal{U}_C(C)Q^{-\gamma} = \mathcal{V}_{y_N}(Y_N, 0) \\ (1-\Phi^F)\mathcal{U}_{C^*}(C^*) = \frac{T^{-n}}{Q^{*1-\gamma}}\mathcal{V}_{y_F}(Y_F, 0), & (1-\Phi^{N^*})\mathcal{U}_{C^*}(C^*)Q^{*-\gamma^*} = \mathcal{V}_{y_{N^*}}(Y_{N^*}, 0) \end{array} \tag{A.5}$$

As mark-ups $\frac{\rho_J}{\rho_J - 1} > 1$ and given Φ^J is small enough, $\tau^J < 0$ such that the tax in sector J operates as a subsidy. The tax perfectly offsets the steady state distortion in labour supply due to the sellers' monopoly power in case $\Phi^J = 0$. Then the subsidy allows firms (and therefore households that own firms) to make labour demand and supply choices as if the steady state equilibrium real rigidities would be absent. The labour income subsidy is fully rebated in order to offset the allocative distortions in the labour supply that derive from monopolistic competition.[5] Note that the sectoral taxes will differ in magnitude in the respective sector depending on the degree of heterogeneity in product supply. When markups coincide, taxes will coincide. Also, the more efficient the fiscal redistribution process, indicated by a lower Φ^J, the higher the subsidy is. In the efficient equilibrium, $\rho_J \to \infty$, monopolistic competition is absent and (A.4) collapses to $\tau^J = \Phi^J$. As by assumption of an efficient equilibrium, the redistribution mechanism is efficient as well, we have that by definition $\Phi^J = 0$. Therefore there is no labour income subsidy needed at all, $\tau^J = 0$.

In the inefficient equilibrium, i.e. where $\rho_J < \infty$, we follow the literature and assume that the labor income subsidy can fully offset the steady state distortion caused by monopolistic competition and the labour market outcome replicates the first best. Then the labour supply decision resembles the decision of households in the first best world, i.e. when nominal and

[4] See appendix A.4.3 for details.

[5] As outlined in the appendix to Benigno (2004) we have to restrict attention to cases where Φ^H, Φ^F, Φ^N, Φ^{N*} are at least of order of the exogenous process in the disutility function in order to replicate the first best.

real rigidities would not exist such that all market imperfections are absent. Therefore, the first best outcome is restored in the steady state given that $\tau^J = 1 - \frac{\rho_J}{\rho_J - 1}$. Consequently, the marginal rate of substitution of leisure for labour equals the real wage that would result in the efficient case.

Combining the labour market equilibria in the tradable sectors across regions, we obtain

$$\frac{\mathcal{U}_C(C)}{\mathcal{U}_{C^*}(C^*)} = T\frac{Q^{\gamma-1}}{(Q^*)^{\gamma^*-1}}\frac{\mathcal{V}_{y_H}(y_H(h),0)}{\mathcal{V}_{y_F}(y_F(f),0)}\frac{(1-\Phi^F)}{(1-\Phi^H)} = TE^{-1}\frac{\mathcal{V}_{y_H}(y_H(h),0)}{\mathcal{V}_{y_F}(y_F(f),0)}\frac{(1-\Phi^F)}{(1-\Phi^H)} \tag{A.6}$$

Combining the labour market equilibrium with risk sharing yields that disutility of labour is linked across regions by

$$\mathcal{V}_{y_F}(y_F(f),0) = T\mathcal{V}_{y_H}(y_H(h),0)\frac{1-\Phi^F}{1-\Phi^H} \tag{A.7}$$

where the efficiency of the fiscal redistribution processes in each region in the tradable sectors together with the steady state terms of trade determine steady state labour supply in each region.[6]

A.3.3 Equilibrium Allocations and Relative Prices

In steady state, all prices are flexible at home and foreign in all sectors and economic choices are guided by relative prices only. For the tradables sector, we obtain that

$$\frac{p_H(h)}{P_H} = 1 = \frac{p_F(f)}{P_F}$$

Also, as we log-linearise around a zero-inflation steady state by assumption, all sectoral inflation rates as well as the CPI rates will equal zero

$$\pi_J = 0,\ \pi = \pi^* = 0$$

without further assumptions about the (constant) levels of P_J, P and P^*. From the sharing rules we obtain the relative price of tradables as being determined by country size and steady state home and foreign tradables consumption levels

$$T = \frac{1-n}{n}\frac{C_H}{C_F} \tag{A.8}$$

We set the elasticity of substitution ν and ν^* to the respective country shares earlier, such that there is no home bias in tradable consumption $v = v^* = n$. For the internal real exchange rates at home Q and foreign Q^*

$$Q = \frac{\gamma}{1-\gamma}\frac{C_N}{C_T}, \qquad Q^* = \frac{\gamma^*}{1-\gamma^*}\frac{C_N^*}{C_T^*} \tag{A.9}$$

Our steady state is not independent of the initial distribution of output per se which was the case in Benigno (2004), i.e. in the absence of non-tradables. Therefore, (some) equilibrium relative prices $\{E, Q, Q^*, T\}$ need to be pinned down to certain levels.[7]. Omitting time subscripts from

[6] We observe that due to the presence of non-tradables, steady state terms of trade will not be equal to unity, as opposed to the one-sector case considered in Benigno (2004).

[7] Further assumptions are not needed in case $Y_H = Y_F$ and $Y_N = Y_{N^*}$, then also $C = C^*$. The result requires that the share of non-tradable consumption expenditure in each region equalises, $1-\gamma = 1-\gamma^*$.

(2.19), we can write the equilibrium real exchange rate as

$$E = \frac{P^*}{P} = \frac{Q^{1-\gamma}}{Q^{*1-\gamma^*}} \tag{A.10}$$

In the following, we pin down the equilibrium to an allocation such that in steady state internal relative prices of goods are equal to one

$$Q = Q^* = 1 \tag{A.11}$$

from which follows that $E = 1$ as well, by using (A.10).[8] Then also $P^U \equiv P^n P^{*(1-n)} = P$, albeit the composition of each basket can and will differ, depending on preferences of households in each region, i.e. depending on the expenditure weights on tradables, γ and γ^*.[9] By using the risk sharing condition (A.3), and as initial wealth was normalised to one, we obtain that overall consumption levels in member states will equalise in steady state

$$C = C^* = C^U \tag{A.13}$$

A.4 Optimality Conditions for a Generic Household

Household j maximises life-time utility

$$U_t^j = \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\mathcal{U}(C_s^j) + \mathcal{N}(\frac{M_t^j}{P_t}, \xi_t) - \mathcal{V}(y_{J,t}^j, A_{J,t}) \right) \tag{A.14}$$

under the budget constraint, written in nominal terms

$$(1-\tau^J) p_{J,t}(j) y_{J,t}(j) + Q_t^j + M_{t-1}^j + D_{t-1,t} \geq P_t C_t^j + M_t^j + E_t \{V_{t,t+1} D_t\} \tag{A.15}$$

where the consumption index (the consumption basket) is given by

$$C_t^j = \frac{(C_{T,t}^j)^\gamma (C_{N,t}^j)^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \tag{A.16}$$

and the tradables consumption index is

$$C_{T,t}^j = \frac{(C_{H,t}^j)^n (C_{F,t}^j)^{1-n}}{n^n (1-n)^{1-n}} \tag{A.17}$$

[8] It follows that the terms of trade T need not be pinned down explicitly.

[9] As a consistency check, union private sector expenditures and public expenditures can be aggregated from country expenditures denoted in steady state union currency

$$P^U C^U + P^U G^U + P^U D^U = nPC + (1-n)P^* C^* + nPG + (1-n)P^* G^* + nPD + (1-n)P^* D^*$$

Also, equilibria in the money and financial markets as well as union current accounts clearing are obtained by omitting time subscripts from (2.60) - (2.62)

$$M^U = nM + (1-n)\, M^*, \qquad 0 = nD + (1-n)\, D^*, \qquad 0 = CA + CA^*$$

Using that in steady state $G = G^* = 0$ as well as zero net savings on the union level, we can write union private sector consumption as

$$\frac{P^U}{P} C^U = nC + (1-n)\frac{P^*}{P} C^* = nC + (1-n)1C = C \tag{A.12}$$

such that $C^U = C$.

In chapter 2, it is assumed that utility from liquidity services obtained from holding real cash balances is zero, $\mathcal{N}(\frac{M_t^j}{P_t}, \xi_t) = 0$, in all periods.

A.4.1 Household Euler Equations

The optimality condition for consumption choices

$$\mathcal{E}_t\left[\beta^{s-t}\mathcal{U}_C(C_t^j) - \lambda_t P_t\right] = 0$$

and the decision on portfolio holdings $D_{t,t+1}$

$$\mathcal{E}_t\left[-\lambda_t V_{t,t+1} + \beta\lambda_{t+1}\right] = 0$$

yields the consumption Euler equation

$$\mathcal{U}_C(C_t^j) = \beta(1+i_t)\mathcal{E}_t\left[\mathcal{U}_C(C_{t+1}^j)\frac{P_t}{P_{t+1}}\right] \tag{A.18}$$

A.4.2 Derivation of the External Balance

The aim is to derive the aggregate external balance, starting with the budget constraint of an arbitrary household j. For household j

$$(1-\tau^J)p_{J,t}(j)y_{J,t}(j) + Q_t^j + M_{t-1}^j + D_{t-1,t}^j \geq P_t C_t^j + M_t^j + E_t\left\{V_{t,t+1}D_{t,t+1}^j\right\}$$

Substitute out output produced by j, $y_{J,t}(j)$, by plugging in the demand function as supply equals demand in equilibrium and markets clear at all dates. In case $J = H$, $y_{H,t}(j) = \left(\frac{p_{H,s}(j)}{P_{H,t}}\right)^{-\rho_H}\left(T_t^{1-n}C_{T,t}^U + G_{H,t}\right)$. If j produces an H good one obtains

$$\begin{aligned}
&\frac{D_{t-1,t}^j}{P_{t-1}}\frac{P_{t-1}}{P_t} + \frac{Q_t^j}{P_t} + \frac{M_{t-1}^j}{P_{t-1}}\frac{P_{t-1}}{P_t} + (1-\tau^H)\frac{p_{H,t}(j)}{P_t}\left(\frac{p_{H,s}(j)}{P_{H,t}}\right)^{-\rho_H}\left(T_t^{1-n}C_{T,t}^U + G_{H,t}\right)\\
\geq\;& \frac{P_{T,t}}{P_t}C_{T,t}^j + \frac{P_{N,t}}{P_t}C_{N,t}^j + \frac{M_t^j}{P_t} + \frac{\mathcal{E}_t D_{t,t+1}^j}{P_t}\frac{1}{1+i_t}
\end{aligned}$$

In case the household produces an N good, accordingly $y_{N,t}(j) = \left(\frac{p_{N,s}(j)}{P_{N,t}}\right)^{-\rho_N}(C_{N,t} + G_{N,t})$ such that

$$\begin{aligned}
&\frac{D_{t-1,t}^j}{P_{t-1}}\frac{P_{t-1}}{P_t} + \frac{Q_t^j}{P_t} + \frac{M_{t-1}^j}{P_{t-1}}\frac{P_{t-1}}{P_t} + (1-\tau^N)\frac{p_{N,t}(j)}{P_t}\left(\frac{p_{N,s}(j)}{P_{N,t}}\right)^{-\rho_N}(C_{N,t} + G_{N,t})\\
\geq\;& \frac{P_{T,t}}{P_t}C_{T,t}^j + \frac{P_{N,t}}{P_t}C_{N,t}^j + \frac{M_t^j}{P_t} + \frac{\mathcal{E}_t D_{t,t+1}^j}{P_t}\frac{1}{1+i_t}
\end{aligned}$$

Now aggregate over all agents in the H region. As agents are homogenous (they only differ whether they produce an H or N good and as insurance schemes are in place), the constraint for an arbitrary household coincides with the constraint for the average household, $C^j = C$. Aggregating over all consumers at home $\int_0^n C^j dj = nC$, hence $C = \frac{1}{n}\int_0^n C^j dj$. Further, for aggregating over all goods produced at home one can write

$$\begin{aligned}
&\int_0^n \left\{ p_{H,t}^{1-\rho_H}(j) \left(\frac{1}{P_{H,t}}\right)^{-\rho_H} \left(T_t^{1-n} C_{T,t}^U + G_{H,t}\right) + p_{N,t}^{1-\rho_N}(j) \left(\frac{1}{P_{N,t}}\right)^{-\rho_N} \left(C_{N,t} + G_{N,t}\right) \right\} dj \\
= \;& \left(T_t^{1-n} C_{T,t}^U + G_{H,t}\right) P_{H,t}^{\rho_H} \int_0^n \left(p_{H,s}(j)\right)^{1-\rho_H} dj + \left(C_{N,t} + G_{N,t}\right) P_{N,t}^{\rho_N} \int_0^n p_{N,t}^{1-\rho_N}(j) dj
\end{aligned}$$

Now by the definition of the price index in sector J, $\left[\frac{1}{n}\int_0^n (p_{J,s}(h))^{1-\rho_J} dh\right]^{\frac{1}{1-\rho_J}} = P_{J,t}$, one obtains that $P_{J,t}^{1-\rho_J} n = \int_0^n (p_{J,s}(j))^{1-\rho_J} dj$, and therefore $\left(T_t^{1-n} C_{T,t}^U + G_{H,t}\right) P_{H,t}^{\rho_H} \int_0^n (p_{H,s}(j))^{1-\rho_H} dj = \left(T_t^{1-n} C_{T,t}^U + G_{H,t}\right) n P_{H,t}$. Analogously for the N sector, one obtains $\left(C_{N,t} + G_{N,t}\right) n P_{N,t}$. Then the aggregated household balance can be written as

$$\begin{aligned}
& n\frac{D_{t-1,t}}{P_{t-1}}\frac{P_{t-1}}{P_t} + \frac{Q_t^{\#}}{P_t} + \frac{M_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t} + (1-\tau^H)\left(T_t^{1-n} C_{T,t}^U + G_{H,t}\right) n \frac{P_{H,t}}{P_t} \\
& +(1-\tau^N) n \frac{P_{N,t}}{P_t}\left(C_{N,t} + G_{N,t}\right) \\
\geq \;& \frac{P_{T,t}}{P_t} n C_{T,t} + \frac{P_{N,t}}{P_t} n C_{N,t} + \frac{M_t}{P_t} + n \frac{\mathcal{E}_t D_{t,t+1}}{P_t}\frac{1}{1+i_t}
\end{aligned}$$

where $\int_0^n M_t^j dj = M_t$ denote aggregate money holdings at home, $\int_0^n Q_t^j dj = Q_t^{\#}$ is home overall lump-sum social security spending. D_{t-1} is the portfolio held by the average household $\frac{1}{n}\int_0^n D_{t,t+1}^j dj = D_{t,t+1}$.

To obtain the government balance, one aggregates income (left hand side) and expenditures (right hand side) over all households

$$\begin{aligned}
& \tau^H \frac{1}{P_t}\int_0^n p_{H,t}(j) y_{H,t}(j) dj + \tau^N \frac{1}{P_t}\int_0^n p_{N,t}(j) y_{N,t}(j) dj + \frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t} \\
= \;& \frac{1}{P_t}\int_0^n p_{H,t}(j) g_{H,t}(j) dj + \frac{1}{P_t}\int_0^n p_{N,t}(j) g_{N,t}(j) dj + \frac{Q_t^{\#}}{P_t}
\end{aligned}$$

Again using the individual demand functions for the private sector and the government for a good j

$$g_{H,t}(h) = \left(\frac{p_{H,s}(h)}{P_{H,t}}\right)^{-\rho_H} G_{H,t}, \quad g_{N,t}(h) = \left(\frac{p_{N,s}(h)}{P_{N,t}}\right)^{-\rho_N} G_{N,t}$$

and again employing the price index property, one can rewrite

$$\begin{aligned}
& \tau^H \frac{1}{P_t} n P_{H,t} Y_{H,t} + \tau^N \frac{1}{P_t} n P_{N,t} Y_{N,t} + \frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t} \\
= \;& \frac{1}{P_t} n P_{H,t} G_{H,t} + \frac{1}{P_t} n P_{N,t} G_{N,t} + \frac{Q_t^{\#}}{P_t}
\end{aligned}$$

Now use the government balance to substitute out real money holdings and transfers. Use aggregate demand functions $Y_{H,t} = T^{1-n} C_{T,t}^W + G_{H,t}$, $Y_{N,t} = C_{N,t} + G_{N,t}$, to substitute out $Y_{H,t}$ and $Y_{N,t}$ which yields that

$$n\frac{D_{t-1,t}}{P_{t-1}}\frac{P_{t-1}}{P_t} + \frac{n}{P_t} P_{H,t} T^{1-n} C_{T,t}^U = \frac{P_{T,t}}{P_t} n C_{T,t} + n \frac{\mathcal{E}_t D_{t,t+1}}{P_t}\frac{1}{1+i_t} \tag{A.19}$$

We then obtain for home

$$n\frac{\mathcal{E}_t D_{t,t+1}}{P_t}\frac{1}{1+i_t} - n\frac{D_{t-1,t}}{P_{t-1}}\frac{P_{t-1}}{P_t} = n\frac{P_{H,t}}{P_t}T_t^{1-n}C^U_{T,t} - n\frac{P_{T,t}}{P_t}C_{T,t} \tag{A.20}$$

We can rewrite price ratios in terms of relative prices. The aggregate price level at home can be written as

$$P_t = (P_{T,t})^\gamma (P_{N,t})^{1-\gamma} = (\frac{P_{T,t}}{P_{N,t}})^\gamma P_{N,t} = Q_t^\gamma P_{N,t}$$

Using the definition of the terms of trade $T_t = \frac{P_{F,t}}{P_{H,t}}$ to simplify the tradables price level

$$P_{T,t} = (P_{H,t})^n (P_{F,t})^{1-n} = T_t^{-n} P_{F,t}$$

By the definition of the internal REER $P_{N,t} = P_{T,t}Q_t^{-1}$, or $P_{N,t} = T_t^{-n}P_{F,t}Q_t^{-1}$. Plug this expression back into the CPI price level

$$P_t = Q_t^\gamma T_t^{-n} P_{F,t} Q_t^{-1} = Q_t^{\gamma-1} T_t^{-n} P_{F,t}$$

Again use that by definition $P_{F,t} = T_t P_{H,t}$

$$\begin{aligned} P_t &= Q_t^{\gamma-1} T_t^{-n} T_t P_{H,t} \\ \frac{P_t}{P_{H,t}} &= Q_t^{\gamma-1} T_t^{1-n} = \frac{T^{1-n}}{Q_t^{1-\gamma}} \end{aligned} \tag{A.21}$$

We use the latter result to substitute out $\frac{P_t}{P_{H,t}}$ in the balance such that

$$\begin{aligned} \frac{\mathcal{E}_t D_{t,t+1}}{P_t}\frac{1}{1+i_t} &= \frac{D_{t-1,t}}{P_{t-1}}\frac{P_{t-1}}{P_t} + \frac{Q_t^{1-\gamma}}{T^{1-n}}T_t^{1-n}C^U_{T,t} - \frac{P_{T,t}}{P_t}C_{T,t} \\ &= \frac{D_{t-1}}{P_{t-1}}\frac{1}{1+\pi_t} + Q_t^{1-\gamma}\left(C^U_{T,t} - C_{T,t}\right) \end{aligned} \tag{A.22}$$

and we also used that $\frac{P_t}{P_{T,t}} = \left(\frac{P_{T,t}}{P_{N,t}}\right)^{\gamma-1} = Q^{\gamma-1}$. (A.22) resembles (2.49) in the text.

A.4.3 Labour Supply and Demand

Labour supply is determined by maximising the utility function of household j with respect to C_t^j and $L_{J,t}^j$ subject to the budget constraint (2.10) which is restated here

$$(1-\tau^J)p_{J,t}(j)y_{J,t}(j) + Q_t^{\#j} + M_{t-1}^j + D_{t-1,t} \geq P_t C_t^j + M_t^j + E_t\left\{V_{t,t+1}D_{t,t+1}\right\}$$

Revenues on selling output $y_{J,t}(j)$ given by $p_{J,t}(j)y_{J,t}(j)$ are distributed as wage income $W_{J,t}^j L_{J,t}^j$ and profit income $\Pi_{J,t}^j$

$$p_{J,t}(j)y_{J,t}(j) = W_{J,t}^j L_{J,t}^j + \Pi_{J,t}^j \tag{A.23}$$

There is no wage differentiation in the model and the nominal wage is same across firms within a sector J

$$W_{J,t}^j = W_{J,t} \tag{A.24}$$

Deflating the budget constraint by P_t, i.e. rewriting in real consumption units

$$(1-\tau^J)\left(\frac{W_{J,t}}{P_t}L_{J,t}^j + \frac{\Pi_{J,t}}{P_t}\right) + \frac{Q_t^{\#j}}{P_t} + \frac{M_{t-1}^j}{P_t} + \frac{D_{t-1,t}}{P_t} \geq C_t^j + \frac{M_t^j}{P_t} + E_t\left\{\frac{V_{t,t+1}}{P_t}D_{t,t+1}\right\}$$

The optimality condition for total consumption was

$$\beta^{s-t}\mathcal{U}_C(C_t^j) - \lambda_t = 0$$

where λ_t denotes the present discounted value Lagrange multiplier. For a household j that produces a single good in the H sector (that supplies labour to a firm h that produces in the H sector) one obtains that

$$-\beta^{s-t}\tilde{\mathcal{V}}_{L_H}(L_{H,t}^j) + \lambda_t(1-\tau^H)\frac{W_{H,t}}{P_t} = 0$$

and the labour supply curve for household j reads

$$MRS_{C,L_H} \equiv \frac{\tilde{\mathcal{V}}_{L_H}(L_{H,t}^j)}{\mathcal{U}_C(C_t^j)} = (1-\tau^H)\frac{W_{H,t}}{P_t}$$

which is similar to equation (2.31) in the text. As a result, the marginal rate of substitution of consumption of goods and services (the basket C_t^j) for leisure (labour $L_{H,t}^j$) is equated to the disposable income generated by providing one hour of work, which is the after-tax real wage. Accordingly, if household j produces in the N sector

$$\frac{\tilde{\mathcal{V}}_{L_N}(L_{N,t}^j)}{\mathcal{U}_C(C_t^j)} = (1-\tau^N)\frac{W_{N,t}}{P_t}$$

Labour demand of a firm in sector H is derived from factor cost minimisation by taking the wage $W_{H,t}$ as well as the aggregate price level $P_{H,t}$ in the sector as given

$$\min_{L_{H,t}^j(h)} \frac{W_{H,t}}{P_{H,t}} L_{H,t}^j(h) \text{ s.t. } y_{H,t}(h) = f(L_{H,t}^j(h))$$

which can be stated as a Lagrangian

$$\mathcal{L} = \frac{W_{H,t}}{P_{H,t}} L_{H,t}^j(h) + MC_t^H(h)\left(y_{H,t}(h) - f(L_{H,t}^j(h))\right)$$

and as a first order condition

$$\mathcal{L}_{L_H^j} = \frac{W_{H,t}}{P_{H,t}} = MC_t^H(h) f'(L_{H,t}^j(h))$$

from which directly follows that

$$W_{H,t} = P_{H,t} MC_t^H(h) f'(L_{H,t}^j(h)) \tag{A.25}$$

$MC_t^H(h)$ denotes the Lagrange multiplier of the problem. Consequently, it denotes the shadow price of increasing output by one unit, hence it is real marginal cost of the production of good h in sector H. Factor costs equal the nominal marginal revenue product which equals the nominal marginal cost times the physical product of labour.

As in Benigno and López-Salido (2002) each differentiated good h in the H sector is produced according to the production function $y_{H,t}(h) = f(L_{H,t}^j(h))$, therefore $L_{H,t}^j(h) = f^{-1}(y_{H,t}(h))$ such that $f'(L_{H,t}^j(h)) = f'(f^{-1}(y_{H,t}(h)))$.[10] Before proceeding, we note that

[10] Note that we follow the exposition of Benigno (2004) here concerning the interpretation of $A_{J,t}$. It denotes a transitory preference shock to labour supply (a shock to factor productivity) whereas in the specification of Woodford (2003), output is produced according to $y_{H,t}(h) = A_{J,t} f(L_{H,t}^j(h))$. $A_{J,t}$ can thus be interpreted as a

households are consumer-producers. Hence they produce output (only) with own hours worked. Therefore $j = h$ for all $h \in n$. and $j = f$ for all $j \in (1-n)$.

We can then proceed analogously to Woodford (2000) and Benigno and López-Salido (2002) and define the function $\mathcal{V}(\cdot)$

$$\mathcal{V}(y_{H,t}(h), A_t^H) \equiv \tilde{\mathcal{V}}(L_{H,t}^j) = \tilde{\mathcal{V}}(f^{-1}(y_{H,t}(h)))$$

from which follows that

$$\tilde{\mathcal{V}}_{y_H}(f^{-1}(y_{H,t}(h))) = \tilde{\mathcal{V}}_{L_H}(L_{H,t}^j) \tag{A.26}$$

We therefore can express the disutility of labour function directly as a disutility of producing output function. Differentiating $\mathcal{V}$ w.r.t. to $y_{H,t}(h)$

$$\begin{aligned} \mathcal{V}_{y_H}(y_{H,t}(h), A_t^H) &\equiv \tilde{\mathcal{V}}_{y_H}(f^{-1}(y_{H,t}(h))) \left[f^{-1}(y_{H,t}(h))\right]' \\ \mathcal{V}_{y_H}(y_{H,t}(h), A_t^H) f' \left[f^{-1}(y_{H,t}(h))\right] &= \tilde{\mathcal{V}}_{y_H}(f^{-1}(y_{H,t}(h))) = \tilde{\mathcal{V}}_{L_H}(L_{H,t}^j) \end{aligned} \tag{A.27}$$

In the preceding equation we have used the law of differentiation for inverse functions which states that for a function g

$$(g^{-1})'(d) = \frac{1}{g'(g^{-1}(d))}$$

Labour market equilibrium in the H sector equates the labour supply with labour demand via the nominal wage $W_{H,t}$

$$\begin{aligned} \frac{\tilde{V}_{L_H}(L_{J,t}^j)}{U_C(C_t)} P_t \frac{1}{(1-\tau^H)} &= W_{H,t} = P_{H,t} MC_t^H(h) f' \left[f^{-1}(y_{H,t}(h))\right] \\ \frac{V_{y_H}(y_{H,t}(h), A_t^H)}{U_C(C_t)} &= (1-\tau^H) \frac{P_{H,t}}{P_t} MC_t^H(h) \end{aligned} \tag{A.28}$$

The consumer price level P_t can be decomposed into relative price contributions T_t and Q_t

$$\begin{aligned} P_t &= P_{T,t}^{\gamma} P_{N,t}^{1-\gamma} = \left(\frac{P_{N,t}}{P_{T,t}}\right)^{1-\gamma} P_{T,t} = \frac{1}{Q_t^{1-\gamma}} P_{H,t}^n P_{F,t}^{1-n} = \frac{1}{Q_t^{1-\gamma}} \left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-n} P_{H,t} \\ \frac{P_{H,t}}{P_t} &= T_t^{n-1} Q_t^{1-\gamma} = \frac{Q^{1-\gamma}}{T_t^{1-n}} \end{aligned} \tag{A.29}$$

Accordingly for foreign, which will be useful later on,

$$\begin{aligned} P_t^* &= \frac{1}{Q_t^{*1-\gamma^*}} \left(\frac{P_{H,t}}{P_{F,t}}\right)^n P_{F,t} = \frac{1}{Q_t^{*1-\gamma^*}} T^{-n} P_{F,t} \\ \frac{P_{F,t}}{P_t^*} &= Q_t^{*1-\gamma^*} T_t^n = \frac{Q_t^{*1-\gamma^*}}{T_t^{-n}} \end{aligned} \tag{A.30}$$

One can rewrite the labour market equilibrium for good h in the H sector in terms of relative prices T_t and Q_t

$$\frac{\mathcal{V}_{y_H}(y_{H,t}(h), A_t^H)}{\mathcal{U}_C(C_t)} = (1-\tau^H) \frac{Q_t^{1-\gamma}}{T_t^{1-n}} MC_t^H(h)$$

At foreign accordingly,

$$\frac{\mathcal{V}_{y_F}(y_{F,t}(h), A_t^F)}{\mathcal{U}_{C^*}(C_t^*)} = (1-\tau^F) \frac{Q_t^{*1-\gamma^*}}{T_t^{-n}} MC_t^F(f)$$

time-varying exogenous technology factor (the level of factor productivity) as in chapters 3 and 4.

In steady state

$$\frac{\mathcal{V}_{y_H}(y_H(h),A^H)}{\mathcal{U}_C(C)}=(1-\tau^H)T^{n-1}Q^{1-\gamma}MC^H(h),\qquad \frac{\mathcal{V}_{y_F}(y_F(h),A^F)}{\mathcal{U}_{C^*}(C^*)}=(1-\tau^F)\frac{Q^{*1-\gamma^*}}{T^{-n}}MC^F(f)$$

$MC_t^H(h)$ is derived from optimal price-setting in order to maximise intertemporal profits. In steady state where prices are flexible we have that real marginal cost for firm h is given by

$$MC^H(h)=\frac{\rho_H-1}{\rho_H}$$

Replacing

$$(1-\tau^H)T^{n-1}Q^{1-\gamma}U_C(C)\frac{\rho_H-1}{\rho_H}=\mathcal{V}_{y_H}(y_H(h),A^H) \tag{A.31}$$

yields

$$\frac{\mathcal{V}_{y_H}(y_H(h),A^H)}{\mathcal{U}_C(C)}=\frac{\rho_H-1}{\rho_H}(1-\tau^H)\frac{Q^{1-\gamma}}{T^{1-n}}$$

The term $\frac{\rho_H-1}{\rho_H}(1-\tau^H)$ drives a wedge between the marginal rate of substitution of consumption for labour and relative prices. Fiscal policy can set the tax rate τ^H in order to obtain an allocation that offsets the equilibrium distortion. Then the household h will choose production as if markup pricing would be absent.

Following the same reasoning, for the N sector, labour market equilibrium is given by

$$\frac{\mathcal{V}_{y_N}(y_{N,t}(h),A_t^N)}{\mathcal{U}_C(C_t)}=(1-\tau^N)\frac{P_{N,t}}{P_t}MC_t^N(h)$$

Using

$$\begin{aligned}P_t &= \left(\frac{P_{T,t}}{P_{N,t}}\right)^{\gamma}P_{N,t}=Q_t^{\gamma}P_{N,t}\\ \frac{P_{N,t}}{P_t} &= Q_t^{-\gamma}\end{aligned}$$

we can write

$$\frac{\mathcal{V}_{y_N}(y_{N,t}(h),A_t^N)}{\mathcal{U}_C(C_t)}=(1-\tau^N)Q_t^{-\gamma}MC_t^N(h)$$

Therefore, differently from the H sector, labour supply for non-tradable goods production only depends on the internal real exchange rate and not the terms of trade. In steady state, using that $MC^N(h)=\frac{\rho_N-1}{\rho_N}$

$$(1-\tau^N)Q^{-\gamma}\mathcal{U}_C(C)=\frac{\rho_N}{\rho_N-1}\mathcal{V}_{y_N}(y_N(h),0) \tag{A.32}$$

A.4.4 Steady State

This paragraph provides further details on the derivation of the steady state presented in section A.3. Union consumption expenditures on tradables are given by

$$P_T^UC_T^U=P_TC_T+P_T^*C_T^*$$

The law of one price guarantees that $P_T=P_T^*=P_T^U$. We thus write

$$\frac{C_T^U}{C}=n\frac{C_T}{C}+(1-n)\frac{C_T^*}{C}=n\gamma+(1-n)\gamma^* \tag{A.33}$$

As $C = C^*$, $\frac{C_T^U}{C^*} = n\frac{C_T}{C^*} + (1-n)\frac{C_T^*}{C^*} = \frac{C_T^U}{C}$. We continue by noting that then the ratios $\frac{C_T}{C_T^U}$ and $\frac{C_T^*}{C_T^U}$ are determined as well

$$\frac{C_T^U}{C_T} = n + (1-n)\frac{C_T^*}{C_T} = n + (1-n)\frac{C/C_T}{C^*/C_T^*} = n + (1-n)\frac{\gamma^*}{\gamma} \tag{A.34}$$

The ratios $\frac{C_T^U}{C}$ and $\frac{C_T^U}{C_T}$ are used later on when aggregating regional welfare to overall union welfare.

As in Benigno (2004) we define

$$(1-\Phi^J) \equiv (1-\tau^J)\frac{\rho_J - 1}{\rho_J}$$

where $\Phi^J = 0$ indicates that fiscal policy can fully offset the steady state distortion caused by monopolistic competition by choosing the appropriate labour supply subsidy. In the efficient equilibrium, $\Phi^H = \Phi^N = 0$, i.e. labor income taxation can per se fully offset the steady state distortion caused by monopolistic competition. In aggregate, for each sector we obtain that

$$(1-\Phi^H)\mathcal{U}_C(C) = \frac{T^{1-n}}{Q^{1-\gamma}}\mathcal{V}_{y^H}(Y_H, 0) \tag{A.35}$$

$$(1-\Phi^N)\mathcal{U}_C(C) = Q^{\gamma}\mathcal{V}_{y_N}(Y_N, 0) \tag{A.36}$$

$$(1-\Phi^F)\mathcal{U}_{C^*}(C^*) = \frac{T^{-n}}{Q^{*1-\gamma^*}}\mathcal{V}_{y_F}(Y_F, 0) \tag{A.37}$$

$$(1-\Phi^{N*})\mathcal{U}_{C^*}(C^*) = Q^{*\gamma^*}\mathcal{V}_{y_{N*}}(Y_{N^*}, 0) \tag{A.38}$$

Define for any variable X_t the gap between its inefficient steady state X and its efficient steady state X^{eff} as

$$x \equiv -\ln\left(X/X^{eff}\right) \tag{A.39}$$

such that $x > 0$, in case there is a steady state distortion, such that X is lower than in the first best case X^{eff}. This definition is also employed in the appendix to Benigno (2004) on page xiii as well as in the appendix to Beetsma and Jensen (2005) on page 9. The inefficiency in the sectoral labour market equilibrium is derived as follows. Take a first order Taylor approximation of (A.35) about its efficient steady state. Marginal utility from consumption in steady state C evaluated around the efficient steady state C^{eff} reads

$$U_C(C) = U_C\left(C^{eff}\right) + U_{CC}\left(C - C^{eff}\right) + (o||\xi||^2)$$

such that as a first order approximation

$$\begin{aligned} U_C(C) &= U_C\left(C^{eff}\right) + U_{CC}(C^{eff})\left(C - C^{eff}\right) \\ \frac{U_C(C)}{U_C\left(C^{eff}\right)} &= 1 + \frac{U_{CC}(C^{eff})}{U_C(C^{eff})}\left(C - C^{eff}\right) \\ &= 1 - \frac{\rho}{C^{eff}}\left(C - C^{eff}\right) = 1 + \rho c \end{aligned}$$

We used that up to second order, the arithmetic percentage change equals the logarithmic percentage change, such that $\frac{C-C^{eff}}{C^{eff}} = \ln C - \ln C^{eff} \equiv -c$. Also, the definition of relative risk aversion $\frac{U_{CC}(C^{eff})}{U_C(C^{eff})}C^{eff} = -\rho$ was used. We can write

$$U_C(C) = U_C\left(C^{eff}\right)(1+\rho c) \tag{A.40}$$

and accordingly at foreign. The disutility of labour effort function can be approximated about the efficient level of output Y_H^{eff}

$$\begin{aligned}
\mathcal{V}_{y_H}(Y_H, 0) &= \mathcal{V}_{y_H}(Y_H^{eff}, 0)\left(1 + \frac{\mathcal{V}_{y_H y_H}(Y_H^{eff}, 0)}{\mathcal{V}_{y_H}(Y_H^{eff}, 0)}\left(Y - Y^{eff}\right)\right) \\
&= \mathcal{V}_{y_H}(Y_H^{eff}, 0)\left(1 + \frac{\eta_H}{Y_H^{eff}}\left(Y_H - Y_H^{eff}\right)\right) \\
\mathcal{V}_{y_H}(Y_H, 0) &= \mathcal{V}_{y_H}(Y_H^{eff}, 0)\,(1 - \eta_H y_H) \qquad \text{(A.41)}
\end{aligned}$$

where the inverse of the labour supply elasticity is given by $\eta_H \equiv \frac{\mathcal{V}_{y_H y_H}(Y_H^{eff},0)Y_H^{eff}}{\mathcal{V}_{y_H}(Y_H^{eff},0)}$. Take log differences of (A.35) about the efficient steady state to obtain that

$$\begin{aligned}
-\Phi^H + \ln\mathcal{U}_C(C) - \ln\mathcal{U}_C(C^{eff}) &= (1-n)\left(\ln T - \ln T^{eff}\right) - (1-\gamma)\left(\ln Q - \ln Q^{eff}\right) \\
&\quad + \ln\mathcal{V}_{y_H}(Y_H, 0) - \ln\mathcal{V}_{y_H}(Y_H^{eff}, 0)
\end{aligned}$$

Denote the relative price gaps between inefficient and efficient steady state allocations as

$$\begin{aligned}
q &\equiv -\ln(Q/Q^{eff}) \\
q^* &\equiv -\ln(Q^*/Q^{*eff}) \\
t &\equiv -\ln(T/T^{eff})
\end{aligned}$$

such that

$$\begin{aligned}
-\Phi^H + \ln\mathcal{U}_C(C) - \ln\mathcal{U}_C(C^{eff}) &= (1-n)(-t) - (1-\gamma)(-q) \\
&\quad + \ln\mathcal{V}_{y_H}(Y_H, 0) - \ln\mathcal{V}_{y_H}(Y_H^{eff}, 0)
\end{aligned}$$

Taking logs of (A.40) and logs of (A.41) and plugging in

$$\begin{aligned}
-\Phi^H + \ln\mathcal{U}_C(C^{eff}) + \rho c - \ln\mathcal{U}_C(C^{eff}) &= (1-n)(-t) - (1-\gamma)(-q) \\
&\quad + \ln\mathcal{V}_{y_H}(Y_H^{eff}, 0) - \eta_H y_H - \ln\mathcal{V}_{y_H}(Y_H^{eff}, 0) \\
-\Phi^H + \rho c &= (1-n)(-t) - (1-\gamma)(-q) - \eta_H y_H \\
\Phi^H &= \rho c + \eta_H y_H + (1-n)\,t - (1-\gamma)\,q \qquad \text{(A.42)}
\end{aligned}$$

Analogously in case of non-tradables and for foreign

$$\Phi^N = \rho c + \eta_N y_N + \gamma q \tag{A.43}$$

and for foreign

$$\begin{aligned}
\Phi^F &= \rho^* c^* + \eta_F y_F - nt - (1-\gamma^*)\,q^* \qquad \text{(A.44)} \\
\Phi^{N^*} &= \rho^* c^* + \eta_{N^*} y_{N^*} + \gamma^* q^* \qquad \text{(A.45)}
\end{aligned}$$

These are the expressions for Φ^J used in the loss function (2.110).

A.4.5 Demand Functions at Home and Foreign

Household j wants to minimise total expenditures for obtaining one unit of the consumption basket $C^j_{k,t} = 1$ where $k = H, N$. The Lagrangian of this problem reads

$$\begin{aligned}
\mathcal{L}_s &= \int_0^n p_{k,s}(h) c^j_{k,s}(h) dh - \lambda_s \left(\left[\left(\frac{1}{n}\right)^{\frac{1}{\rho_k}} \int_0^n c(h)^{\frac{\rho_k - 1}{\rho_k}} dh \right]^{\frac{\rho_k}{\rho_k - 1}} - C^j_{k,s} \right) \\
\mathcal{L}_{c^j_{k,s}(h)} &= p_{k,s}(h) - \lambda_s \frac{\rho_k}{\rho_k - 1} \left(\left[\left(\frac{1}{n}\right)^{\frac{1}{\rho_k}} \int_0^n c(h)^{\frac{\rho_k - 1}{\rho_k}} dh \right]^{\frac{\rho_k}{\rho_k - 1} - 1} \frac{\rho_k - 1}{\rho_k} c(h)^{\frac{\rho_k - 1}{\rho_k} - 1} \right) = 0 \\
\left(\frac{p_{k,s}(h)}{\lambda_s}\right)^{-\rho_k} &= \left(\frac{1}{n}\right)^{-1} (C^j_{k,s})^{-1} (c^j_{k,s}(h))
\end{aligned}$$

Demand for good h is then $c^j_{k,s}(h) = \frac{1}{n} \left(\frac{p_{k,s}(h)}{\lambda_s}\right)^{-\rho_H} C^j_{k,t}$. The respective price index can be obtained by plugging the individual demand function in the consumption aggregator

$$\begin{aligned}
\left[\left(\frac{1}{n}\right)^{\frac{1}{\rho_k}} \int_0^n (c^j_{k,s}(k))^{\frac{\rho_k - 1}{\rho_k}} dh \right]^{\frac{\rho_k}{\rho_k - 1}} &= C^j_{k,s} \\
\left[\left(\frac{1}{n}\right)^{\frac{1}{\rho_k}} \int_0^n \left(\frac{1}{n} \left(\frac{p_{k,s}(h)}{\lambda_s}\right)^{-\rho_H} C^j_{k,s} \right)^{\frac{\rho_k - 1}{\rho_k}} dh \right]^{\frac{\rho_k}{\rho_k - 1}} &= C^j_{k,s} \\
\left[\frac{1}{n} \int_0^n (p_{k,s}(h))^{1-\rho_k} dh \right] &= \lambda_s^{1-\rho_k} \\
\left[\frac{1}{n} \int_0^n (p_{k,s}(h))^{1-\rho_k} dh \right]^{\frac{1}{1-\rho_k}} &= \lambda_s \equiv P_{k,s} \qquad \text{(A.46)}
\end{aligned}$$

And analogously

$$\left[\frac{1}{1-n} \int_n^1 (p_{F,s}(f))^{1-\rho_F} df \right]^{\frac{1}{1-\rho_F}} = \lambda_s \equiv P_{F,s}, \qquad \left[\frac{1}{1-n} \int_n^1 (p^*_{F,s}(f))^{1-\rho^*_F} df \right]^{\frac{1}{1-\rho^*_F}} = \lambda_s \equiv P^*_{F,s} \qquad \text{(A.47)}$$

such that equations (2.7)-(2.9) in the text are obtained. λ_s is by definition the increase in the objective function (total expenditures) when relaxing the constraint by one unit. This is just the definition of a price index. Accordingly

$$c^j_{k,s}(h) = \frac{1}{n} \left(\frac{p_{k,s}(h)}{P_k}\right)^{-\rho_k} C^j_{k,s}, \qquad c^{j*}_{k,s}(f) = \frac{1}{1-n} \left(\frac{p^*_{k,s}(f)}{P^*_k}\right)^{-\rho^*_k} C^{j*}_{k,s} \qquad \text{(A.48)}$$

Individual Demand Functions

We assume that the price elasticity for a home produced good is same whether consumed at Home or Foreign $\rho_H = \rho^*_H$. Further we assume that the share of Home produced tradables consumption is same at Home and Foreign $v = v^*$ (no home bias) Also, the share corresponds to the size of the country of origin of the produced good $v = n$. Then

$$\begin{aligned}
y_{H,t}(h) &= \left(\frac{p_{H,s}(h)}{P_{H,t}}\right)^{-\rho_H} \int_0^1 \left\{ \begin{array}{c} \frac{1}{n}\left(\frac{p_{H,s}(h)}{P_{H,t}}\right)^{-\rho_H} n\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-n} C^j_{T,t} \\ +\frac{1}{n}\left(\frac{p^*_{H,s}(f)}{P^*_{H,t}}\right)^{-\rho^*_H} n\left(\frac{P^*_{F,t}}{P^*_{H,t}}\right)^{1-n} C^{*j}_{T,t} \end{array} \right\} dj + \left(\frac{p_{H,s}(h)}{P_{H,t}}\right)^{-\rho_H} G_{H,t} \\
&= \left(\frac{p_{H,s}(h)}{P_{H,t}}\right)^{-\rho_H} \left(\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-n} \int_0^n C^j_{T,t} dj + \left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-v} \int_n^1 C^{*j}_{T,t} dj + G_{H,t}\right)
\end{aligned}$$

Due to insurance schemes in each country, all idiosyncratic consumption risk can be hedged, such that

$$C^j_{T,t} = C_{T,t}, \qquad C^j_{T^*,t} = C^*_{T,t}$$

and

$$\begin{aligned}
y_{H,t}(h) &= \left(\frac{p_{H,s}(h)}{P_{H,t}}\right)^{-\rho_H} \left(\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-n} \int_0^n C_{T,t} dj + \left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-n} \int_n^1 C^*_{T,t} dj + G_{H,t}\right) \\
y_{H,t}(h) &= \left(\frac{p_{H,s}(h)}{P_{H,t}}\right)^{-\rho_H} T_t^{1-n} C^U_{T,t} + G_{H,t} \qquad \text{(A.49)}
\end{aligned}$$

In the last step, we used the definition of the terms of trade for Home, defined as the relative price of one unit of the tradable good $\frac{P_{F,t}}{P_{H,t}} = T_t$ as well as the definition of world tradable consumption $C^W_{T,t} = nC_{T,t} + (1-n)\, C_{T^*,t}$. For the non-tradable demand we obtain

$$y_{N,t}(h) = \left(\frac{p_{N,t}(h)}{P_{N,t}}\right)^{-\rho_N} (C_{N,t} + G_{N,t}) \qquad \text{(A.50)}$$

For the foreign produced tradable good, accordingly

$$\begin{aligned}
y^*_{F,t}(f) &= \int_0^1 \left\{ c^j_{F,t}(f) + \int_n^1 c^{j,*}_{F,t}(f) \right\} dj + \int_n^1 g^*_{F,t}(f) dj \\
&= \int_0^1 \left\{ \begin{array}{c} \frac{1}{1-n}\left(\frac{p_{F,s}(f)}{P_{F,t}}\right)^{-\rho_F} (1-v)\left(\frac{P_{F,t}}{P_{H,t}}\right)^{-v} C^j_{T,t} \\ +\frac{1}{1-n}\left(\frac{p^*_{F,s}(f)}{P^*_{F,t}}\right)^{-\rho^*_F} (1-v^*)\left(\frac{P^*_{F,t}}{P^*_{H,t}}\right)^{-v^*} C^{*j}_{T,t} \end{array} \right\} dj + \left(\frac{p^*_{F,s}(h)}{P^*_{F,t}}\right)^{-\rho^*_F} G^*_{F,t} \\
y^*_{F,t}(f) &= \left(\frac{p^*_{F,s}(h)}{P^*_{F,t}}\right)^{-\rho^*_F} T_t^{-n} C^U_{T,t} + G^*_{F,t} \qquad \text{(A.51)}
\end{aligned}$$

In the last step we transformed $\frac{p_{F,s}(h)}{P_{F,t}} = \frac{p^*_{F,s}(h)}{P^*_{F,t}}$ by using the LOP as well as $v = v^* = n$.

Aggregate Demand Functions

Aggregate demand functions are obtained by summing over all goods

$$\begin{aligned}
Y^H_t &\equiv \left[\frac{1}{n}\int_0^n (y_{H,t}(h))^{\frac{\rho_H - 1}{\rho_H}} dh\right]^{\frac{\rho_H}{\rho_H - 1}} \\
&= \left[\frac{1}{n}\int_0^n \left(\left(\frac{p_{H,s}(h)}{P_{H,t}}\right)^{-\rho_H} \left((T_t)^{1-n} C^U_{T,t} + G_{H,t}\right)\right)^{\frac{\rho_H - 1}{\rho_H}} dh\right]^{\frac{\rho_H}{\rho_H - 1}} \\
Y^H_t &= T_t^{1-n} C^U_{T,t} + G_{H,t} \qquad \text{(A.52)}
\end{aligned}$$

For foreign, $y^*_{F,t}(f) = \left(\frac{p^*_{F,s}(h)}{P^*_{F,t}}\right)^{-\rho^*_F} T_t^{-n} C^U_{T,t} + G^*_{F,t}$

$$Y_t^F \equiv \left[\frac{1}{1-n}\int_n^1 \left(y^*_{F,t}(f)\right)^{\frac{\rho^*_F-1}{\rho^*_F}} df\right]^{\frac{\rho^*_F}{\rho^*_F-1}} = T_t^{-n} C^U_{T,t} + G^*_{F,t} \tag{A.53}$$

For non-tradables at home and foreign

$$Y_{N,t} = C_{N,t} + G_{N,t}, \quad Y^*_{N^*,t} = C^*_{N,t} + G^*_{N,t} \tag{A.54}$$

A.5 Log-Linear Fluctuations under Flexible Prices

A.5.1 Labour Market Equilibrium in Tradable Sectors

Under flexible prices, output price is set as mark up over real marginal cost and all firms can reset prices. Note that as stressed in section 2.3.1, we will further assume that the steady state under flexible prices is efficient. The exposition here considers the more general case where $\rho_H < \infty$. (A.28) then becomes, by using $MC_t^H(h) = \frac{\rho_H - 1}{\rho_H}\frac{p^*_t(h)}{P^*_{H,t}} = \frac{\rho_H-1}{\rho_H}$

$$\frac{\mathcal{V}_{y_H}(y_{H,t}(h), A_t^H)}{\mathcal{U}_C(C_t)} = (1-\tau^H)\frac{P_{H,t}}{P_t}\frac{\rho_H - 1}{\rho_H} \tag{A.55}$$

We can write $\frac{P_{H,t}}{P_t} = \frac{(Q_t)^{1-\gamma}}{(T_t)^{1-\nu}} = \frac{(Q_t)^{1-\gamma}}{(T_t)^{1-n}}$ where we have imposed that the share of home tradables in the consumption basket equals the country size of the home region (the origin of production of these goods), $v = n$. Hence

$$\begin{aligned} \frac{\mathcal{V}_{y_H}(y_{H,t}(h), A_t^H)}{\mathcal{U}_C(C_t)} &= (1-\tau^H)\frac{(Q_t)^{1-\gamma}}{(T_t)^{1-n}}\frac{\rho_H - 1}{\rho_H} \\ (1-\Phi^H)\mathcal{U}_C(C_t) &= \frac{T_t^{1-n}}{Q_t^{1-\gamma}}\mathcal{V}_{y_H}(y_{H,t}(h), A_t^H) \end{aligned} \tag{A.56}$$

Log-linearise the left hand side, denoting deviations from the flex-price steady state by a tilde

$$(1-\Phi^H)\mathcal{U}_C(C)(1 + \tilde{U}_C(C_t))$$

where $\tilde{U}_C(C_t) = \frac{\mathcal{U}_{CC}(C)C}{\mathcal{U}_C(C)}\tilde{C}_t \equiv -\rho\tilde{C}_t$. Hence $(1-\Phi^H)\mathcal{U}_C(C)(1-\rho\tilde{C}_t)$. Log-linearise the right hand side

$$\frac{T^{1-n}}{Q^{1-\gamma}}\mathcal{V}_{y_H}(Y_H, 0)\left(1 + (1-n)\tilde{T}_t - (1-\gamma)\tilde{Q}_t + \tilde{\mathcal{V}}_{y_H}(y_{H,t}(h), A_t^H)\right)$$

Write $\tilde{\mathcal{V}}_{y_H}(y_{H,t}(h), A_t^H) \equiv \ln\frac{\mathcal{V}_{y_H}(y_{H,t}(h), A_t^H)}{\mathcal{V}_{y_H}(Y_H, 0)}$. Then first-order Taylor approximate about a steady state with $(Y_H, 0)$

$$\begin{aligned} \mathcal{V}_{y_H}(y_{H,t}(h), A_t^H) &= \mathcal{V}_{y_H}(Y_H, 0) + \mathcal{V}_{y_H y_H}(Y_H, 0)(y_{H,t} - Y_H) + \mathcal{V}_{y_H A_H}(Y_H, 0)\,(A_{H,t} - 0) \\ \frac{\mathcal{V}_{y_H}(y_{H,t}(h), A_t^H)}{\mathcal{V}_{y_H}(Y_H, 0)} &= 1 + \frac{\mathcal{V}_{y_H y_H}(Y_H, 0)}{\mathcal{V}_{y_H}(Y_H, 0)}(y_{H,t} - Y_H) + \frac{\mathcal{V}_{y_H A_H}(Y_H, 0)}{\mathcal{V}_{y_H}(Y_H, 0)}A_{H,t} \\ \ln\frac{\mathcal{V}_{y_H}(y_{H,t}(h), A_t^H)}{\mathcal{V}_{y_H}(Y_H, 0)} &= \frac{\mathcal{V}_{y_H y_H}(Y_H, 0)}{\mathcal{V}_{y_H}(Y_H, 0)}(y_{H,t} - Y_H) + \frac{\mathcal{V}_{y_H A_H}(Y_H, 0)}{\mathcal{V}_{y_H}(Y_H, 0)}A_{H,t} \\ \tilde{\mathcal{V}}_{y_H}(y_{H,t}(h), A_t^H) &= \frac{\mathcal{V}_{y_H y_H}(Y_H, 0)}{\mathcal{V}_{y_H}(Y_H, 0)}(y_{H,t} - Y_H) - \frac{\mathcal{V}_{y_H y_H} Y_H}{\mathcal{V}_{y_H}(Y_H, 0)}S_t^H \end{aligned} \tag{A.57}$$

by defining the supply shock $S_t^H \equiv -\frac{\mathcal{V}_{y_H A^H}}{\mathcal{V}_{y_H y_H}} \frac{1}{Y_H} A_t^H$. The log-deviation of $y_{H,t}$ about Y_H is $\tilde{y}_{H,t} \simeq \frac{y_{H,t}-Y_H}{Y_H}$, hence $y_{H,t} - Y_H = Y_H \tilde{y}_{H,t}$. Define η_H as the inverse of the elasticity of labour supply

$$\eta_H \equiv \frac{\mathcal{V}_{y_H y_H}(Y_H, 0) Y_H}{\mathcal{V}_{y_H}(Y_H, 0)}$$

$\frac{\mathcal{V}_{y_H y_H}(Y_H,0)}{\mathcal{V}_{y_H}(Y_H,0)}$ determines the curvature of the labour supply function. Then

$$\begin{aligned} \tilde{\mathcal{V}}_{y_H}(y_{H,t}(h), A_t^H) &= \eta_H \left(\tilde{y}_{H,t} - S_t^H\right) && \text{(A.58)} \\ &= \eta_H \left(\tilde{Y}_{H,t} - S_t^H\right) && \text{(A.59)} \end{aligned}$$

as under flexible prices $y_{H,t}(h) = \left(\frac{p_{H,t}}{P_{H,t}}\right)^{-\rho_H} (C_t^U T_t^{1-n} + G_t^H) = 1(C_t^U T_t^{1-n} + G_t^H) = Y_{H,t}$, and therefore $\tilde{y}_{H,t} = \tilde{Y}_{H,t}$. Analogously for the non-traded goods sector. Putting the left and right hand side together

$$(1 - \Phi^H)\mathcal{U}_C(C)(1 - \rho\tilde{C}_t) = \frac{T^{1-\nu}}{Q^{1-\gamma}} \mathcal{V}_{y_H}(Y_H, 0) \left\{1 + (1-n)\tilde{T}_t - (1-\gamma)\tilde{Q}_t + \eta_H \left(\tilde{Y}_{H,t} - S_t^H\right)\right\}$$

In steady state $(1 - \Phi^H)\mathcal{U}_C(C) = \frac{T^{1-n}}{Q^{1-\gamma}} \mathcal{V}_{y_H}(Y_H, 0)$. Hence

$$-\rho\tilde{C}_t = (1-n)\tilde{T}_t - (1-\gamma)\tilde{Q}_t + \eta_H \left(\tilde{Y}_{H,t} - S_t^H\right) \tag{A.60}$$

For the share of tradables $\gamma = \gamma^* = 1$ we obtain that $\tilde{C}_t^* = \tilde{C}_t$ i.e. real consumption fluctuates one for one across regions and movements of the real exchange rate will not matter. Subtracting the foreign terms from home terms, in order to obtain an expression for the terms of trade under flexible prices $\tilde{T}_t$

$$\begin{aligned} -\rho\tilde{C}_t &= (1-n)\tilde{T}_t - (1-\gamma)\tilde{Q}_t + \eta_H \left(\tilde{Y}_{H,t} - S_t^H\right) \\ &\quad - \left\{-\rho^*\tilde{C}_t^* = -n\tilde{T}_t - (1-\gamma^*)\tilde{Q}_t^* + \eta_F^* \left(\tilde{Y}_{F,t} - S_t^F\right)\right\} \end{aligned}$$

which yields

$$\tilde{T}_t = (1-\gamma)\tilde{Q}_t - (1-\gamma^*)\tilde{Q}_t^* - \eta_H \left(\tilde{Y}_{H,t} - S_t^H\right) + \eta_F \left(\tilde{Y}_{F,t} - S_t^F\right) - \rho\tilde{C}_t + \rho^*\tilde{C}_t^*$$

By assuming that $\eta_H = \eta_F = \eta$ and using the perfect nominal risk sharing condition $-\rho\tilde{C}_t + \tilde{E}_t = -\rho^*\tilde{C}_t$, the expression simplifies to

$$\tilde{T}_t = (1-\gamma)\tilde{Q}_t - (1-\gamma^*)\tilde{Q}_t^* - \eta \left(\tilde{Y}_{H,t} - \tilde{Y}_{F,t} - \left(S_t^H - S_t^F\right)\right) - \tilde{E}_t \tag{A.61}$$

Now use that due to the law of one price in the tradable sector, the real exchange rate can be written as

$$E_t = \frac{Q^{1-\gamma}}{Q^{*1-\gamma^*}}$$

such that the terms of trade are independent of the internal real exchange rates

$$\tilde{T}_t = -\eta \left(\tilde{Y}_{H,t} - \tilde{Y}_{F,t} - \left(S_t^H - S_t^F\right)\right)$$

and further using the sectoral resource constraints under flexible prices

$$\begin{aligned}\tilde{T}_t &= -\eta\left((1-n)\tilde{T}_t + \tilde{C}^U_{T,t} + G_{H,t} - \left(-n\tilde{T}_t + \tilde{C}^U_{T,t} + G_{F,t}\right) - \left(S^H_t - S^F_t\right)\right) \\ \tilde{T}_t &= (-\eta\left(\tilde{T}_t + (G_{H,t} - G_{F,t}) - \left(S^H_t - S^F_t\right)\right)\end{aligned}$$

Using the definition that any relative variable in levels $X^R_t \equiv X^F_t - X^H_t$ one obtains that

$$\tilde{T}_t = \frac{\eta}{1+\eta}\left(G^R_t - S^R_t\right) \tag{A.62}$$

which is equal to the expression for the terms of trade in Benigno (2004, p. 304) for the one-sector case and also equation (2.71) in the text. Note that this expression is a direct consequence of complete risk sharing in nominal consumption spending.

Using the sharing rules between tradable and non-tradable consumption yields the fluctuations in the internal real exchange rate under flexible prices

$$\tilde{Q}_t = \tilde{P}_{T,t} - \tilde{P}_{N,t} = \tilde{C}_{N,t} - \tilde{C}_{T,t} \tag{A.63}$$

$$\tilde{Q}^*_t = \tilde{C}^*_{N^*,t} - \tilde{C}^*_{T,t} \tag{A.64}$$

where $\tilde{P}_{T,t}$ and $\tilde{P}_{N,t}$ denote the log-linear deviations from the period t steady state price levels in the efficient case $\tilde{P}_{T,t} = \ln\left(P_{T,t}/P^{eff}_{T,t}\right)$ and $\tilde{P}_{N,t} = \ln\left(P_{N,t}/P^{eff}_{N,t}\right)$, respectively. We write total home consumption as

$$\tilde{C}_t = \gamma\tilde{C}_{T,t} + (1-\gamma)(\tilde{Q}_t + \tilde{C}_{T,t}) = \tilde{C}_{T,t} + (1-\gamma)\tilde{Q}_t$$

and at foreign

$$\tilde{C}^*_t = \tilde{C}^*_{T,t} + (1-\gamma^*)\tilde{Q}^*_t$$

Continue with (A.60) and use that due to market clearing $\tilde{Y}_{H,t} = (1-n)\tilde{T}_t + \tilde{C}^U_{T,t} + g_{H,t}$

$$\begin{aligned}-\rho\tilde{C}_t &= (1-n)\tilde{T}_t - (1-\gamma)\tilde{Q}_t + \eta_H\left((1-n)\tilde{T}_t + \tilde{C}^U_{T,t} + g_{H,t} - S^H_t\right) \\ -\rho\left(\tilde{C}_{T,t} + (1-\gamma)\tilde{Q}_t\right) &= (1+\eta_H)(1-n)\tilde{T}_t - (1-\gamma)\tilde{Q}_t + \eta_H\tilde{C}^U_{T,t} + \eta_H g_{H,t} - \eta_H S^H_t\end{aligned}$$

Hence home tradables consumption under flexible prices fluctuates according to

$$\tilde{C}_{T,t} = \frac{1-\rho}{\rho}(1-\gamma)\tilde{Q}_t - \frac{1+\eta_H}{\rho}(1-n)\tilde{T}_t - \frac{\eta_H}{\rho}\tilde{C}^U_{T,t} + \frac{\eta_H}{\rho}\left(S^H_t - g^H_t\right) \tag{A.65}$$

which is equation (2.67) in the text. For the foreign tradable sector, start with

$$\frac{\mathcal{V}_{y^*_F}(y^*_{F,t}(f), A_{F,t})}{\mathcal{U}_{C^*}(C^*_t)} = (1-\tau^F)\frac{\rho_F - 1}{\rho_F}\frac{P_{F,t}}{P^*_t}$$

analogously to (A.28). Hence foreign tradables under flexible prices becomes

$$\tilde{C}^*_{T,t} = \frac{1-\rho^*}{\rho^*}(1-\gamma^*)\tilde{Q}^*_t - \frac{1+\eta^*_F}{\rho^*}(-n)\tilde{T}_t - \frac{\eta^*_F}{\rho^*}\tilde{C}^U_{T,t} + \frac{\eta^*_F}{\rho^*}\left(S^F_t - g^F_t\right) \tag{A.66}$$

which is equation (2.68).

A.5.2 Labour Market Equilibrium in Non-Tradable Sectors

Recall the labour market equilibrium in the N sector

$$\frac{\mathcal{V}_{y_N}(y_{N,t}(h), A_t^N)}{\mathcal{U}_C(C_t)} = (1-\tau^N)\frac{P_{N,t}}{P_t}MC_t^N(h)$$

Using that $P_t = P_{T,t}^{\gamma}P_{N,t}^{1-\gamma} = \left(\frac{P_{T,t}}{P_{N,t}}\right)^{\gamma}P_{N,t} = Q_t^{\gamma}P_{N,t}$, $\frac{P_{N,t}}{P_t} = Q_t^{-\gamma}$ and as prices are flexible $MC_t^N(h) = MC_t^N = \frac{\rho_N - 1}{\rho_N}$. Then

$$\begin{aligned}\frac{\mathcal{V}_{y_N}(y_{N,t}(h), A_t^N)}{\mathcal{U}_C(C_t)} &= (1-\tau^N)Q_t^{-\gamma}\frac{\rho_N - 1}{\rho_N} \\ (1-\Phi^N)\mathcal{U}_C(C_t) &= Q_t^{\gamma}\mathcal{V}_{y_N}(y_{N,t}(h), A_t^N)\end{aligned}$$

where we used that $(1-\Phi^N) \equiv (1-\tau^N)\frac{\rho_N-1}{\rho_N}$. Log-linearising yields

$$-\rho\tilde{C}_t = \gamma\tilde{Q}_t + \eta_N\left(\tilde{Y}_{N,t} - S_t^N\right) \tag{A.67}$$

To obtain similar expressions as in case of tradables, (A.63) can be used in the definition for $\tilde{C}_t$

$$\tilde{C}_t = \gamma\tilde{C}_{T,t} + (1-\gamma)\tilde{C}_{N,t} = \gamma\left(\tilde{C}_{N,t} - \tilde{Q}_t\right) + (1-\gamma)\tilde{C}_{N,t} = \tilde{C}_{N,t} - \gamma\tilde{Q}_t$$

As $\tilde{Y}_{N,t} = \tilde{C}_{N,t} + g_{N,t}$, one obtains

$$\begin{aligned}-\rho\left(\tilde{C}_{N,t} - \gamma\tilde{Q}_t\right) &= \gamma\tilde{Q}_t + \eta_N\left(\tilde{C}_{N,t} + g_t^N - S_t^N\right) \\ \tilde{C}_{N,t} &= \gamma\frac{\rho-1}{\rho+\eta_N}\tilde{Q}_t + \frac{\eta_N}{\rho+\eta_N}\left(S_t^N - g_t^N\right) \qquad \text{(A.68)}\end{aligned}$$

and accordingly at foreign

$$\tilde{C}_{N,t}^* = \gamma^*\frac{\rho^*-1}{\rho^*+\eta_{N^*}}\tilde{Q}_t^* + \frac{\eta_{N^*}}{\rho^*+\eta_{N^*}}\left(S_t^{N^*} - g_t^{N^*}\right) \tag{A.69}$$

yielding (2.70) in the text. Weighing the home Euler equation under flexible prices $\tilde{C}_t = E_t\tilde{C}_{t+1} - \frac{1}{\rho}\tilde{\imath}_t$ with the foreign one $\tilde{C}_t^* = E_t\tilde{C}_{t+1}^* - \frac{1}{\rho^*}\tilde{\imath}_t$ by their respective region sizes n and $1-n$, one obtains the Euler equation in the union

$$\tilde{C}_t^U = E_t\tilde{C}_{t+1}^U - \left(n\frac{1}{\rho} + (1-n)\frac{1}{\rho^*}\right)\tilde{\imath}_t$$

Inverting, one obtains $\tilde{\imath}_t = \left(n\frac{1}{\rho} + (1-n)\frac{1}{\rho^*}\right)^{-1}\left(E_t\tilde{C}_{t+1}^U - \tilde{C}_t^U\right)$ which is the efficient/natural rate log-deviation in the interest rate in the absence of nominal and real rigidities.

A.5.3 Price Setting of Firms

Under flexible prices, the optimal pricing decision of an arbitrary firm in the J sector is given by

$$\frac{p_{J,t}^o(z)}{P_{J,t}} = \frac{\rho_J}{\rho_J - 1}MC_t^J$$

and $\frac{p_{J,t}^o(z)}{P_{J,t}} = 1$. Due to the assumed absence of nominal rigidities, also the share of price-indexing firm vanishes $\varpi_J \to 0$ and $P_{J,t}$ denotes the aggregate of optimally set prices. Hence under flexible prices, the output price is a constant markup over real marginal cost (whereas in the text we explicitly considered the special case where $\rho_J \to \infty$). Real marginal cost will therefore not deviate from steady state under flexible prices, such that

$$\widetilde{MC}_t^J = 0$$

We will need this result later on for derivation of the Phillips curve that will be stated in terms of the inefficiency gap $\widehat{MC}_t^J - \widetilde{MC}_t^J$. Log-linearise the labour market equilibrium under flexible prices

$$\begin{aligned}
\frac{\mathcal{V}_{y_H}(y_{H,t}(h), A_t^H)}{\mathcal{U}_C(C_t)} &= (1-\tau^H)\frac{P_{H,t}}{P_t} MC_t^H(h) \\
\frac{\mathcal{V}_{y_H}(y_{H,t}(h), A_t^H)}{\mathcal{U}_C(C_t)} \left((1-\tau^H)\frac{Q_t^{1-\gamma}}{T_t^{1-n}}\right)^{-1} &= MC_t^H(h) \\
MC_t^H(h) &= \frac{\mathcal{V}_{y_H}(y_{H,t}(h), A_t^H)}{\mathcal{U}_C(C_t)} T_t^{1-n} \frac{1}{1-\tau^H} \frac{1}{Q_t^{1-\gamma}}
\end{aligned}$$

Log-linearise the left hand side, using $\widetilde{MC}_t^H(h) = 0$. Log-linearise the right hand side

$$\begin{aligned}
\frac{\mathcal{V}_{y_H}(y_H(h), 0)}{\mathcal{U}_C(C)} \left((1-\tau^H)\frac{Q^{1-\gamma}}{T^{1-n}}\right)^{-1} \begin{pmatrix} \tilde{\mathcal{V}}_{y_H}(y_{H,t}(h), A_t^H) - \tilde{\mathcal{U}}_C(C_t) \\ +(1-n)\tilde{T}_t - (1-\gamma)\tilde{Q}_t \end{pmatrix} &= MC^H(1 + \widetilde{MC}_t^H(h)) \\
\tilde{\mathcal{V}}_{y_H}(y_{H,t}(h), A_t^H) - \tilde{\mathcal{U}}_C(C_t) + (1-n)\tilde{T}_t - (1-\gamma)\tilde{Q}_t &= \widetilde{MC}_t^H(h) \\
\eta_H\left(\tilde{y}_{H,t}(h) - S_t^H\right) + \rho\tilde{C}_t + (1-n)\tilde{T}_t - (1-\gamma)\tilde{Q}_t &= \widetilde{MC}_t^H(h) \qquad \text{(A.70)}
\end{aligned}$$

Log-linearising the demand function under flexible prices yields

$$\tilde{y}_{H,t} = -\rho_H\left(\ln p_{H,t}(h) - \ln p_{H,t}^{ss,flex}(h) - \left(\ln P_{H,t} - \ln P_{H,t}^{ss,flex}\right)\right) + \ln Y_{H,t} - \ln Y_{H,t}^{ss,flex} \quad \text{(A.71)}$$

Note that under flexible prices $p_{H,t}(h) = P_{H,t}$, such that $\tilde{y}_{H,t} = \tilde{Y}_{H,t}$. The individual equilibrium for a firm coincides with the aggregate equilibrium. Therefore

$$\begin{aligned}
0 &= \eta_H\left(\tilde{Y}_{H,t} - S_t^H\right) + \rho\tilde{C}_t + (1-n)\tilde{T}_t - (1-\gamma)\tilde{Q}_t \\
0 &= \frac{1+\eta_H}{1+\rho_H\eta_H}(1-n)\tilde{T}_t - \frac{1-\gamma}{1+\rho_H\eta_H}\tilde{Q}_t + \frac{\eta_H}{1+\rho_H\eta_H}\tilde{C}_{T,t}^U \\
&\quad + \frac{\eta_H}{1+\rho_H\eta_H}\left(g_{H,t} - S_t^H\right) + \frac{\rho}{1+\rho_H\eta_H}\tilde{C}_t
\end{aligned}$$

Bringing real shocks $g_{H,t} - S_t^H$ to the left hand side, one obtains

$$\begin{aligned}
\frac{\eta_H}{1+\rho_H\eta_H}\left(g_{H,t} - S_t^H\right) &= -\frac{1+\eta_H}{1+\rho_H\eta_H}(1-n)\tilde{T}_t + \frac{1-\gamma}{1+\rho_H\eta_H}\tilde{Q}_t \\
&\quad - \frac{\eta_H}{1+\rho_H\eta_H}\tilde{C}_{T,t}^U - \frac{\rho}{1+\rho_H\eta_H}\tilde{C}_t \qquad \text{(A.72)}
\end{aligned}$$

A.6 Log-Linear Fluctuations under Sticky Prices

A.6.1 Euler Equations and National Accounts

Log-linearising the Euler equations under sticky prices yields

$$\hat{C}_t = E_t\hat{C}_{t+1} - \frac{1}{\rho}(\hat{\imath}_t - E_t\pi_{t+1}), \qquad \hat{C}_t^* = E_t\hat{C}_{t+1}^* - \frac{1}{\rho^*}(\hat{\imath}_t - E_t\pi_{t+1}^*)$$

and union overall consumption expenditures are

$$\hat{C}_t^U = \frac{nPC}{P^UC^U}\hat{C}_t + \frac{(1-n)\,P^*C^*}{P^UC^U}\hat{C}_t^* = n\hat{C}_t + (1-n)\,\hat{C}_t^* \tag{A.73}$$

using that by $C = C^*$, $P = P^* = P^U$. Note that this result justifies the aggregation of union variables concerning decisions of households by region size in case of both levels and log-deviations. Union tradable consumption fluctuations are given by

$$\hat{C}_{T,t}^U = \frac{\gamma n}{n\gamma + (1-n)\,\gamma^*}\hat{C}_{T,t} + \frac{(1-n)\,\gamma^*}{n\gamma + (1-n)\,\gamma^*}\hat{C}_{T,t}^*$$

using that the LOP holds for each good in the tradables basket and hence for the overall index and therefore $P = P^* = P^U$. Further, we used the steady state relationship $\frac{C_T^U}{C_T} = n + (1-n)\,\frac{\gamma^*}{\gamma}$ derived under (A.34) and accordingly $\frac{C_T^U}{C_T^*} = n\frac{C_T}{C_T^*} + (1-n) = n\frac{C_T/C}{C_T^*/C^*} + (1-n) = n\frac{\gamma}{\gamma^*} + (1-n)$. Also note that $\frac{C_T^*}{C_T^U} = \left(1 - n\frac{C_T}{C_T^U}\right)\frac{1}{1-n}$. When the share of tradables in overall consumption is same across regions $\gamma^* = \gamma$, we would obtain that $\hat{C}_{T,t}^U = n\hat{C}_{T,t} + (1-n)\hat{C}_{T,t}^*$.

For aggregate output or real union GDP (GDP at constant prices), we obtain that fluctuations are given by

$$\hat{Y}_t^U + \hat{P}^U = \frac{PY}{P^UY^U}\left(\hat{Y}_t + \hat{P}\right) + \left(1 - \frac{PY}{P^UY^U}\right)\left(\hat{Y}_t^* + \hat{P}^*\right) = n\hat{Y}_t + (1-n)\,\hat{Y}_t^* \tag{A.74}$$

where we assumed that economic weight and country size coincide, such that

$$\frac{PY}{P^UY^U} = n \tag{A.75}$$

and that for a variable that is constant through time, its deviation $\hat{X}^U \equiv \ln X^U - \ln X^U = 0$. Within each region, fluctuations in real GDP are given by

$$\begin{aligned}
\hat{Y}_t + \hat{P} &= \hat{Y}_t = \frac{P_HY_H}{PY}\hat{Y}_{H,t} + \left(1 - \frac{P_HY_H}{PY}\right)\hat{Y}_{N,t} \\
\hat{Y}_t^* + \hat{P}^* &= \frac{P_F^*Y_F}{P^*Y^*}\hat{Y}_{F,t} + \left(1 - \frac{P_F^*Y_F}{P^*Y^*}\right)\hat{Y}_{N^*,t}
\end{aligned}$$

where $\frac{P_HY_H}{PY}$ is the share of industry gross value added in overall gross value added in the H region, and $\frac{P_F^*Y_F}{P^*Y^*}$ accordingly for the F region. We also used that constant price levels imply that $\hat{P} = \hat{P}^* = 0$. As a consistency check, union supply needs to equal private sector and public consumption expenditures as well as aggregate savings in levels

$$\begin{aligned}
PY_t + P^*Y_t^* &= nPC_t + (1-n)P^*C_t^* + nPG_t + (1-n)\,P^*G_t^* + nPD_t + (1-n)P^*D_t^* \\
&= nPC_t + (1-n)P^*C_t^* + nPG_t + (1-n)\,P^*G_t^* = P^UC_t^U + P^UG_t^U
\end{aligned}$$

using that $E = 1$ implies $P = P^*$ and that aggregate portfolio holdings are in zero net supply $nPD_t + (1-n)P^*D_t^* = 0$. In steady state we obtained that income equals private sector expenditures

$$P^U Y^U = nPC + (1-n)P^*C^*$$

Therefore

$$\frac{P^*C^*}{P^UY^U} = \left(1 - n\frac{PC}{P^UY^U}\right)\frac{1}{1-n}$$

The aggregate equilibrium can then be written as

$$\begin{aligned}
\hat{Y}_t^U + \hat{P}^U &= \frac{PC}{P^UY^U}n\left(\hat{C}_t + \hat{P}\right) + \left(1 - n\frac{PC}{P^UY^U}\right)\frac{1}{1-n}(1-n)\left(\hat{C}_t^* + \hat{P}^*\right) \\
&\quad + \frac{nP}{P^UY^U}\left(G_t + \hat{P}\right) + \frac{(1-n)P^*}{P^UY^U}\left(G_t^* + \hat{P}^*\right) \\
\hat{Y}_t^U &= n\hat{C}_t + (1-n)\hat{C}_t^* + ng_t + (1-n)g_t^* \qquad \text{(A.76)}
\end{aligned}$$

We used that as $C = C^* = C^U$ and as government shocks are zero in steady state, $Y^U = C^U$. Note that government expenditures are shocks and are therefore already linear and no log-linearisation takes place for these items. Also, the asset market equilibrium on the union level implies $nPD_t + (1-n)P^*D_t^* = 0$. Hence overall public demand in each region is

$$g_t = \frac{P_HY_H}{PY}g_{H,t} + \left(1 - \frac{P_HY_H}{PY}\right)g_{N,t}, \qquad g_t^* = \frac{P_FY_F}{PY}g_{F,t}^* + \left(1 - \frac{P_FY_F}{PY}\right)g_{N^*,t}^*$$

where $g_{H,t} = \frac{G_{H,t}}{Y_{H,t}}$, $g_{N,t} = \frac{G_{N,t}}{Y_{N,t}}$, $g_{F,t}^* = \frac{G_{F,t}^*}{Y_{F,t}}$, $g_{N^*,t}^* = \frac{G_{N^*,t}^*}{Y_{N^*,t}}$.

A.6.2 Real Marginal Cost and Price Setting

Performing the same steps as above, we obtain

$$\begin{aligned}
\hat{\mathcal{V}}_{y_H}(y_{H,t}(h), A_t^H) - \hat{U}_C(C_t) + (1-n)\hat{T}_t - (1-\gamma)\hat{Q}_t &= \widehat{MC}_t^H(h) \\
\eta\left(\hat{y}_{H,t}(h) - S_t^H\right) + \rho\hat{C}_t + (1-n)\hat{T}_t - (1-\gamma)\hat{Q}_t &= \widehat{MC}_t^H(h)
\end{aligned}$$

Under sticky prices, individual output $\hat{y}_{H,t}(h)$ and aggregate output $\hat{Y}_{H,t}$ will diverge due to price dispersion that leads firms to choose different levels of output as an optimum induced by the Calvo pricing mechanism and the presence of rule-of-thumb behaviour by some firms. We have from the demand function for an arbitrary h good

$$\begin{aligned}
\hat{y}_{H,t}(h) &= -\rho_H(\ln p_{H,t}(h) - \ln p_{H,t}^{ss}(h) - (\ln P_{H,t} - \ln P_{H,t}^{ss})) + \hat{Y}_{H,t} \\
&= -\rho_H\left(\hat{p}_{H,t}(h) - \hat{P}_{H,t}\right) + \hat{Y}_{H,t}
\end{aligned}$$

Now follow Beetsma and Jensen (2005a) and define $\hat{p}_{H,t,t+k}(h) \equiv \ln p_{H,t}^o(h) - \ln P_{H,t+k}$ where $p_{H,t}^o(h)$ denotes the price set by an optimising firm in period t, such that for $k = 0$

$$\hat{p}_{H,t,t}(h) \equiv \ln p_{H,t}^o(h) - \ln P_{H,t}$$

and for $k = s$

$$\hat{p}_{H,t,t+s}(h) = \hat{p}_{H,t,t}(h) - \sum_{k=0}^{s}\pi_{H,t+k} \qquad \text{(A.77)}$$

Hence the demand function reads

$$\hat{y}_{J,t,t+s}(h) = -\rho_H \left(\hat{p}_{H,t}(h) - \hat{P}_{H,t,t+s} \right) + \hat{Y}_{H,t} = -\rho_H \hat{p}^o_{H,t,t+s}(h) + \hat{Y}_{H,t}$$

An optimising firm z in sector J maximises the present discounted value of expected profit streams

$$\max_{p^o_{J,t}(z)} \mathcal{E}_t \sum_{s=0}^{\infty} (\theta_J)^s m_{t,t+s} \left[\frac{p^o_{J,t}(z)}{P_{J,t+s}} y_{J,t,t+s}(z) - MC_{H,t,t+s}(z) y_{J,t,t+s}(z) \right]$$

taking into account the demand function for its good $y_{J,t,t+s}(z) = \left(\frac{p_{H,s}(h)}{P_{H,t,t+s}} \right)^{-\rho_H} Y_{H,t,t+s}$ and evolvement of the aggregate price index. Note that as the problem is stated in real, not nominal terms, we used the real stochastic discount factor $m_{t,t+s} = V_{t,t+s} \frac{P_{t,t+s}}{P_t}$ and accordingly $MC_{H,t,t+s}(z)$ is real marginal cost. We obtain that

$$\mathcal{E}_t \sum_{s=0}^{\infty} (\theta_J)^s m_{t,t+s} \left\{ \begin{array}{c} (1 - \rho_H) \frac{y_{J,t,t+s}(z)}{P_{J,t+s}} \\ -MC_{H,t,t+s}(z) \left(-\rho_H \left(\frac{p^o_{H,s}(h)}{P_{H,t,t+s}} \right)^{-\rho_H - 1} \right) Y_{H,t+s} \end{array} \right\} = 0$$

where we have replaced $p_{H,t}(z) \equiv p^o_{H,t}(z)$ in order to indicate the optimality of the price chosen in period t. Rewrite

$$-\mathcal{E}_t \sum_{s=0}^{\infty} (\theta_H) m_{t,t+s} (1 - \rho_H) \frac{y_{J,t,t+s}(z)}{P_{J,t+s}} = \mathcal{E}_t \sum_{s=0H}^{\infty} (\theta_H) m_{t,t+s} \rho_H MC_{H,t,t+s}(z) \frac{y_{J,t,t+s}(z)}{p^o_{H,s}(h)}$$

Multiplying by $p^o_{H,t}(z)$ on both sides

$$-p^o_{H,t}(z)(1 - \rho_H) \mathcal{E}_t \sum_{s=0}^{\infty} (\theta_H)^s m_{t,t+s} \frac{y_{J,t,t+s}(z)}{P_{J,t+s}} = \rho_H \mathcal{E}_t \sum_{s=0H}^{\infty} (\theta_H)^s m_{t,t+s} MC_{H,t,t+s}(z) y_{J,t,t+s}(z)$$

from which we obtain as a first intermediate result

$$p^o_{H,t}(z) = \frac{\rho_H}{\rho_H - 1} \frac{\mathcal{E}_t \sum_{s=0}^{\infty} (\theta_H)^s m_{t,t+s} MC_{H,t,t+s}(z) y_{J,t,t+s}(z)}{\mathcal{E}_t \sum_{s=0}^{\infty} (\theta_H)^s m_{t,t+s} \frac{y_{J,t,t+s}(z)}{P_{J,t+s}}}$$

Plugging in the real stochastic discount factor $m_{t,t+s} = \beta^s \frac{\mathcal{U}_C(C_{t+s})}{\mathcal{U}_C(C_s)}$ yields equation (2.34) in the text

$$p^o_{H,t}(z) = \frac{\mathcal{E}_t \sum_{s=0}^{\infty} (\theta_H \beta)^s \mathcal{U}_C(C_{t+s}) MC_{H,t,t+s}(z) y_{J,t,t+s}(z)}{\mathcal{E}_t \sum_{s=0}^{\infty} (\theta_H \beta)^s \mathcal{U}_C(C_{t+s}) \frac{y_{J,t,t+s}(z)}{P_{J,t+s}}} \tag{A.78}$$

Real marginal cost for a firm z in sector J at home is defined by

$$MC_{J,t,t+s}(z) = \frac{\mathcal{V}_{y_J}(y_{J,t+s}(h), A^J_{t+s})}{\mathcal{U}_C(C_{t+s})} \frac{1}{1 - \tau^J} \frac{P_{t+s}}{P_{J,t+s}}$$

We follow Benigno and López-Salido (2006) and define *average* real marginal cost as

$$MC_{J,t+s} = \frac{\mathcal{V}_{y_J}(Y_{J,t+s}, A^J_{t+s})}{\mathcal{U}_C(C_{t+s})} \frac{1}{1 - \tau^J} \frac{P_{t+s}}{P_{J,t+s}}$$

obtained by aggregating $MC_{J,t,t+s}(z)$ over all firms in sector J. We need average cost, as we want to state the Phillips curve for the economy as a whole, and hence aggregate over marginal cost of all firms. We note the relation

$$y_{J,t,t+s}(z) = \left(\frac{p_{H,s}(h)}{P_{H,t}} \frac{P_{H,t}}{P_{H,t+s}}\right)^{-\rho_H} Y_{H,t+s}$$

Continuing with above expression and using the labour market equilibrium

$$\begin{aligned}
\mathcal{E}_t \sum_{s=0}^{\infty} (\theta_J \beta)^s \left\{ \left[\frac{p^o_{J,t}(z)}{P_{J,t+s}} \mathcal{U}_c\left(C_{t+s}\right) - \frac{\rho_J}{\rho_J - 1} MC_{J,t,t+s}(z) \mathcal{U}_c\left(C_{t+s}\right) \right] y_{J,t,t+s}(z) \right\} &= 0 \\
\mathcal{E}_t \sum_{s=0}^{\infty} (\theta_J \beta)^s \left\{ \left[\frac{p^o_{J,t}(z)}{P_{J,t+s}} \mathcal{U}_c\left(C_{t+s}\right) - \frac{\rho_J}{\rho_J - 1} \mathcal{V}_{y_J}(y_{J,t+s}(h), A^J_{t+s}) \frac{1}{1-\tau^J} \frac{P_{t+s}}{P_{J,t+s}} \right] y_{J,t,t+s}(z) \right\} &= 0
\end{aligned}$$

Take a log-linear approximation about a steady state where $\frac{p^o_{J,t}(z)}{P_{J,t}} = 1$ and $\frac{P_{J,t+s}}{P_{J,t}} = 1$. Following Beetsma and Jensen (2005a, appendix, p. 15) define $\hat{p}^o_{J,t,t+s}(h) = \ln p^o_{J,t}(h) - \ln P_{J,t+s}$ and use the definition for $\hat{y}_{J,t,t+s}(h)$. Also steady state terms will drop out.

$$\begin{aligned}
0 &= \mathcal{E}_t \sum_{s=0}^{\infty} (\theta_J \beta)^s \left\{ \begin{array}{c} \hat{p}^o_{H,t,t+s}(h) - \rho \hat{C}_{t+k} + \hat{y}_{J,t,t+s}(h) - \eta_H \left[\hat{y}_{J,t,t+s}(h) - S_{H,t}\right] \\ - \left(\hat{P}_{t+s} - \hat{P}_{J,t+s}\right) - \hat{y}_{J,t,t+s}(h) \end{array} \right\} \\
0 &= \mathcal{E}_t \sum_{s=0}^{\infty} (\theta_J \beta)^s \left\{ \hat{p}^o_{H,t,t+s}(h) - \rho \hat{C}_{t+k} - \eta_H \left[\hat{y}_{J,t,t+s}(h) - S_{H,t}\right] - \left(\hat{P}_{t+s} - \hat{P}_{J,t+s}\right) \right\}
\end{aligned}$$

Using $\hat{y}_{J,t,t+s}(h) = -\rho_H \hat{p}^o_{H,t,t+s}(h) + \hat{Y}_{H,t}$ and $\hat{p}^o_{H,t,t+s}(h) = \hat{p}_{H,t,t}(h) - \sum_{k=0}^{s} \pi_{H,t+k}$ such that eventually

$$\frac{\hat{p}_{H,t,t}(h)}{1 - \theta_J \beta} = \mathcal{E}_t \sum_{k=0}^{\infty} (\theta_J \beta)^s \left(\begin{array}{c} \left(\hat{P}_{t+s} - \hat{P}_{J,t+s}\right) + \frac{\rho}{1+\eta_H \rho_H} \hat{C}_{t+k} \\ + \frac{\eta_H}{1+\eta_H \rho_H} \left[\hat{Y}_{H,t+k} - S_{H,t+k}\right] \end{array} \right) + \mathcal{E}_t \sum_{k=1}^{\infty} (\theta_J \beta)^s \pi_{H,t+k}$$

Log-linearise the average real marginal cost term

$$\widehat{MC}_{J,t+s} = \eta_H \left[\hat{Y}_{H,t+k} - S_{H,t+k}\right] + \rho \hat{C}_{t+k} + \left(\hat{P}_{t+s} - \hat{P}_{J,t+s}\right)$$

Hence the expression in brackets can be written in terms of the log-approximation of average real marginal cost as

$$\frac{\hat{p}_{H,t,t}(h)}{1 - \theta_J \beta} = \mathcal{E}_t \sum_{s=0}^{\infty} (\theta_J \beta)^s \frac{\widehat{MC}_{J,t+s}}{1 + \eta_J \rho_J} + \mathcal{E}_t \sum_{s=1}^{\infty} (\theta_J \beta)^s \pi_{J,t+k} \tag{A.79}$$

In the following, define $mc_{J,t} \equiv \frac{\widehat{MC}_{J,t+s}}{1+\eta_J \rho_J}$. Further, exogenous shocks g^H_t, S^H_t are same whether a flex or a sticky-price approximation is taken. We therefore can employ the expression for $\frac{\eta_H}{1+\rho_H \eta_H}\left(g^H_t - S^H_t\right)$ for flexible prices derived under (A.72) to obtain that in case $J = H$, $\hat{P}_{t+s} - \hat{P}_{H,t+s} = (1-n)\hat{T}_t - (1-\gamma)\hat{Q}_t$. Hence

$$\begin{aligned}
mc_{H,t} &= \frac{1+\eta_H}{1+\rho_H \eta_H}(1-n)(\hat{T}_t - \tilde{T}) - \frac{1-\gamma}{1+\rho_H \eta_H}(\hat{Q}_t - \tilde{Q}_t) \\
&\quad + \frac{\eta_H}{1+\rho_H \eta_H}(\hat{C}^U_{T,t} - \tilde{C}^U_{T,t}) + \frac{\rho}{1+\rho_H \eta_H}\left(\hat{C}_t - \tilde{C}_t\right)
\end{aligned} \tag{A.80}$$

Analogously at foreign

$$
\begin{aligned}
mc_{F,t} &= -n\frac{1+\eta_F}{1+\rho_F\eta_F}(\hat{T}_t-\tilde{T})-\frac{1-\gamma^*}{1+\rho_F\eta_F}(\hat{Q}^*_t-\tilde{Q}^*_t) \\
&\quad+\frac{\eta_F}{1+\rho_F\eta_F}(\hat{C}^U_{T,t}-\tilde{C}^U_{T,t})+\frac{\rho^*}{1+\rho_F\eta_F}(\hat{C}^*_t-\tilde{C}^*_t)
\end{aligned}
\tag{A.81}
$$

Again substituting out expressions that are same under flex- and sticky prices $\frac{\eta_N}{1+\rho_N\eta_N}\left(g^N_t-S^N_t\right)$ at home and accordingly at foreign, one obtains

$$
\begin{aligned}
mc_{N,t} &= \frac{\eta_N}{1+\rho_N\eta_N}(\hat{C}_{N,t}-\tilde{C}_{N,t})+\frac{\rho}{1+\rho_N\eta_N}\left(\hat{C}_t-\tilde{C}_t\right)+\frac{\gamma}{1+\rho_N\eta_N}(\hat{Q}_t-\tilde{Q}_t) \\
mc_{N^*,t} &= \frac{\eta_{N^*}}{1+\rho_{N^*}\eta_{N^*}}(\hat{C}^*_{N^*,t}-\tilde{C}^*_{N^*,t})+\frac{\rho^*}{1+\rho_{N^*}\eta_{N^*}}\left(\hat{C}^*_t-\tilde{C}^*_t\right)+\frac{\gamma^*}{1+\rho_{N^*}\eta_{N^*}}(\hat{Q}^*_t-\tilde{Q}^*_t)
\end{aligned}
$$

The equations resemble (2.75) to (2.78) in the text.

A.6.3 Hybrid Inflation Dynamics

The price set by a backward-looking firm in sector H is

$$P^b_{H,t}=P^{\#}_{H,t-1}\frac{P_{t-1}}{P_{t-2}} \tag{A.82}$$

The aggregate price level then evolves according to

$$P_{H,t}=\left[(1-\theta_H)P^{\#,1-\rho_H}_{H,t}+\theta_H P^{1-\rho_H}_{H,t-1}\right]^{\frac{1}{1-\rho_H}} \tag{A.83}$$

$1-\theta_H$ is the exogenous probability that a firm can re-set prices in t which is then split between firms that optimise and those that do not when selecting a new output price. Analogous relationships hold in the N sector and for tradables production at foreign. $P^{\#}_{H,t}$ is an index of newly set prices by forward- and backward-looking firms given by

$$P^{\#}_{H,t}=\left[(1-\omega_H)P^{f,1-\rho_H}_{H,t}+\omega_H P^{b,1-\rho_H}_{H,t}\right]^{\frac{1}{1-\rho_H}} \tag{A.84}$$

The derivation of the New Keynesian Phillips-curve follows Holmsberg (2006). See Amato and Laubach (2003) for the definition of the indices used here. Log-linearising (A.82), (A.83), and (A.84) around P_t yields

$$
\begin{aligned}
p^b_{H,t} &= p^{\#}_{H,t-1}+\pi_{H,t-1}-\pi_{H,t} && \text{(A.85)}\\
p_{H,t} &= (1-\theta_H)\,p^{\#}_{H,t}+\theta_H p_{H,t-1} && \text{(A.86)}\\
p^{\#}_{H,t} &= (1-\omega_H)\,p^f_{H,t}+\omega_H p^b_{H,t} && \text{(A.87)}
\end{aligned}
$$

Further, as $\ln P_{H,t}=\theta_H\ln P_{H,t-1}+(1-\theta_H)\ln P^{\#}_{H,t}$ and as we denote $p^{\#}_{H,t}=\ln P^{\#}_{H,t}-\ln P_{H,t}$, we obtain

$$p^{\#}_{H,t}=\frac{\theta_H}{1-\theta_H}\pi_{H,t}$$

Forward-looking firms set prices à la Calvo (1983)

$$p^f_{H,t}=(1-\theta_H\beta)(mc^H_t+p_{H,t})+\theta_H\beta E_t[p^f_{H,t+1}] \tag{A.88}$$

where mc_t^H denotes the log-deviation of real marginal cost under constant returns to scale. We used that

$$p_{H,t}^f = \hat{V}_t + p_{H,t} = \ln \frac{P_{H,t}^f}{P_{H,t}} + p_{H,t} = p_{H,t}^f$$

$\hat{V}_t$ is obtained from the aggregate price level as in Walsh (2003, appendix to chapter 5). Start with

$$\begin{aligned} P_{H,t} &= \left[(1-\theta_H) \left(P_{H,t}^{\#}(z) \right)^{1-\rho_H} + \theta_H P_{H,t-1}^{1-\rho_H} \right]^{\frac{1}{1-\rho_H}} \\ 1 &= (1-\theta_H) V_t^{(1-\rho_H)} + \theta_H (\frac{P_{H,t-1}}{P_{H,t}})^{(1-\rho_H)} \end{aligned}$$

The optimal price $P_{H,t}^{\#}(z)$ set by a forward-looking firm is labeled $P_{H,t}^{\#}(z) = P_{H,t}^f(z)$. Hence, by log-linearising around a constant steady state price level $P_t = P$ one obtains $\hat{V}_t = p_{H,t}^f(z) - p_{H,t}$. Newly set prices consist of a share of ϖ_H prices set by backward-looking firms denoted by $p_{H,t}^b$ and a share $1-\varpi_H$ of prices set by Ricardian firms denoted by $p_{H,t}^f$

$$p_{H,t}^{\#} = (1-\omega_H) p_{H,t}^f + \omega_H p_{H,t}^b \tag{A.89}$$

For backward-looking firms

$$p_{H,t}^b - p_{H,t} = -\pi_{H,t} + \frac{1}{1-\theta_H} \pi_{H,t-1} \tag{A.90}$$

Solve (A.86) for $p_{H,t}^{\#}$ and combining with (A.89)

$$\frac{\theta_H}{1-\theta_H} \pi_{H,t} = (1-\omega_H) \left(p_{H,t}^f - p_{H,t} \right) + \omega_H \left(p_{H,t}^b - p_{H,t} \right) \tag{A.91}$$

From (A.88) it follows that

$$p_{H,t}^f - p_{H,t} = (1-\theta_H \beta) mc_t^H - \theta_H \beta p_{H,t} + \theta_H \beta E_t [p_{H,t+1}^f] \tag{A.92}$$

Solving (A.91) for $p_{H,t}^f$, leading the equation one period and plugging in (A.90)

$$\mathcal{E}_t p_{H,t+1}^f = \frac{\theta_H + \omega_H (1-\theta_H)}{(1-\theta_H)(1-\omega_H)} E_t \pi_{H,t+1} - \frac{\omega_H}{1-\omega_H} \frac{1}{1-\theta_H} \pi_{H,t} + E_t p_{H,t+1}$$

Plugging this expression in (A.92)

$$\begin{aligned} p_{H,t}^f - p_{H,t} &= (1-\theta_H \beta) mc_t^H \\ &\quad + \theta_H \beta \left(1 + \frac{\theta_H + \omega_H (1-\theta_H)}{(1-\theta_H)(1-\omega_H)} \right) \mathcal{E}_t \pi_{H,t+1} - \frac{\omega_H}{1-\omega_H} \frac{\theta_H \beta}{1-\theta_H} \pi_{H,t} \end{aligned} \tag{A.93}$$

Using (A.93) and (A.90) in (A.91) leads to

$$\begin{aligned} \pi_{H,t} &= \frac{\omega_H}{\theta_H + \omega_H (1-\theta_H(1-\beta))} \pi_{H,t-1} + \frac{(1-\theta_H)(1-\omega_H)(1-\theta_H \beta)}{\theta_H + \omega_H (1-\theta_H(1-\beta)} mc_t^H \\ &\quad + \frac{1-\theta_H}{\theta_H + \omega_H (1-\theta_H(1-\beta))} (1-\omega_H) \theta_H \beta \left(1 + \frac{\theta_H + \omega_H (1-\theta_H)}{(1-\theta_H)(1-\omega_H)} \right) \mathcal{E}_t \pi_{H,t+1} \end{aligned}$$

Hence

$$\begin{aligned}\pi_{H,t} &= \frac{\omega_H}{\theta_H+\omega_H\left(1-\theta_H(1-\beta)\right)}\pi_{H,t-1}+\frac{\left(1-\theta_H\right)\left(1-\omega_H\right)(1-\theta_H\beta)}{\theta_H+\omega_H\left(1-\theta_H(1-\beta)\right)}mc_t^H \\ &+\frac{\theta_H\beta}{\theta_H+\omega_H\left(1-\theta_H(1-\beta)\right)}\mathcal{E}_t\pi_{H,t+1}\end{aligned}$$

We end up with with the hybrid Phillips curves

$$\pi_{H,t} = \lambda_H^b\pi_{H,t-1}+\lambda_H^{mc}mc_t^H+\lambda_H^f\mathcal{E}_t\pi_{H,t+1} \tag{A.94}$$

$$\pi_{N,t} = \lambda_N^b\pi_{N,t-1}+\lambda_N^{mc}mc_t^N+\lambda_N^f\mathcal{E}_t\pi_{N,t+1} \tag{A.95}$$

which is a sectoral version of equation A.29 in Holmsberg (2006). Reduced and structural parameters are linked by $\lambda_{H,t}^b=\frac{\omega_H}{\theta_H+\omega_H(1-\theta_H(1-\beta))}$, $\lambda_{H,t}^{mc_H}=\frac{(1-\omega_H)(1-\theta_H)(1-\theta_H\beta)}{\omega_H(1-\theta_H+\theta_H\beta)+\theta_H}$, $\lambda_{H,t}^f=\frac{\beta\theta_H}{\omega_H}\lambda_{H,t}^b$ and analogously in the N sector. For the share of backward-looking firms $\omega_H,\omega_N\to 0$ we obtain the forward-looking New Keynesian Phillips curves.

CPI Inflation

The CPI price level at home is given by (2.5). We link home CPI inflation with the ToT and the internal real exchange rate

$$\begin{aligned}\frac{P_{H,t}}{P_t} &= \frac{(Q_t)^{1-\gamma}}{(T_t)^{1-\nu}}=\frac{(Q_t)^{1-\gamma}}{(T_t)^{1-n}} \\ \hat{P}_{H,t}-\hat{P}_t &= (1-\gamma)\hat{Q}_t-(1-n)\hat{T}_t \\ \pi_{H,t}-\pi_t &= (1-\gamma)(\hat{Q}_t-\hat{Q}_{t-1})-(1-n)(\hat{T}_t-\hat{T}_{t-1})\end{aligned}$$

A.6.4 Log-Linearising the Current Account

The home region's external balance is given by

$$\frac{\mathcal{E}_tD_{t,t+1}}{P_t}\frac{1}{1+i_t}=\frac{D_{t-1,t}}{P_t}+Q_t^{1-\gamma}\left[C_{T,t}^U-C_{T,t}\right]$$

and therefore

$$\mathcal{E}_tD_{t,t+1}^r\frac{1}{1+i_t}=D_{t-1,t}^r\frac{1}{1+\pi_t}+Q_t^{1-\gamma}\left[C_{T,t}^U-C_{T,t}\right]$$

In a steady state where $Q=1$, $\pi=0$ and $\frac{1}{1+i}=\beta$, one obtains

$$D^r=\frac{1}{\beta-1}\left(C_T^U-C_T\right) \tag{A.96}$$

Log-linearise about this steady state. Use that $\widehat{\frac{1}{1+i_t}}\equiv\ln\left[1+i_t\right]^{-1}-\ln\left[1+i\right]^{-1}=-\left(i_t-i\right)=-\hat{\imath}$ which denotes the deviation of the interest rate from its steady state in levels (in percentage points instead of percent). Analogously in case of the inflation rate $\widehat{\frac{1}{1+\pi_t}}=\ln\left[1+\pi_t\right]^{-1}-\ln\left[1+\pi\right]^{-1}=-\pi_t+0=-\pi_t$. Hence eventually

$$\begin{aligned}\mathcal{E}_t\hat{D}_{t,t+1}^r &= \hat{\imath}+\frac{1}{\beta}\left(\hat{D}_{t-1,t}^r-\pi_t\right)+\frac{\beta-1}{\beta}\left(1-\frac{C_T}{C_T^U}\right)^{-1}\hat{C}_{T,t}^U \\ &-\frac{\beta-1}{\beta}\left(\left(\frac{C_T}{C_T^U}\right)^{-1}-1\right)^{-1}\hat{C}_{T,t}+\frac{\beta-1}{\beta}(1-\gamma)\hat{Q}_t\end{aligned} \tag{A.97}$$

where $\frac{C_T}{C_T^U} = \left(n + (1-n)\frac{\gamma^*}{\gamma}\right)^{-1}$ for the steady state assumptions $Q = Q^* = 1$ as well as $C = C^*$. As the financial market needs to clear on the union level, we can determine foreign's real external assets from the union asset market equilibrium and deflating by the Home CPI index P_t

$$0 = n\frac{\mathcal{E}_t D_{t,t+1}}{P_t} + (1-n)\frac{\mathcal{E}_t D^*_{t,t+1}}{P^*_t}\frac{P^*_t}{P_t}$$

One obtains that

$$0 = n\frac{D^H}{P}\mathcal{E}_t\hat{D}^r_{t,t+1} + (1-n)\frac{D^F}{P^*}E\left(\mathcal{E}_t\hat{D}^{r*}_{t,t+1} + \hat{E}_t\right)$$

leading to $\mathcal{E}_t\hat{D}^{r*}_{t,t+1} + \hat{E}_t = \mathcal{E}_t\hat{D}^r_{t,t+1}$ Note that under incomplete financial markets when there are only available non-contingent bonds B_t and B^*_t, the same derivation would go through with $\mathcal{E}_t D_{t,t+1} = B_t$ and $\mathcal{E}_t D^*_{t,t+1} = B^*_t$. The log-deviation in the real exchange rate is given by

$$\hat{E}_t = (1-\gamma)\hat{Q}_t - (1-\gamma^*)\hat{Q}^*_t \tag{A.98}$$

As shown before, fluctuations in the real exchange rate are driven by fluctuations in internal real exchange rates. From the risk sharing condition we obtain that

$$-\rho\hat{C}_t + \hat{E}_t = -\rho^*\hat{C}^*_t \tag{A.99}$$

where the degree of overall risk aversion within each region is given by ρ and ρ^*. Movements in the real exchange rate $\hat{E}_t$ prevent fluctuations in overall consumption from equalising across regions.

A.7 Welfare Analysis

A.7.1 Expanding the Utility of Consumption Term

In general, a second-order Taylor approximation for a function f around x_0 is given by

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2}(x-x_0)^2 + (o||\xi||^3)$$

hence for the first argument in (2.1)

$$\mathcal{U}(C_t) = \mathcal{U}(C) + \mathcal{U}_C(C)(C_t - C) + \frac{\mathcal{U}_{CC}(C)}{2}(C_t - C)^2 + (o||\xi||^3) \tag{A.100}$$

were C is the level of consumption in the non-stochastic steady state. We make repeated use of two approximations. Let X_t be a generic variable, restricted to assume positive values. The arithmetic percentage change in X_t has the following relation to the logarithmic percentage change:

$$\frac{X_t - X}{X} = \frac{dX_t}{X} \simeq \hat{X}_t + \frac{1}{2}\hat{X}^2_t, \qquad \hat{X}_t \equiv \ln(X_t/X)$$

Let $\tilde{X}_t = X_t - X$ denote the deviation of X_t from its steady state value X. By definition $\hat{X}_t \equiv \ln\frac{X_t}{X}$ is the log deviation of X_t about X. Note that we could - up to first order - also write $\hat{X}_t \equiv d\ln X_t = dX_t/X$ when evaluating at $X_t = X$ (see for example Obstfeld and Rogoff (1996, 208)). Write the change in the level of X_t as $\tilde{X}_t$

$$\tilde{X}_t = X_t - X = X(\frac{X_t - X}{X})$$

Take a second-order Taylor approximation of $\ln X_t$ about $\ln X$

$$\begin{aligned}\ln X_t &= \ln X + \frac{d}{dX_t}[\ln X_t]_{X_t=X}(X_t - X) + \frac{1}{2}\frac{d}{dX_t^2}[\ln X_t]_{X_t=X}(X_t - X)^2 + (o||\xi||^3)\\ \hat{X}_t &= \frac{X_t - X}{X} - \frac{1}{2}(\frac{X_t - X}{X})^2 + (o||\xi||^3) \qquad \text{(A.101)}\end{aligned}$$

as $\frac{d}{dX_t}[\ln X_t]_{X_t=X} = \frac{1}{X}$, $\frac{d}{dX_t^2}[\ln X_t]_{X_t=X} = -\frac{1}{X^2}$ where we evaluated at $X_t = X$. Therefore, apply this approximation twice in above definition of X_t

$$X_t = X\left(1 + \hat{X}_t + \frac{1}{2}\hat{X}_t^2\right) + (o||\xi||^3) \simeq X\left(1 + \hat{X}_t + \frac{1}{2}\hat{X}_t^2\right) \qquad \text{(A.102)}$$

where terms $\frac{1}{2}\hat{X}_t(\frac{X_t-X}{X})^2$, $\frac{1}{4}(\frac{X_t-X}{X})^4$, and terms involving products between $\hat{X}_t$, $\frac{1}{2}(\frac{X_t-X}{X})^2$, and $(o||\xi||^3)$ are of order higher than two. Hence applied to the level of consumption

$$C_t = C(1 + \hat{C}_t + \frac{1}{2}\hat{C}_t^2) + (o||\xi||^3) \qquad \text{(A.103)}$$

Substituting (A.103) in (A.100)

$$\mathcal{U}(C_t) = \mathcal{U}_C(C)C\left(\hat{C}_t + \frac{1}{2}\hat{C}_t^2 + \frac{1}{2}\frac{\mathcal{U}_{CC}(C)}{\mathcal{U}_C(C)}C\hat{C}_t^2\right) + t.i.p. + (o||\xi||^3)$$

We can define the (negative) of the relative parameter of risk aversion as $\frac{\mathcal{U}_{CC}(C)}{\mathcal{U}_C(C)}C = -\rho$ to eventually obtain

$$\mathcal{U}(C_t) = \mathcal{U}_C(C)C\left(\hat{C}_t + \frac{1}{2}\hat{C}_t^2(1-\rho)\right) + t.i.p. + (o||\xi||^3) \qquad \text{(A.104)}$$

A.7.2 Expanding the Disutility of Labour Effort Terms

For the approximation of the other terms of the utility function in (A.100) note that for a scalar rational function $f(x_t, z_t)$

$$\begin{aligned}f(x_t, z_t) &= f(x, z) + f_{x_t}(x_t - x) + f_{z_t}(z_t - z) + \frac{1}{2}f_{x_t x_t}(x_t - x)^2 + \\ &+\frac{1}{2}f_{x_t z_t}(x_t - x)(z_t - z) + \frac{1}{2}f_{z_t x_t}(x_t - x)(z_t - z) + \frac{1}{2}f_{z_t z_t}(z_t - z)^2 + (o||\xi||^3)\end{aligned}$$

note that $f_{x_t z_t} = f_{z_t x_t}$ by Young's theorem. Expanding $v(y_{H,t}(h), A_t^H)dh$ around a steady state where $y_H(h) = Y_H$ and the productivity shock $A^H = 0$

$$\begin{aligned}& v(y_{H,t}(h), A_t^H)\\ =\;& v(y_H, 0) + v_{y_H}(y_{H,t}(h) - Y^H) + v_{A^H}A_t^H + \frac{1}{2}v_{y_H y_H}(y_{H,t}(h) - Y^H)^2\\ & + v_{y_H A^H}(y_{H,t}(h) - Y^H)A_t^H + \frac{1}{2}v_{A^H A^H}(A_t^H)^2 + (o||\xi||^3)\\ =\;& v_{y_H}y_{H,t}(h) + \frac{1}{2}v_{y_H y_H}(y_{H,t}(h) - Y^H)^2 + v_{y_H A^H}y_{H,t}(h)A_t^H\\ & + t.i.p. + (o||\xi||^3) \qquad \text{(A.105)}\end{aligned}$$

Aggregate demand for good h in the H sector, $y_{H,t}(h)$ consists of private sector demand and public sector demand

$$y_{H,t}(h) \equiv y_{H,t}^d(h) + y_{H,t}^g(h) \qquad \text{(A.106)}$$

where the specific demand functions

$$y^d_{H,t}(h) = \left(\frac{p_{H,t}(h)}{P_{H,t}}\right)^{-\rho_H} (T_t)^{1-v} C^W_t, \qquad y^g_{H,t}(h) = \left(\frac{p_{H,t}(h)}{P_{H,t}}\right)^{-\rho_H} G_{H,t}$$

In deriving government demand, it is assumed that government minimises the cost of producing $G_{H,t}$. Take a second order approximation of $y_{H,t}(h)$ about Y_H

$$y_{H,t}(h) = Y_H \left(1 + \hat{y}_{H,t}(h) + \frac{1}{2}(\hat{y}_{H,t}(h))^2\right) + (o||\xi||^3)$$

Substituting out $y_{H,t}(h)$ in (A.105) leads to

$$\begin{aligned} v(y_{H,t}(h), A^H_t) &= v_{y_H} Y_H \left(\hat{y}_{H,t}(h) + \frac{1}{2}(\hat{y}_{H,t}(h))^2 + \frac{1}{2}\frac{v_{y_H y_H}}{v_{y_H}} Y^H (\hat{y}_{H,t}(h))^2 + \frac{v_{y_H A^H}}{v_{y_H}} \hat{y}_{H,t}(h) A^H_t\right) \\ &\quad + t.i.p. + (o||\xi||^3) \end{aligned}$$

As $\eta_H \equiv \frac{V_{y_H y_H}(Y_H,0)}{V_{y_H}(Y_H,0)} Y_H$ and as in steady state $v_{y_H} = \mathcal{V}_{y_H}$ and therefore $v_{y_H y_H} = \mathcal{V}_{y_H y_H}(Y_H, 0)$

$$\begin{aligned} &v(y_{H,t}(h), A^H_t) = v_{y_H} Y_H \left(\hat{y}_{H,t}(h) + \frac{1}{2}(\hat{y}_{H,t}(h))^2 + \frac{\eta_H}{2}(\hat{y}_{H,t}(h))^2 + \frac{v_{y_H A^H}}{v_{y_H}} \hat{y}_{H,t}(h) A^H_t\right) \\ &+ t.i.p. + (o||\xi||^3) \end{aligned}$$

Earlier, we defined a supply shock in the home tradable sector S^H_t to be proportional to the TFP shock A^H_t

$$S^H_t \equiv -\frac{v_{y_H A^H}}{v_{y_H y_H}} \frac{1}{Y_H} A^H_t$$

Therefore $v_{y_H A^H} = -S^H_t Y_H v_{y_H y_H} \frac{1}{A^H_t}$ and

$$\frac{v_{y_H A^H}}{v_{y_H}} \hat{y}_{H,t}(h) A^H_t = -S^H_t Y_H \frac{v_{y_H y_H}}{v_{y_H}} \hat{y}_{H,t}(h) = -\eta_H S^H_t \hat{y}_{H,t}(h)$$

Hence

$$\begin{aligned} &v(y_{H,t}(h), A^H_t) \\ =\; & v_{y_H} Y_H \left(\hat{y}_{H,t}(h) + \frac{1+\eta_H}{2}(\hat{y}_{H,t}(h))^2 - \eta_H S^H_t \hat{y}_{H,t}(h)\right) + t.i.p. + (o||\xi||^3) \end{aligned}$$

and analogously for the non-traded sector. Use the steady state labour market equilibrium given by (A.35) and (A.36)

$$(1-\Phi^H) T^{n-1} Q^{1-\gamma} U_C(C) = V_{y_H}(y_H(h), A^H), \qquad (1-\Phi^N) Q^{-\gamma} U_C(C) = V_{y_N}(y_N(h), A^N) \tag{A.107}$$

to substitute out marginal disutility of labour in each sector

$$\begin{aligned} v(y_{H,t}(h), A^H_t) &= \mathcal{U}_C(C) T^{n-1} Q^{1-\gamma} Y_H \begin{pmatrix} (1-\Phi^H)\hat{y}_{H,t}(h) + \frac{1}{2}(\hat{y}^d_{H,t}(h))^2 \\ + \frac{\eta_H}{2}(\hat{y}_{H,t}(h))^2 - \eta_H S^H_t \hat{y}_{H,t}(h) \end{pmatrix} \\ &\quad + t.i.p + (o||\xi||^3) \end{aligned}$$

where the terms $T^{n-1}Q^{1-\gamma}\Phi^H\frac{1+\eta_H}{2}(\hat{y}_{H,t}(h))^2U_C(C)C_T$ and $T^{n-1}Q^{1-\gamma}\Phi^H\eta_H S_t^H\hat{y}_{H,t}(h)$ are at least of order $(o||\xi||^3)$. Integrating the preceding expressions over the home population

$$\begin{aligned}
&\frac{1}{n}\int_0^n v(y_{H,t}(h), A_t^H)dh \\
=\ & \mathcal{U}_C(C)T^{n-1}Q^{1-\gamma}Y_H\begin{pmatrix} (1-\Phi^H)\frac{1}{n}\int_0^n \hat{y}_{H,t}(h)dh \\ +\frac{1+\eta_H}{2}\frac{1}{n}\int_0^n(\hat{y}_{H,t}(h))^2dh - \eta_H S_t^H\frac{1}{n}\int_0^n \hat{y}_{H,t}(h)dh \end{pmatrix} \\
&+t.i.p + (o||\xi||^3)
\end{aligned}$$

By definition $\frac{1}{n}\int_0^n \hat{y}_{H,t}(h)dh$ equals the unconditional expectations operator $E_h\hat{y}_{H,t}(h)$ where it is aggregated over all varieties h. Further $Var_h\hat{y}_{H,t}(h) = E_h(\hat{y}_{H,t}(h))^2 - [E_h\hat{y}_{H,t}(h)]^2 = \frac{1}{n}\int_0^n(\hat{y}_{H,t}(h))^2dh - [E_h\hat{y}_{H,t}(h)]^2$. Hence

$$\begin{aligned}
&\frac{1}{n}\int_0^n v(y_{H,t}(h), A_t^H)dh \\
=\ & \mathcal{U}_C(C)Y_HT^{n-1}Q^{1-\gamma}\begin{pmatrix} (1-\Phi^H)E_h\hat{y}_{H,t}(h) \\ +\frac{1+\eta_H}{2}\left(Var_h\hat{y}_{H,t}(h) + [E_h\hat{y}_{H,t}(h)]^2\right) - \eta_H S_t^H E_h\hat{y}_{H,t}(h) \end{pmatrix} \\
&+t.i.p + (o||\xi||^3) \qquad \text{(A.108)}
\end{aligned}$$

In case of non-tradables,

$$\begin{aligned}
&\frac{1}{n}\int_0^n v(y_{N,t}(h), A_t^N)dh \\
=\ & \mathcal{U}_C(C)Y_NQ^{-\gamma}\begin{pmatrix} (1-\Phi^N)E_h\hat{y}_{N,t}(h) \\ +\frac{1+\eta_N}{2}\left(Var_h\hat{y}_{N,t}(h) + [E_h\hat{y}_{N,t}(h)]^2\right) - \eta_N S_t^N E_h\hat{y}_{N,t}(h) \end{pmatrix} \\
&+t.i.p + (o||\xi||^3) \qquad \text{(A.109)}
\end{aligned}$$

Note that we can write $\frac{Q^{1-\gamma}}{T^{1-n}}\mathcal{U}_C(C)Y_H = \mathcal{U}_C(C)CQ^{1-\gamma}\frac{C_T^U}{C}$ where we used that in steady state $Y_H = T^{1-n}C_T^U$. For foreign $\frac{(Q^*)^{1-\gamma^*}}{T^{-n}}\mathcal{U}_{C^*}(C^*)Y_F = \mathcal{U}_{C^*}(C^*)C^*\,(Q^*)^{1-\gamma^*}\,\frac{C_T^U}{C}$ and for non-tradables

$$\mathcal{U}_C(C)Y_NQ^{-\gamma} = \mathcal{U}_C(C)C\frac{C_N}{C}Q^{-\gamma}, \qquad \mathcal{U}_{C^*}(C^*)Y_{N^*}Q^{*-\gamma^*} = \mathcal{U}_{C^*}(C^*)C^*\frac{C_N^*}{C^*}Q^{*-\gamma^*}$$

Aggregating private sector demand over all h goods in the H sector gives the Dixit-Stiglitz aggregator for private aggregate demand in the H sector

$$Y_{H,t}^d = \left(\frac{1}{n}\int_0^n y_{H,t}^d(h)^{\frac{\rho_H-1}{\rho_H}}dh\right)^{\frac{\rho_H}{\rho_H-1}}$$

Recall that aggregate demand in the H sector is composed of private aggregate demand and public sector demand $Y_{H,t} = Y_{H,t}^d + Y_{H,t}^g$. Take a second order expansion of $\ln\frac{Y_{H,t}^d}{Y_H^d}$ about Y_H^d. Start with

$$y_{H,t}^d(h)^{\frac{\rho_H-1}{\rho_H}} = (Y_H^d)^{\frac{\rho_H-1}{\rho_H}}\exp[\frac{\rho_H-1}{\rho_H}\hat{y}_{H,t}^d(h)]$$

Do a second order Taylor approximation

$$y_{H,t}^d(h)^{\frac{\rho_H-1}{\rho_H}} = (Y_H^d)^{\frac{\rho_H-1}{\rho_H}}\left(1+\frac{\rho_H-1}{\rho_H}\hat{y}_{H,t}^d(h) + \frac{1}{2}(\frac{\rho_H-1}{\rho_H})^2(\hat{y}_{H,t}^d(h))^2\right) + (o||\xi||^3)$$

Sum over all agents at home and take the average

$$\begin{aligned}
&\frac{1}{n}\int_0^n y_{H,t}^d(h)^{\frac{\rho_H-1}{\rho_H}}dh\\
=\;&(Y_H^d)^{\frac{\rho_H-1}{\rho_H}}\left(1+\frac{\rho_H-1}{\rho_H}\frac{1}{n}\int_0^n \hat{y}_{H,t}^d(h)dh+\frac{1}{2}(\frac{\rho_H-1}{\rho_H})^2\frac{1}{n}\int_0^n(\hat{y}_{H,t}^d(h))^2dh\right)+(o||\xi||^3)\\
=\;&(Y_H^d)^{\frac{\rho_H-1}{\rho_H}}\left(1+\frac{\rho_H-1}{\rho_H}E_h\hat{y}_{H,t}(h)+\frac{1}{2}(\frac{\rho_H-1}{\rho_H})^2Var_h\hat{y}_{H,t}^d(h)\right)+t.i.p.+(o||\xi||^3)
\end{aligned}$$

such that

$$\begin{aligned}
\frac{Y_{H,t}^d}{Y_H^d} &= \left[\left(1+\frac{\rho_H-1}{\rho_H}E_h\hat{y}_{H,t}(h)+\frac{1}{2}(\frac{\rho_H-1}{\rho_H})^2Var_h\hat{y}_{H,t}^d(h)\right)+t.i.p.+(o||\xi||^3)\right]^{\frac{\rho_H}{\rho_H-1}}\\
\ln\frac{Y_{H,t}^d}{Y_H^d} &= \frac{\rho_H}{\rho_H-1}\left(\frac{\rho_H-1}{\rho_H}E_h\hat{y}_{H,t}(h)+\frac{1}{2}(\frac{\rho_H-1}{\rho_H})^2Var_h\hat{y}_{H,t}^d(h)\right)+t.i.p.+(o||\xi||^3)
\end{aligned}$$

As a result,

$$\hat{Y}_{H,t}^d=E_h\hat{y}_{H,t}^d(h)+\frac{1}{2}(\frac{\rho_H-1}{\rho_H})Var_h\hat{y}_{H,t}^d(h)+t.i.p.+(o||\xi||^3)$$

And for aggregate demand

$$\hat{Y}_{H,t}=E_h\hat{y}_{H,t}(h)+\frac{1}{2}(\frac{\rho_H-1}{\rho_H})Var_h\hat{y}_{H,t}(h)+t.i.p.+(o||\xi||^3)$$

Insert the value for average demand for a good h, $E_h\hat{y}_{H,t}^d(h)$ by a household into (A.108)

$$\begin{aligned}
&\frac{1}{n}\int_0^n v(y_{H,t}(h),A_t^H)dh\\
=\;&\frac{Q^{1-\gamma}}{T^{1-n}}\mathcal{U}_C(C)Y_H\left\{\begin{array}{c}(1-\Phi^H)\hat{Y}_{H,t}-\frac{1}{2}[\frac{\rho_H-1}{\rho_H}-(1+\eta_H)]Var_h\hat{y}_{H,t}(h)\\ +\frac{1+\eta_H}{2}(\hat{Y}_{H,t})^2-\eta_H S_t^H\hat{Y}_{H,t}^d\}+t.i.p+(o||\xi||^3)\end{array}\right\}\\
=\;&\frac{Q^{1-\gamma}}{T^{1-n}}\mathcal{U}_C(C)Y_H\left\{\begin{array}{c}(1-\Phi^H)\hat{Y}_{H,t}-\frac{1}{2}[-\frac{1}{\rho_H}-\eta_H)]Var_h\hat{y}_{H,t}(h)-\eta_H S_t^H\hat{Y}_{H,t}^d\\ +\frac{1+\eta_H}{2}(\hat{Y}_{H,t})^2\end{array}\right\}+t.i.p+(o||\xi||^3)
\end{aligned}$$

where $\Phi^H Var_h\hat{y}_{H,t}(h)$ and $\hat{Y}_{H,t}Var_h\hat{y}_{H,t}(h)$ and $(Var_h\hat{y}_{H,t}(h))^2$, $S_t^H Var_h\hat{y}_{H,t}(h)$ are at least of order $(o||\xi||^3)$. Further, by simplifying $-\frac{1}{2}[\frac{\rho_H-1}{\rho_H}-(1+\eta_H)]=\frac{1}{2}[(\rho_H)^{-1}+\eta_H]$

$$\begin{aligned}
&\frac{1}{n}\int_0^n v(y_{H,t}(h),A_t^H)dh\\
=\;&\frac{Q^{1-\gamma}}{T^{1-n}}\mathcal{U}_C(C)Y_H\left\{\begin{array}{c}(1-\Phi^H)\hat{Y}_{H,t}^d+\frac{1}{2}(\hat{Y}_{H,t}^d)^2+\frac{\eta_H}{2}\hat{Y}_{H,t}^2\\ +\frac{1}{2}[(\rho_H)^{-1}+\eta_H]var_h\hat{y}_{H,t}(h)-\eta_H S_t^H\hat{Y}_{H,t}^d\end{array}\right\}\\
&+t.i.p+(o||\xi||^3)
\end{aligned}\tag{A.110}$$

which is analogous to Benigno (2004 appendix, xii equation E.17). Analogously for non-tradables

$$\begin{aligned}
\frac{1}{n}\int_0^n v(y_{N,t}(h),A_t^N)dh &= \mathcal{U}_C(C)C\frac{C_N}{C}Q^{-\gamma}\{\left\{\begin{array}{c}(1-\Phi^N)\hat{Y}_{N,t}^d+\frac{1}{2}(\hat{Y}_{N,t}^d)^2+\frac{\eta_N}{2}\hat{Y}_{N,t}^2\\ +\frac{1}{2}[(\rho_N)^{-1}+\eta_N]Var_h\hat{y}_{N,t}(h)-\eta_N S_t^N\hat{Y}_{H,t}^d\end{array}\right\}\\
&\quad +t.i.p+(o||\xi||^3)
\end{aligned}\tag{A.111}$$

Similar expressions apply for Foreign.

A.7.3 Derivation of Variances

Derivation of $Var_h \hat{y}_{H,t}(h)$ and $Var_h \hat{y}_{N,t}(h)$

We focus on the derivation of $Var_h \hat{y}_{H,t}(h)$ where the variance in the N sector is derived accordingly. Start with the demand function for good h given by (A.106)

$$y_{H,t}(h) = \left(\frac{p_{H,t}(h)}{P_{H,t}}\right)^{-\rho_H} \left(T_t^{1-n} C_t^U + G_{H,t}\right)$$

We derive the variance of $\ln y_{H,t}(h)$. As $\hat{y}_{H,t}(h) \equiv \ln y_{H,t}(h) - \ln Y_H$, the variance of $\ln y_{H,t}(h)$ is equal to the variance of $\hat{y}_{H,t}(h)$ as the steady state is non-stochastic. Take natural logs

$$\ln y_{H,t}(h) = -\rho_H \left[\ln p_{H,t}(h) - \ln P_{H,t}\right] + \ln\left[(T_t)^{1-n} C_t^W + G_{H,t}\right]$$

Calculate the variance of $\ln y_{H,t}(h)$ by summing over all goods, conditional on period t information

$$\begin{aligned}
var_h \ln y_{H,t}(h) &\equiv E_h[(\ln y_{H,t} - E_h \ln y_{H,t})^2] \\
&\simeq E_h[(-\rho_H \left[\ln p_{H,t}(h) - E_h \ln p_{H,t}(h)\right])^2] \\
&= E_h[(-\rho_H \left[\ln p_{H,t}(h) - \bar{P}_{H,t}\right])^2] \\
&= \rho_H^2 var_h \ln p_{H,t}(h) \qquad \text{(A.112)}
\end{aligned}$$

where the term $\ln\left[T_t^{1-n} C_t^U + G_{H,t}\right]$ is non-stochastic w.r.t. h (but T_t, C_t^U, and $G_{H,t}$ are random variables w.r.t. supply and demand shocks). Relate $var_h \ln p_{H,t}(h)$ to the average inflation rate across all firms. We used the definition of the average price level in the traded goods sector

$$\bar{P}_{H,t} \equiv E_h \ln p_{H,t}(h)$$

Use the rule that for two independent variables X and Y

$$var(X - Y) = varX + varY$$

Further

$$var_h \bar{P}_{H,t-1} \equiv E_h[\left(\bar{P}_{H,t-1} - E_h \bar{P}_{H,t-1}\right)^2] = 0$$

using $\bar{P}_{H,t-1} = E_h \bar{P}_{H,t-1}$ as the aggregate price level is not depending on the variety h. Then

$$var_h \ln p_{H,t}(h) = var_h \ln p_{H,t}(h) + var_h \bar{P}_{H,t-1} = var_h(\ln p_{H,t}(h) - \bar{P}_{H,t-1})$$

By the definition of a variance

$$\begin{aligned}
var_h \ln p_{H,t}(h) &= E_{h,t}[\left(\ln p_{H,t}(h) - \bar{P}_{H,t-1}\right)^2] - \left(E_{h,t} \ln p_{H,t}(h) - \bar{P}_{H,t-1}\right)^2 \\
&= E_{h,t}[\left(\ln p_{H,t}(h) - \bar{P}_{H,t-1}\right)^2] - \left(\bar{P}_{H,t} - \bar{P}_{H,t-1}\right)^2 \qquad \text{(A.113)}
\end{aligned}$$

To expand the first term on the right hand side, use the assumptions of the Calvo pricing model. Each period, the distribution of prices $\{p_{H,t}(h)\}$ consists of prices in the previous period plus a fraction of size $(1 - \theta_H)$ at the price $p_{H,t}^{\#}$ that is chosen at date t. The aggregate price index of newly set prices $P_{H,t}^{\#}$ is a composite of prices that are set in a forward- and backward-looking way

$$P_{H,t}^{\#} = \left[(1 - \omega_H) P_{H,t}^{o,1-\rho_H} + \omega_H P_{H,t}^{b,1-\rho_H}\right]^{\frac{1}{1-\rho_H}}$$

Log-linearise about $P_{H,t}$ such that $\hat{p}^{\#}_{H,t} \equiv \ln P^{\#}_{H,t} - \ln P_{H,t}$, $\hat{p}^{o}_{H,t} \equiv \ln P^{o}_{H,t} - \ln P_{H,t}$ and $\hat{p}^{b}_{H,t} \equiv \ln P^{b}_{H,t} - \ln P_{H,t}$. Note that this is different from log-linearising about the steady state price level of that period, $P^{ss}_{H,t}$ (as performed otherwise).

$$
\begin{aligned}
P^{\#}_{H,t}\left(1+\ln P^{\#}_{H,t}-P_{H,t}\right) &= \left((1-\omega_H)P^{1-\rho_H}_{H,t}+\omega_H P^{1-\rho_H}_{H,t}\right)^{\frac{1}{1-\rho_H}} \\
&\quad \left(1+\frac{(1-\rho_H)}{1-\rho_H}\left[\begin{array}{c}(1-\omega_H)\left(\ln P^{o}_{H,t}-\ln P_{H,t}\right)\\ +\omega_H\left(\ln P^{b}_{H,t}-\ln P_{H,t}\right)\end{array}\right]\right) \\
&= P_{H,t}\left(1+\left[\begin{array}{c}(1-\omega_H)\left(\ln P^{o}_{H,t}-\ln P_{H,t}\right)\\ +\omega_H\left(\ln P^{b}_{H,t}-\ln P_{H,t}\right)\end{array}\right]\right) \\
\ln P^{\#}_{H,t}-\ln P_{H,t} &= (1-\omega_H)\left(\ln P^{o}_{H,t}-\ln P_{H,t}\right)+\omega_H\left(\ln P^{b}_{H,t}-\ln P_{H,t}\right) \\
\hat{p}^{\#}_{H,t} &= (1-\omega_H)\hat{p}^{o}_{H,t}+\omega_H\hat{p}^{b}_{H,t}
\end{aligned}
$$

Further note that by adding $\ln P_{H,t}$ on both sides, we obtain

$$\ln P^{\#}_{H,t} = (1-\omega_H)\ln P^{o}_{H,t}+\omega_H \ln P^{b}_{H,t}$$

A similar derivation goes through for $P_{H,t}$ such that

$$\ln P_{H,t} = (1-\theta_H)\ln P^{\#}_{H,t}+\theta_H \ln P_{H,t-1}$$

Plug in the definitions of the Dixit-Stiglitz aggregators

$$
\begin{aligned}
\ln\left[\frac{1}{n}\int_0^n (p_{H,t}(h))^{1-\rho_H}dh\right]^{\frac{1}{1-\rho_H}} &= (1-\theta_H)\ln\left[\frac{1}{n}\int_0^n (p^{\#}_{H,t}(h))^{1-\rho_H}dh\right]^{\frac{1}{1-\rho_H}} \\
&\quad +\theta_H \ln\left[\frac{1}{n}\int_0^n (p_{H,t-1}(h))^{1-\rho_H}dh\right]^{\frac{1}{1-\rho_H}} \\
\ln\int_0^n (p_{H,t}(h))^{1-\rho_H}dh &= (1-\theta_H)\ln\int_0^n (p^{\#}_{H,t}(h))^{1-\rho_H}dh \\
&\quad +\theta_H \ln\int_0^n (p_{H,t-1}(h))^{1-\rho_H}dh
\end{aligned}
$$

Concentrating on the h^{th} firm, the expression collapses to

$$\ln(p_{H,t}(h))^{1-\rho_H} = (1-\theta_H)\ln(p^{\#}_{H,t}(h))^{1-\rho_H}+\theta_H\ln(p_{H,t-1}(h))^{1-\rho_H}$$

We therefore have that the expected price set by firm h in period t given by $p_{H,t}(h)$ is

$$p_{H,t}(h) = \begin{cases} p^{\#}_{H,t}(h) \text{ with probability } 1-\theta_H \\ p_{H,t-1}(h) \text{ with probability } \theta_H \end{cases}$$

from which follows that $p_{H,t}(h)$ is a random variable and we can write

$$p_{H,t}(h) \equiv E_{s,t}\ln p_{H,t}(h,s)$$

Therefore, the price expected to be set by firm h becomes[11]

$$E_{s,t}\left[(\ln p_{H,t}(h,s))\right] = (1-\theta_H)E_{s,t}\ln p^{\#}_{H,t}(h,s) + \theta_H \ln p_{H,t-1}(h) \tag{A.115}$$

Note that this is an expectation over the two states for an arbitrary firm h. Hence one needs to differentiate between summing across states and across firms. Further, $p^{\#}_{H,t}(h)$ is a random variable as well and reveals whether a firm is forward- or backward-looking in price setting

$$p^{\#}_{H,t}(h) \equiv E_{s,t}\ln p^{\#}_{H,t}(h,s)$$

To obtain the aggregate in the sector H, one still needs to sum over all goods h. Applying the expectations operator for (A.115)

$$\begin{aligned} E_{h,t}\mu'_1 &= (1-\theta_H)E_{h,t}\left[E_{s,t}\ln p^{\#}_{H,t}(h,s)\right] + \theta_H E_{h,t}\ln p_{H,t-1}(h) \qquad \text{(A.116)} \\ \mu'_1 &= (1-\theta_H)E_{h,t}\left[E_{s,t}\ln p^{\#}_{H,t}(h,s)\right] + \theta_H E_{h,t}\ln p_{H,t-1}(h) \\ E_{h,t}\left[\ln p_{H,t}(h)\right] &= (1-\theta_H)E_{h,t}\left[E_{s,t}\ln p^{\#}_{H,t}(h,s)\right] + \theta_H E_{h,t}\ln p_{H,t-1}(h) \end{aligned}$$

As by definition $E_{h,t}\ln p_{H,t}(h) = \bar{P}_{H,t}$, one can rewrite

$$\bar{P}_{H,t} = (1-\theta_H)E_{h,t}\left[E_{s,t}\ln p^{\#}_{H,t}(h,s)\right] + \theta_H \bar{P}_{H,t-1}$$

Centering above equation about $\bar{P}_{H,t-1}$ by subtracting $\bar{P}_{H,t-1}$ on each side, one obtains

$$E_{h,t}\left[\ln p_{H,t}(h) - \bar{P}_{H,t-1}\right] = (1-\theta_H)E_{h,t}\left[E_{s,t}\ln p^{\#}_{H,t}(h,s) - \bar{P}_{H,t-1}\right] + \theta_H E_{h,t}\left[\ln p_{H,t-1}(h) - \bar{P}_{H,t-1}\right]$$

Different from Woodford (2003, p. 695), the expectations operator preceding $\ln p^{\#}_{H,t}(h)$ cannot be removed yet, as there are two types of firms whose optimally set prices will differ. $\ln p^{\#}_{H,t}(h)$ is therefore still depending on an individual firm as a newly set price can be set by an optimising or backward-looking firm. For the second raw moment of the random variable $\ln p_{H,t}(h)$ one obtains

$$E_{h,t}\left[(\ln p_{H,t}(h))^2\right] = (1-\theta_H)E_{h,t}\left[\left(E_{s,t}\ln p^{\#}_{H,t}(h,s)\right)^2\right] + \theta_H E_{h,t}\left[(\ln p_{H,t-1}(h))^2\right]$$

[11] The result on $E_{s,t}\left[(\ln p_{H,t}(h,s))\right]$ is a straightforward application of the methods on raw moments. Recall that the n^{th} raw moment of a probability function $P(X)$ is obtained from

$$\mu'_n = \sum_{s=1}^{S} x_s^n P(X = x_s) \tag{A.114}$$

hence $E_{s,t}\left[(\ln p_{H,t}(h,s))^n\right]$ is the n^{th} raw moment of the random variable $\ln p_{H,t}(h)$ over all states $s \in S$. By definition, with probability θ_H a firm cannot reset its price which is therefore $p_{H,t}(h) = p_{H,t-1}(h)$ and if it resets, it sets $p^{\#}_{H,t}(h)$ with probability $1-\theta_H$. Then $S = 2$ and for the first moment $n = 1$ where $x_1 = \ln p_{H,t}(h,1) = \ln p^{\#}_{H,t}(h)$, $x_2 = \ln p_{H,t}(h,2) = \ln p_{H,t-1}(h)$, $P(X = x_1) = 1-\theta_H$, and $P(X = x_2) = \theta_H$

$$\mu'_1 = (1-\theta_H)E_{s,t}\ln p^{\#}_{H,t}(h,s) + \theta_H \ln p_{H,t-1}(h)$$

One arrives at

$$
\begin{aligned}
& E_{h,t}\left[\left(\ln p_{H,t}(h)-\bar{P}_{H,t-1}\right)^2\right] \\
\equiv\ & \theta_H E_{h,t}[\left(\ln p_{H,t-1}(h)-\bar{P}_{H,t-1}\right)^2]+(1-\theta_H)E_{h,t}\left[\left(E_{s,t}\ln p^{\#}_{H,t}(h,s)-\bar{P}_{H,t-1}\right)^2\right]
\end{aligned}
$$

The probability that a firm is selected to reset prices does not depend on whether the firm is optimising or backward-looking. When selected, only a share $1-\varpi_H$ of firms is optimising and sets prices in a forward-looking fashion where the price of a firm is $p^o_{H,t}(h)$. Backward-looking firms set prices to $p^b_{H,t}(h)=P^b_{H,t}$ and prices of unselected firms remain on their previous level. Hence in any point in time, the average aggregate index of newly set prices is given by

$$
\begin{aligned}
E_{s,t}\ln p^{\#}_{H,t}(h,s) &= \varpi_H \ln p^b_{H,t}(h)+(1-\varpi_H)\ln p^o_{H,t}(h) \\
E_{h,t}\ln p^{\#}_{H,t}(h) &= \varpi_H E_{h,t}\ln p^b_{H,t}(h)+(1-\varpi_H)E_{h,t}\ln p^o_{H,t}(h) \\
\ln p^{\#}_{H,t} &= \varpi_H \ln p^b_{H,t}+(1-\varpi_H)\ln p^o_{H,t} \qquad \text{(A.117)}
\end{aligned}
$$

We again used the properties of raw moments and that when integrating over all firms h, we can omit the expectations operator, as the price set within each group is not depending on h, $p^b_{H,t}(h)=p^b_{H,t}$ and $p^o_{H,t}(h)=\ln p^o_{H,t}$. Further, observe that $p^{\#}_{H,t}(h)$ no longer depends on h, and therefore also $E_{h,t}\ln p^{\#}_{H,t}(h)=\ln p^{\#}_{H,t}$ (the average newly set price does not depend on a specific firm). When there is no dispersion of prices across firms, the individual price equals the aggregate price index. Therefore, analogously to above, the second raw moment of $\ln p^{\#}_{H,t}(h)$ centered at $\bar{P}_{H,t-1}$ is obtained as

$$
\begin{aligned}
& E_{h,t}\left[\left(\ln p^{\#}_{H,t}(h)-\bar{P}_{H,t-1}\right)^2\right] \\
=\ & \varpi_H E_{h,t}\left[\left(\ln p^b_{H,t}(h)-\bar{P}_{H,t-1}\right)^2\right]+(1-\varpi_H)E_{h,t}\left[\left(\ln p^o_{H,t}(h)-\bar{P}_{H,t-1}\right)^2\right] \\
=\ & \varpi_H\left(\ln p^b_{H,t}-\bar{P}_{H,t-1}\right)^2+(1-\varpi_H)\left(\ln p^o_{H,t}-\bar{P}_{H,t-1}\right)^2 \qquad \text{(A.118)}
\end{aligned}
$$

Then, as in Amato and Laubach (2003), the variance of prices in period t is obtained by plugging (A.118) in (A.113)

$$
\begin{aligned}
var_h \ln p_{H,t}(h) &= \theta_H var_h \ln p_{H,t-1}(h)+(1-\theta_H)\varpi_H\left(\ln p^b_{H,t}-\bar{P}_{H,t-1}\right)^2 \\
&\quad +(1-\theta_H)(1-\varpi_H)\left(\ln p^o_{H,t}-\bar{P}_{H,t-1}\right)^2-\left(\bar{P}_{H,t}-\bar{P}_{H,t-1}\right)^2 \qquad \text{(A.119)}
\end{aligned}
$$

The average price level in the traded sector, derived from the Calvo assumption was

$$
\bar{P}_{H,t}=(1-\theta_H)E_{h,t}\ln p^{\#}_{H,t}(h)+\theta_H\bar{P}_{H,t-1}
$$

Hence

$$
\begin{aligned}
\bar{P}_{H,t}-\bar{P}_{H,t-1} &= (1-\theta_H)E_{h,t}\ln p^{\#}_{H,t}(h)-(1-\theta_H)\bar{P}_{H,t-1} \\
(1-\theta_H)\left(E_{h,t}\ln p^{\#}_{H,t}(h)-\bar{P}_{H,t-1}\right) &= \bar{P}_{H,t}-\bar{P}_{H,t-1} \\
E_{h,t}\ln p^{\#}_{H,t}(h)-\bar{P}_{H,t-1} &\simeq \frac{1}{1-\theta_H}\pi_{H,t} \qquad \text{(A.120)}
\end{aligned}
$$

Note that we used in the last step that the change in the average aggregate price level (the average price dispersion across firms) equals the aggregate inflation rate up to a term of second order

$$\begin{aligned}
\bar{P}_{H,t} - \bar{P}_{H,t-1} &= \pi_{H,t} + (o||\xi||^2) \equiv \ln P_{H,t} - \ln P_{H,t-1} + (o||\xi||^2) \\
\bar{P}_{H,t} - \bar{P}_{H,t-1} &\simeq \pi_{H,t} \equiv \ln P_{H,t} - \ln P_{H,t-1}
\end{aligned}$$

Therefore the average aggregate price index is related to the Dixit-Stiglitz index through the log-linear approximation

$$\bar{P}_{H,t} = \ln P_{H,t} + (o||\xi||^2) \tag{A.121}$$

see also Woodford (2003, p. 695). The price level of optimally set prices consists of a share $1 - \varpi_H$ of firms that sets prices in a forward-looking manner and a portion ϖ_H that sets prices by backward-looking behaviour, using a rule of thumb. Firm h, if non-optimising, chooses

$$p^b_{H,t}(h) = p^{\#}_{H,t-1}(h)\frac{P_{H,t-1}}{P_{H,t-2}}, \qquad p^b_{H,t} = p^{\#}_{H,t-1}\frac{P_{H,t-1}}{P_{H,t-2}} \tag{A.122}$$

where $\frac{P_{H,t-1}}{P_{H,t-2}}$ denotes the gross sectoral inflation and $p^{\#}_{H,t-1}(h)$ is the price index composed of prices reset in period $t-1$ relevant for firm h. Note that we can omit the firm specific index from the equation as $p^{\#}_{H,t-1}$ does not depend on h. Take logs (do not log-linearise) of the rule of thumb and use (A.120). Also use that $p^b_{H,t}(h) = p^b_{H,t}$ and $p^{\#}_{H,t-1}(h) = p^{\#}_{H,t-1}$. Then also $E_{h,t} \ln p^{\#}_{H,t}(h) = \ln p^{\#}_{H,t}$ and therefore $\ln p^{\#}_{H,t-1} - \bar{P}_{H,t-2} \simeq \frac{1}{1-\theta_H}\pi_{H,t-1}$

$$\begin{aligned}
\ln p^b_{H,t}(h) &= \ln p^{\#}_{H,t-1}(h) + \ln P_{H,t-1} - \ln P_{H,t-2} \\
\ln p^b_{H,t} - \ln P_{H,t-1} &= \ln p^{\#}_{H,t-1} - \ln P_{H,t-2} \\
\ln p^b_{H,t} - \bar{P}_{H,t-1} &= \frac{1}{1-\theta_H}\pi_{H,t-1} \qquad \text{(A.123)}
\end{aligned}$$

see also Amato and Laubach (2003, p. 826). Using the definition of the distribution of newly set prices $\left\{\ln p^{\#}_{H,t}\right\}$ which is invariant of the firm (therefore not indexed by the firm index i) and (A.120) as well as (A.122) yields

$$\ln p^b_{H,t} - \bar{P}_{H,t-1} = \frac{1}{1-\theta_H}\pi_{H,t-1} \tag{A.124}$$

$$\ln p^{\#}_{H,t} - \bar{P}_{H,t-1} = \frac{1}{1-\theta_H}\pi_{H,t} \tag{A.125}$$

Plug the preceding two results in the definition for $\ln p^{\#}_{H,t}$ given by (A.117)

$$\begin{aligned}
\ln p^{\#}_{H,t} &= (1-\varpi_H)\ln p^o_{H,t} + \varpi_H \ln p^b_{H,t} \\
\ln p^o_{H,t} - \bar{P}_{H,t-1} &= \frac{1}{1-\theta_H}\frac{1}{1-\varpi_H}\pi_{H,t} - \frac{\varpi_H}{1-\varpi_H}\frac{1}{1-\theta_H}\pi_{H,t-1} \qquad \text{(A.126)}
\end{aligned}$$

which is equation A.21 in Amato and Laubach (2003). Plug the results for $\ln p^b_{H,t}(h) - \bar{P}_{H,t-1}$ and $\ln p^o_{H,t}(h) - \bar{P}_{H,t-1}$ in the variance term (A.119)

$$\begin{aligned}
var_h \ln p_{H,t}(h) &= \theta_H var_h \ln p_{H,t-1}(h) \\
&\quad + \frac{\theta_H}{1-\theta_H}(\pi_{H,t})^2 + \frac{1}{1-\theta_H}\left[\begin{array}{c} \varpi_H (\pi_{H,t-1})^2 \\ +\frac{1}{1-\varpi_H}(\pi_{H,t} - \varpi_H \pi_{H,t-1})^2 - (\pi_{H,t})^2 \end{array}\right]
\end{aligned}$$

The terms in brackets can be further simplified

$$\varpi_H \pi_{H,t-1}^2 + \frac{1}{1-\varpi_H}\left(\pi_{H,t} - \varpi_H \pi_{H,t-1}\right)^2 - \pi_{H,t}^2 = \frac{\varpi_H}{1-\varpi_H}\left[\pi_{H,t} - \pi_{H,t-1}\right]^2$$

Therefore

$$var_h \ln p_{H,t}(h) \simeq \theta_H var_h \ln p_{H,t-1}(h) + \frac{\theta_H}{1-\theta_H}\left(\pi_{H,t}\right)^2 + \frac{1}{1-\theta_H}\frac{\varpi_H}{1-\varpi_H}\left[\pi_{H,t} - \pi_{H,t-1}\right]^2 \tag{A.127}$$

such that

$$var_h \ln p_{H,t}(h) \simeq \sum_{s=0}^{t}\left(\theta_H\right)^{t-s}\left[\frac{\theta_H}{1-\theta_H}\pi_{H,t}^2 + \frac{1}{1-\theta_H}\frac{\varpi_H}{1-\varpi_H}\left[\pi_{H,t} - \pi_{H,t-1}\right]^2\right] + t.i.p.$$

Since we consider evaluating policies adopted at time 0, the term $\left(\theta_H\right)^{t+1} var_h \ln p_{H,-1}(h)$ is treated as independent of the policy under consideration. Take expectations conditional on period t information and and sum to obtain

$$\begin{aligned}
&\mathcal{E}_t \sum_{s=0}^{\infty} \beta^s var_h \ln p_{H,t+s}(h) \\
=\; & \frac{1}{1-\beta\theta_H}\mathcal{E}_t \sum_{s=0}^{\infty} \beta^s \left\{\frac{\theta_H}{1-\theta_H}\pi_{H,s}^2 + \frac{1}{1-\theta_H}\frac{\varpi_H}{1-\varpi_H}\left[\pi_{H,s} - \pi_{H,s-1}\right]^2\right\}
\end{aligned}$$

which is equation A.23 in Amato and Laubach (2003). Therein, the share of optimising firms is given by $\lambda = 1 - \varpi_H$ and in each period a fraction $1 - \alpha = 1 - \theta_H$ of suppliers is offered the opportunity to choose a new price, while the remaining suppliers have to maintain whichever price they charged before. The dispersion in individual demands can eventually be linked to dispersion in prices

$$\begin{aligned}
&E_t \sum_{s=0}^{\infty} \beta^s var_h \hat{y}_{H,t+s}(h) \\
=\; & \frac{\rho_H^2}{1-\beta\theta_H} E_t \sum_{s=0}^{\infty} \beta^s \left\{\frac{\theta_H}{1-\theta_H}\pi_{H,s}^2 + \frac{1}{1-\theta_H}\frac{\varpi_H}{1-\varpi_H}\left[\pi_{H,s} - \pi_{H,s-1}\right]^2\right\}
\end{aligned} \tag{A.128}$$

and analogously for N, F, N^* sectors. Note that in case of foreign non-tradables, we will define $P_{N^*,t}^* \equiv P_{N^*,t}$. As the LOP holds in case of tradables, $P_{H,t}^* \equiv P_{H,t}$ and $P_{F,t}^* \equiv P_{F,t}$, we can drop the superscript asterisk when describing inflation dynamics in the sectors at Foreign.

A.7.4 Home Welfare Function

Welfare $\mathcal{W}$ for the average household in the home economy is derived as a second-order approximation of life-time utility defined in (2.1)

$$\frac{\mathcal{W}}{U_C(C)C} = \mathcal{E}_t \sum_{s=0}^{\infty} \beta^s w_{t+s}$$

Combining (A.104) with (A.110) and (A.111) together with (A.128) for each sector yields the period welfare function $w_{t+s} = [\mathcal{P}_{t+s} - \mathcal{Q}_{t+s} - \mathcal{R}_{t+s}] + t.i.p + (o||\xi||^3)$ where

$$\begin{aligned}
\mathcal{P}_{t+s} &= \hat{C}_t + \frac{1-\rho}{2}\hat{C}_t^2 \\
\mathcal{Q}_{t+s} &= \left(Q^{1-\gamma}\frac{C_T^U}{C}\right)\left\{\begin{array}{c} \left(1-\Phi^H\right)\hat{Y}_{H,t}^d + \frac{1}{2}\left(\hat{Y}_{H,t}^d\right)^2 - \eta_H S_t^H \hat{Y}_{H,t}^d + \frac{\eta_H}{2}\hat{Y}_{H,t}^2 \\ +\frac{1}{2}[\rho_H^{-1}+\eta_H]\frac{\rho_H^2}{1-\beta\theta_H}\frac{\theta_H}{1-\theta_H}\left\{\pi_{H,t}^2 + \frac{1}{\theta_H}\frac{\varpi_H}{1-\varpi_H}\left[\pi_{H,t}-\pi_{H,t-1}\right]^2\right\} \end{array}\right\} \\
\mathcal{R}_{t+s} &= \frac{C_N}{C}Q^{-\gamma}\left\{\begin{array}{c} (1-\Phi^N)\hat{Y}_{N,t}^d + \frac{1}{2}\left(\hat{Y}_{N,t}^d\right)^2 + \frac{\eta_N}{2}\hat{Y}_{N,t}^2 - \eta_N S_t^N \hat{Y}_{N,t}^d \\ +\frac{1}{2}[\rho_N^{-1}+\eta_N]\frac{\rho_N^2}{1-\beta\theta_N}\frac{\theta_N}{1-\theta_N}\left\{\pi_{N,t}^2 + \frac{1}{\theta_N}\frac{\varpi_N}{1-\varpi_N}\left[\pi_{N,t}-\pi_{N,t-1}\right]^2\right\} \end{array}\right\}
\end{aligned}$$

Foreign Welfare Function

Accordingly at foreign

$$\frac{\mathcal{W}^*}{U_{C^*}(C^*)C^*} = E_t\sum_{s=0}^{\infty}\beta^s w_{t+s}^* \tag{A.129}$$

$$w_{t+s}^* = \mathcal{P}_{C,t+s}^* - \mathcal{Q}_{F,t+s}^* - \mathcal{R}_{N,t+s}^* \tag{A.130}$$

where

$$\mathcal{P}_{C,t+s}^* = \hat{C}_t^* + \frac{1-\rho^*}{2}\hat{C}_t^{*2}$$

as well as

$$\begin{aligned}
\mathcal{Q}_{F,t+s}^* &= Q^{*1-\gamma^*}\frac{C_T^U}{C^*} \\
&\left\{\begin{array}{c} \left(1-\Phi^F\right)\hat{Y}_{F,t}^{d*} + \frac{1}{2}\left(\hat{Y}_{F,t}^{d*}\right)^2 + \frac{\eta_F}{2}\hat{Y}_{F,t}^{*2} - \eta_F S_t^F \hat{Y}_{F,t}^{*d} \\ +\frac{1}{2}[\rho_F^{-1}+\eta_F]\frac{\rho_F^2}{1-\beta\theta_F}\frac{\theta_F}{1-\theta_F}\left\{\pi_{F,t}^2 + \frac{1}{\theta_F}\frac{\varpi_F}{1-\varpi_F}\left[\pi_{F,t}-\pi_{F,t-1}\right]^2\right\} \end{array}\right\}
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{R}_{N^*,t+s}^* &= \frac{C_{N^*}^*}{C^*}Q^{*-\gamma^*} \\
&\left\{\begin{array}{c} \left(1-\Phi^{N^*}\right)\hat{Y}_{N^*,t}^{d*} + \frac{1}{2}\left(\hat{Y}_{N^*,t}^{d*}\right)^2 + \frac{\eta_{N^*}}{2}\hat{Y}_{N^*,t}^{*2} - \eta_{N^*} S_t^{N^*} \hat{Y}_{N^*,t}^{*d} \\ +\frac{1}{2}[\rho_{N^*}^{-1}+\eta_{N^*}]\frac{\rho_{N^*}^2}{1-\beta\theta_{N^*}}\frac{\theta_{N^*}}{1-\theta_{N^*}}\left\{\pi_{N^*,t}^2 + \frac{1}{\theta_N^*}\frac{\varpi_{N^*}}{1-\varpi_{N^*}}\left[\pi_{N^*,t}-\pi_{N^*,t-1}\right]^2\right\} \end{array}\right\}
\end{aligned}$$

A.7.5 Union Welfare Function

Union welfare can be obtained component-wise as we take a Utilitarian perspective. Welfare of the average agent times the region size yields welfare in each region. Hence, $\mathcal{P}_{t+s}^U = n\mathcal{P}_{t+s} + (1-n)\mathcal{P}_{t+s}^*$, $\mathcal{Q}_{t+s}^U = n\mathcal{Q}_{t+s} + (1-n)\mathcal{Q}_{t+s}^*$, and $\mathcal{R}_{t+s}^U = n\mathcal{R}_{t+s} + (1-n)\mathcal{R}_{t+s}^*$. As $C = C^*$ will not arise as a necessary equilibrium allocation derived from labour market equilibrium, we needed to pin down that equilibrium among the many possible allocations. We required that $Q = Q^* = 1$. The assumption of complete markets at the interregional and intraregional level allows then to equilibrate overall real consumption in steady state at home and foreign $C = C^*$, thus

$$\begin{aligned}
C^U &\equiv nC + (1-n)C^* = C \\
U_{C^U}(C^U) &= U_C(C) = U_{C^*}(C^*)
\end{aligned}$$

We can then aggregate welfare across regions as all equilibrium consumption flow is expressed in the same basket $C = C^U$

$$\frac{\mathcal{W}_t^U}{U_{C^U}(C^U)} = n\frac{\mathcal{W}_t^U}{U_{C^U}(C^U)} + (1-n)\frac{\mathcal{W}_t^U}{U_{C^U}(C^U)}$$

Period union welfare is given by $w_{t+s}^U = \mathcal{P}_{t+s}^U - \mathcal{Q}_{t+s}^U - \mathcal{R}_{t+s}^U$ where $\mathcal{P}_{t+s}^U$

$$\mathcal{P}_{t+s}^U = n\hat{C}_t + n\frac{1-\rho}{2}\hat{C}_t^2 + (1-n)\hat{C}_t^* + (1-n)\frac{1-\rho^*}{2}\hat{C}_t^{*2}$$

and $\mathcal{Q}_{t+s}^U$

$$\begin{aligned}\mathcal{Q}_{t+s}^U &= n\frac{Q^{1-\gamma}}{T^{1-n}}\frac{Y_H}{C}\left[\begin{array}{c}\hat{Y}_{H,t}^d - \Phi^H\hat{Y}_{H,t}^d - \eta_H S_t^H\hat{Y}_{H,t}^d + \frac{1}{2}\left(\hat{Y}_{H,t}^d\right)^2 + \frac{\eta_H}{2}\hat{Y}_{H,t}^2\\ +\lambda_{\pi_H}\pi_{H,t}^2 + \lambda_{\triangle\pi_H}\left(\pi_{H,t} - \pi_{H,t-1}\right)^2\end{array}\right]\\ &+(1-n)\frac{Q^{*1-\gamma^*}}{T^{-n}}\frac{Y_F^*}{C^*}\left[\begin{array}{c}\hat{Y}_{F,t}^{d*} - \Phi^F\hat{Y}_{F,t}^{d*} - \eta_F S_t^F\hat{Y}_{F,t}^{*d} + \frac{1}{2}\left(\hat{Y}_{F,t}^d\right)^2\\ +\frac{\eta_F}{2}\hat{Y}_{F,t}^{*2} + \lambda_{\pi_F}\pi_{F,t}^2 + \lambda_{\triangle\pi_F}\left(\pi_{F,t} - \pi_{F,t-1}\right)^2\end{array}\right]\end{aligned}$$

as well as $\mathcal{R}_{t+s}^U$

$$\begin{aligned}\mathcal{R}_{t+s}^U &= n\frac{C_N}{C}Q^{-\gamma}\left[\begin{array}{c}\hat{Y}_{N,t}^d - \Phi^N\hat{Y}_{N,t}^d - \eta_N S_t^N\hat{Y}_{N,t}^d + \frac{1}{2}\left(\hat{Y}_{N,t}^d\right)^2 + \frac{\eta_N}{2}\hat{Y}_{N,t}^2\\ +\lambda_{\pi_N}\pi_{N,t}^2 + \lambda_{\triangle\pi_N}\left(\pi_{N,t} - \pi_{N,t-1}\right)^2\end{array}\right]\\ &+(1-n)\frac{C_{N^*}}{C^*}Q^{*-\gamma^*}\left[\begin{array}{c}\hat{Y}_{N^*,t}^{d*} - \Phi^{N^*}\hat{Y}_{N^*,t}^{d*} - \eta_{N^*} S_t^{N^*}\hat{Y}_{N^*,t}^{*d} + \frac{1}{2}\left(\hat{Y}_{N^*,t}^{d*}\right)^2\\ +\frac{\eta_{N^*}}{2}\hat{Y}_{N^*,t}^2 + \lambda_{\pi_{N^*}}\pi_{N^*,t}^2 + \lambda_{\triangle\pi_{N^*}}\left(\pi_{N^*,t} - \pi_{N^*,t-1}\right)^2\end{array}\right]\end{aligned}$$

We further defined

$$\lambda_{\pi_J} = \frac{1}{2}[\rho_J^{-1} + \eta_J]\frac{\rho_J^2}{1-\beta\theta_J}\frac{\theta_J}{1-\theta_J}, \qquad \lambda_{\triangle\pi_J} = \lambda_{\pi_J}\frac{1}{\theta_J}\frac{\varpi_J}{1-\varpi_J}$$

for $J = H, N, F, N^*$. We observe that intrinsic inflation persistence brought about the presence of rule-of-thumb firms introduces speed limit terms on inflation in the welfare function.

Cancelling First-Order Terms

In order to accurately evaluate welfare, first order terms must not be present in the welfare objective other than in the *t.i.p.* part (that contains the welfare evaluation of the steady state not subject to policy intervention). We obtain that due to market clearance on the union level, this is indeed the case.[12] Using that the government spending shock is zero in steady state $G_H = G_F = 0$

$$\begin{aligned}&n\hat{C}_t + (1-n)\hat{C}_t^*\\ &-n\frac{Q^{1-\gamma}}{T^{1-n}}\left(\frac{C_T^U T^{1-n} + G_H}{C}\right)\hat{Y}_{H,t}^d - (1-n)\frac{Q^{*1-\gamma^*}}{T^{-n}}\frac{C_T^U T^{-n} + G_F}{C^*}\hat{Y}_{F,t}^{d*}\\ &-n\frac{C_N}{C}Q^{-\gamma}\hat{Y}_{N,t}^d - (1-n)\frac{C_{N^*}^*}{C^*}Q^{*-\gamma^*}\hat{Y}_{N^*,t}^{d*}\end{aligned}$$

[12] It becomes clear that these terms would remain in the expressions, if one would derive the loss functions for each country separately. Benigno and Woodford (2006) present a method to eliminate nevertheless linear terms also in that case.

where $\frac{Q^{-\gamma}C_N}{C} = 1-\gamma$, $\frac{Q^{*-\gamma^*}C^*_{N^*}}{C^*} = 1-\gamma^*$. Private sector aggregate demand functions are written as $\hat{Y}^d_{H,t} = \hat{C}^U_{T,t} + (1-n)\hat{T}_t$ and $\hat{Y}^{d*}_{F,t} = \hat{C}^U_{T,t} + -n\hat{T}_t$, as well as $\hat{Y}^d_{N,t} = \hat{C}_{N,t}$ and $\hat{Y}^{d*}_{N^*,t} = \hat{C}^*_{N^*,t}$. Further, use that $Q = Q^* = 1$ and $C = C^* = 1$ as derived under A.3. Then

$$\begin{aligned}
&\Leftrightarrow \quad n\gamma\hat{C}_{T,t} + (1-n)\gamma^*\hat{C}^*_{T,t} - n\frac{C^U_T}{C}\left(\hat{C}^U_{T,t} + (1-n)\hat{T}_t\right) - (1-n)\frac{C^U_T}{C}\left(\hat{C}^U_{T,t} + -n\hat{T}_t\right) \\
&\Leftrightarrow \quad n\gamma\hat{C}_{T,t} + (1-n)\gamma^*\hat{C}^*_{T,t} - n\frac{C^U_T}{C}\hat{C}^U_{T,t} - (1-n)\frac{C^U_T}{C}\hat{C}^U_{T,t} \\
&\Leftrightarrow \quad n\gamma\hat{C}_{T,t} + (1-n)\gamma^*\hat{C}^*_{T,t} - \frac{C^U_T}{C}\hat{C}^U_{T,t}
\end{aligned}$$

As $\frac{C^U_T}{C} = n\frac{C_T}{C} + (1-n)\frac{C^*_T}{C} = n\gamma + (1-n)\gamma^*$ it follows that

$$\Leftrightarrow n\gamma\hat{C}_{T,t} + (1-n)\gamma^*\hat{C}^*_{T,t} - (n\gamma + (1-n)\gamma^*)\hat{C}^U_{T,t}$$

Using that $\hat{C}^U_{T,t} = \frac{\gamma n}{n\gamma+(1-n)\gamma^*}\hat{C}_{T,t} + \frac{(1-n)\gamma^*}{n\gamma+(1-n)\gamma^*}\hat{C}^*_{T,t}$ eventually

$$\begin{aligned}
\Leftrightarrow \quad & n\gamma\hat{C}_{T,t} + (1-n)\gamma^*\hat{C}^*_{T,t} \\
& -(n\gamma + (1-n)\gamma^*)\left(\frac{\gamma n}{n\gamma + (1-n)\gamma^*}\hat{C}_{T,t} + \frac{(1-n)\gamma^*}{n\gamma + (1-n)\gamma^*}\hat{C}^*_{T,t}\right) \\
\Leftrightarrow \quad & n\gamma\hat{C}_{T,t} + (1-n)\gamma^*\hat{C}^*_{T,t} \\
& -\gamma n\hat{C}_{T,t} - (1-n)\gamma^*\hat{C}^*_{T,t} \\
= \quad & 0
\end{aligned}$$

A.7.6 Differences from Flexible Price Gaps

Concentrate on the parts involving consumption and output and define auxiliary parameters A_i

$$A_1 = n\frac{Q^{1-\gamma}}{T^{1-n}}\frac{Y_H}{C}, \qquad A_2 = (1-n)\frac{Q^{*1-\gamma^*}}{T^{-n}}\frac{Y_F}{C^*}, \qquad A_3 = n\frac{C_N}{C}Q^{-\gamma}, \qquad A_4 = (1-n)\frac{C^*_{N^*}}{C^*}Q^{*-\gamma^*}$$

therefore

$$\begin{aligned}
& n\frac{1-\rho}{2}\hat{C}^2_t + (1-n)\frac{1-\rho^*}{2}\hat{C}^{*2}_t \\
& -A_1\left[-\Phi^H\hat{Y}^d_{H,t} - \eta_H S^H_t\hat{Y}^d_{H,t} + \frac{1}{2}\left(\hat{Y}^d_{H,t}\right)^2 + \frac{\eta_H}{2}\hat{Y}^2_{H,t}\right] \\
& -A_2\left[-\Phi^F\hat{Y}^{d*}_{F,t} - \eta_F S^F_t\hat{Y}^d_{F,t} + \frac{1}{2}\left(\hat{Y}^d_{F,t}\right)^2 + \frac{\eta_F}{2}\hat{Y}^2_{F,t}\right] \\
& -A_3\left[-\Phi^N\hat{Y}^d_{N,t} - \eta_N S^N_t\hat{Y}^d_{N,t} + \frac{1}{2}\left(\hat{Y}^d_{N,t}\right)^2 + \frac{\eta_N}{2}\hat{Y}^2_{N,t}\right] \\
& -A_4\left[-\Phi^{N^*}\hat{Y}^{d*}_{N^*,t} - \eta_{N^*} S^{N^*}_t\hat{Y}^{*d}_{N^*,t} + \frac{1}{2}\left(\hat{Y}^{d*}_{N^*,t}\right)^2 + \frac{\eta_{N^*}}{2}\hat{Y}^2_{N,t}\right]
\end{aligned}$$

The inflation terms are already defined as gaps and need no further rearrangement. Inflation under flexible prices is zero throughout, $\tilde{\pi}_{J,t} = 0$ as there will be no movements in the aggregate price index under such circumstances. Substitute out supply shocks in order to obtain expressions for flexible sectoral output gaps. Rewrite, using $\hat{Y}^d_{J,t} = \hat{Y}_{J,t} - g^J_t$ and note that products involving only shocks can be put in the *t.i.p.* part, as they cannot be affected by stabilisation policies. Also, $S_t^J \equiv S_{J,t}$ and $g_t^J \equiv g_{J,t}$ to be in line with the notation used in the main text. Rewriting in gaps from fluctuations under flexible prices yields

$$\begin{aligned}
& n\frac{1-\rho}{2}\hat{C}_t^2 + (1-n)\frac{1-\rho^*}{2}\hat{C}_t^{*2} \\
& -A_1\frac{\eta_H+1}{2}\left(\hat{Y}_{H,t} - \tilde{\Psi}_{H,t}\right)^2 - A_2\frac{\eta_{F+1}}{2}\left(\hat{Y}_{F,t}^2 - \tilde{\Psi}_{F,t}\right)^2 \\
& -A_3\frac{\eta_N+1}{2}\left(\hat{Y}_{N,t} - \tilde{\Psi}_{N,t}\right)^2 - A_4\frac{\eta_{N^*}+1}{2}\left(\hat{Y}_{N^*,t} - \tilde{\Psi}_{N^*,t}\right)^2 \\
& +t.i.p.
\end{aligned}$$

where

$$\begin{aligned}
\tilde{\Psi}_{H,t} &= \frac{\Phi^H + \eta_H S_{H,t}^* + g_{H,t}}{\eta_{H+1}}, \qquad \tilde{\Psi}_{F,t} = \frac{\left(\Phi^F + \eta_F S_{F,t} + g_{F,t}\right)}{\eta_{F+1}} \\
\tilde{\Psi}_{N,t} &= \frac{\left(\Phi^N + \eta_N S_{N,t} + g_{N,t}\right)}{\eta_N + 1}, \qquad \tilde{\Psi}_{N^*,t} = \frac{\left(\Phi^{N^*} + \eta_{N^*} S_{N^*,t} + g_{N^*,t}\right)}{\eta_{N^*} + 1}
\end{aligned}$$

$\tilde{\Psi}_{J,t}$ summarises all terms that affect sectoral output under flexible prices as well as steady state distortions Φ^J that arise from real rigidities. Note that we derived under (A.4.4) that Φ^J terms are determined by

$$\begin{aligned}
\Phi^H &= \rho c + \eta_H y_H + (1-n)\,t - (1-\gamma)\,q \\
\Phi^N &= \rho c + \eta_N y_N + \gamma q \\
\Phi^F &= \rho^* c^* + \eta_F y_F - nt - (1-\gamma^*)\,q^* \\
\Phi^{N^*} &= \rho^* c^* + \eta_{N^*} y_{N^*} + \gamma^* q^*
\end{aligned}$$

When $\Phi^J = 0$, there is the same steady state referred to whether under flexible or sticky prices, as all gaps between inefficient and efficient steady state allocations are closed, $X = X^{eff}$. Under this condition, there arises no equilibrium inflationary bias in monetary policy setting which would be triggered in case $\Phi^J > 0$ where $\Phi^J > 0$ indicates that steady state allocations are inefficiently low[13] The effort in monetary policy to bring steady state allocations C, C^* and Y_J closer to their first best values C^{eff}, C^{*eff}, Y_J^{eff} would be foreseen by households forming rational expectations. In the welfare analyses, we assume that the fiscal redistribution scheme is fully efficient such that all profits from monopolistic competition are rebated and $\Phi^J = 0$ in all sectors. Depending on the degree of product heterogeneity, this requires sector-specific values for τ^J.

Finally, we rewrite overall consumption in terms of union and relative consumption gaps. Use that for any variable that can be defined as $X^U \equiv nX + (1-n)X^*$ it also holds that $nX^2 + (1-n)\left(X^*\right)^2 \equiv \left(X^U\right)^2 + n(1-n)(X - X^*)^2$. By assuming that $\rho = \rho^*$ we can further simplify the consumption terms

$$\begin{aligned}
n\frac{1-\rho}{2}\hat{C}_t^2 + (1-n)\frac{1-\rho^*}{2}\hat{C}_t^{*2} &= \frac{1-\rho}{2}\left(n\hat{C}_t^2 + (1-n)\hat{C}_t^{*2}\right) \\
&= \frac{1-\rho}{2}\left(\left(\hat{C}_t^U\right)^2 + n(1-n)\left(\hat{C}_t - \hat{C}\right)^2\right)
\end{aligned}$$

[13] Note that $x = -\ln(X/X^{eff})$ such that when $X < X^{eff}$, $x > 0$.

Now rewrite the welfare function altogether as $w^U_{t+s} = \mathcal{P}^U_{t+s} - \mathcal{Q}^U_{t+s} - \mathcal{R}^U_{t+s}$

$$\begin{aligned}
w^U_t &= \lambda_C \left(\hat{C}^U_t\right)^2 + \lambda_C n(1-n)\left(\hat{C}_t - \hat{C}^*_t\right)^2 \\
&\quad -\frac{\eta_H+1}{2}\lambda_H\left(\hat{Y}_{H,t} - \tilde{\Psi}_{H,t}\right)^2 - \frac{\eta_{F+1}}{2}\lambda_F\left(\hat{Y}_{F,t} - \tilde{\Psi}_{F,t}\right)^2 \\
&\quad -\frac{\eta_N+1}{2}\lambda_N\left(\hat{Y}_{N,t} - \tilde{\Psi}_{N,t}\right)^2 - \frac{\eta_{N^*}+1}{2}\lambda_{N^*}\left(\hat{Y}_{N^*,t} - \tilde{\Psi}_{N^*,t}\right)^2 \\
&\quad -\lambda_H\left(\lambda_{\pi_H}\pi^2_{H,t} + \lambda_{\triangle\pi_H}\left(\triangle\pi_{H,t}\right)^2\right) - \lambda_F\left(\lambda_{\pi_{F^*}}\pi^2_{F^*,t} + \lambda_{\triangle\pi_{F^*}}\left(\triangle\pi_{F^*,t}\right)^2\right) \\
&\quad -\lambda_N\left(\lambda_{\pi_N}\pi^2_{N,t} + \lambda_{\triangle\pi_N}\left(\triangle\pi_{N,t}\right)^2\right) - \lambda_{N^*}\left(\lambda_{\pi_{N^*}}\pi^2_{N^*,t} + \lambda_{\triangle\pi_{N^*}}\left(\triangle\pi_{N^*,t}\right)^2\right) \\
&\quad +t.i.p.
\end{aligned}$$

which is the expression (2.110) in the text, where $L^U_t \equiv -w^U_t$ and $\triangle$ expresses the difference operator. Deep parameters are collected in

$$\begin{aligned}
\lambda_C &= \frac{1-\rho}{2}, \qquad \lambda_H = n\frac{Q^{1-\gamma}}{T^{1-n}}\frac{Y_H}{C}, \qquad \lambda_F = (1-n)\frac{Q^{*1-\gamma^*}}{T^{-n}}\frac{Y^*_F}{C^*}, \\
\lambda_N &= n\frac{C_N}{C}Q^{-\gamma}, \qquad \lambda_{N^*} = (1-n)\frac{C^*_{N^*}}{C^*}Q^{*-\gamma^*}
\end{aligned}$$

Consumption shares weighted by relative prices can be further simplified. Use that in steady state $Q = Q^* = 1$ and that the expenditure share on non-tradable goods is $\frac{P_N C_N}{PC} = 1-\gamma$. The latter result is obtained from rewriting the consumption basket for C_t and using that from the optimal sharing rules $Q = \frac{\gamma}{1-\gamma}\frac{C_N}{C_T}$

$$\begin{aligned}
C &= C^\gamma_T C^{1-\gamma}_N \frac{1}{\gamma^\gamma(1-\gamma)^{1-\gamma}} \\
\frac{PC}{P_N C_N} &= \frac{P}{P_N}\left(\frac{C_T}{C_N}\right)^\gamma \frac{1}{\gamma^\gamma(1-\gamma)^{1-\gamma}} = \frac{P}{P_N}\left(\frac{\gamma}{1-\gamma}Q^{-1}\right)^\gamma \frac{1}{\gamma^\gamma(1-\gamma)^{1-\gamma}}
\end{aligned}$$

As $P_t = (P_{T,t})^\gamma (P_{N,t})^{1-\gamma} = (\frac{P_{T,t}}{P_{N,t}})^\gamma P_{N,t} = Q^\gamma_t P_{N,t}$ one arrives at

$$\frac{PC}{P_N C_N} = \frac{\gamma^\gamma}{(1-\gamma)^\gamma}\frac{1}{\gamma^\gamma(1-\gamma)^{1-\gamma}} = \frac{1}{1-\gamma}$$

and therefore $\frac{Q^{-\gamma}C_N}{C} = 1-\gamma$, $\frac{Q^{*-\gamma^*}C^*_{N^*}}{C^*} = 1-\gamma^*$. Accordingly for tradables

$$\begin{aligned}
\frac{Q^{1-\gamma}}{T^{1-n}}\frac{Y_H}{C} &= \frac{Q^{1-\gamma}}{T^{1-n}}\frac{T^{1-n}C^U_T + G_H}{C} = Q^{1-\gamma}\frac{C^U_T}{C} \\
\frac{Q^{*1-\gamma^*}C^U_T}{C^*} &= \frac{Q^{*1-\gamma*}}{T^{-n}}\frac{T^{-n}C^U_T + G_F}{C^*} = Q^{*1-\gamma*}\frac{C^U_T}{C^*} = Q^{*1-\gamma*}\frac{C^U_T}{C}
\end{aligned}$$

In the latter step we used that $C = C^*$.by full risk sharing. $\frac{C^U_T}{C}$ is given by

$$\frac{C^U_T}{C} = n\frac{C_T}{C} + (1-n)\frac{C^*_T}{C} = n\gamma + (1-n)\gamma^*$$

where we again used that for the expenditure shares $\frac{P_T C_T}{PC} = \gamma$ as well as $\frac{P^*_T C^*_T}{P^* C^*} = \gamma^*$.

A.8 The Model in Log-Linearised Form

A.8.1 Non-Expectational Equations

Goods market equilibrium under sticky and flexible prices is given by

$$\begin{aligned}
\hat{Y}_{H,t} &\equiv (1-n)\hat{T}_t + \hat{C}^U_{T,t} + g_{H,t} && \text{(A.131)}\\
\tilde{Y}_{H,t} &\equiv (1-n)\tilde{T}_t + \tilde{C}^U_{T,t} + g_{H,t} && \text{(A.132)}\\
\hat{Y}_{N,t} &\equiv \hat{C}_{N,t} + g_{N,t} && \text{(A.133)}\\
\tilde{Y}_{N,t} &\equiv \tilde{C}_{N,t} + g_{N,t} && \text{(A.134)}\\
\hat{Y}_{F,t} &\equiv -n\hat{T}_t + \hat{C}^U_{T,t} + g^*_{F,t} && \text{(A.135)}\\
\tilde{Y}_{F,t} &\equiv -n\tilde{T}_t + \tilde{C}^U_{T,t} + g^*_{F,t} && \text{(A.136)}\\
\hat{Y}_{N^*,t} &\equiv \hat{C}^*_{N^*,t} + g^*_{N^*,t} && \text{(A.137)}\\
\tilde{Y}_{N^*,t} &\equiv \tilde{C}^*_{N^*,t} + g^*_{N^*,t} && \text{(A.138)}
\end{aligned}$$

Evolvement of sectoral consumption under flexible prices reads

$$\begin{aligned}
\tilde{C}_{T,t} &= \frac{1-\rho}{\rho}(1-\gamma)\tilde{Q}_t - \frac{1+\eta_H}{\rho}(1-n)\tilde{T}_t - \frac{\eta_H}{\rho}\tilde{C}^U_{T,t} + \frac{\eta_H}{\rho}\left(S_{H,t} - g_{H,t}\right) && \text{(A.139)}\\
\tilde{C}^*_{T,t} &= \frac{1-\rho^*}{\rho^*}(1-\gamma^*)\tilde{Q}^*_t - \frac{1+\eta^*_F}{\rho^*}(-n)\tilde{T}_t - \frac{\eta_F}{\rho^*}\tilde{C}^U_{T,t} + \frac{\eta_F}{\rho^*}\left(S_{F,t} - g_{F,t}\right) && \text{(A.140)}\\
\tilde{C}_{N,t} &= \gamma\frac{\rho-1}{\rho+\eta_N}\tilde{Q}_t + \frac{\eta_N}{\rho+\eta_N}\left(S_{N,t} - g_{N,t}\right) && \text{(A.141)}\\
\tilde{C}^*_{N^*,t} &= \gamma^*\frac{\rho^*-1}{\rho^*+\eta_{N^*}}\tilde{Q}^*_t + \frac{\eta_{N^*}}{\rho^*+\eta_{N^*}}\left(S_{N^*,t} - g_{N^*,t}\right) && \text{(A.142)}
\end{aligned}$$

Real marginal cost dynamics are given by

$$\begin{aligned}
\widehat{MC}^H_t(h) &= (1-n)(1+\eta_H)(\hat{T}_t - \tilde{T}) - (1-\gamma)(\hat{Q}_t - \tilde{Q}_t)\\
&\quad +\eta_H(\hat{C}^U_{T,t} - \tilde{C}^U_{T,t}) + \rho\left(\hat{C}_t - \tilde{C}_t\right) && \text{(A.143)}\\
\widehat{MC}^F_t(f) &= -n(1+\eta_F)(\hat{T}_t - \tilde{T}) - (1-\gamma^*)(\hat{Q}^*_t - \tilde{Q}^*_t)\\
&\quad +\eta_F(\hat{C}^U_{T,t} - \tilde{C}^U_{T,t}) + \rho^*(\hat{C}^*_t - \tilde{C}^*_t) && \text{(A.144)}\\
\widehat{MC}_{N,t}(h) &= \eta_N(\hat{C}_{N,t} - \tilde{C}_{N,t}) + \rho\left(\hat{C}_t - \tilde{C}_t\right) + \gamma(\hat{Q}_t - \tilde{Q}_t) && \text{(A.145)}\\
\widehat{MC}_{N^*,t}(h) &= \eta_{N^*}(\hat{C}^*_{N^*,t} - \tilde{C}^*_{N^*,t}) + \rho^*\left(\hat{C}^*_t - \tilde{C}^*_t\right) + \gamma^*(\hat{Q}^*_t - \tilde{Q}^*_t) && \text{(A.146)}
\end{aligned}$$

Intranational and international relative prices evolve according to

$$\begin{aligned}
\hat{T}_t &= \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t} && \text{(A.147)}\\
\tilde{Q}_t &= \tilde{C}_{N,t} - \tilde{C}_{T,t} && \text{(A.148)}\\
\hat{Q}_t &= \hat{C}_{N,t} - \hat{C}_{T,t} && \text{(A.149)}\\
\hat{Q}_t &= \hat{Q}_{t-1} + \pi_{T,t} - \pi_{N,t} && \text{(A.150)}\\
\tilde{Q}^*_t &= \tilde{C}^*_{N^*,t} - \tilde{C}^*_{T,t} && \text{(A.151)}\\
\hat{Q}^*_t &= \hat{C}^*_{N^*,t} - \hat{C}^*_{T,t} && \text{(A.152)}\\
\hat{Q}^*_t &= \hat{Q}^*_{t-1} + \pi_{T,t} - \pi_{N^*,t} && \text{(A.153)}
\end{aligned}$$

The interest rate rule is given by

$$\begin{aligned}\hat{\imath}_t &= r_i \hat{\imath}_{t-1} + (1-r_i)\left\{\bar{\pi}_t + r_\pi\left(\pi^U_{t-1} - \bar{\pi}_t\right) + r_Y\left(\hat{Y}^U_t - \tilde{Y}^U_t\right)\right\} \\ &\quad + r_{\triangle\pi}\left(\pi^U_t - \pi^U_{t-1}\right) + r_{\triangle Y}\left(\hat{Y}^U_t - \tilde{Y}^U_t - \left(\hat{Y}^U_{t-1} - \tilde{Y}^U_{t-1}\right)\right) \qquad \text{(A.154)}\end{aligned}$$

Identities

$$\hat{C}_t = \gamma \hat{C}_{T,t} + (1-\gamma)\,\hat{C}_{N,t} \qquad \text{(A.155)}$$
$$\hat{C}^*_t = \gamma^* \hat{C}^*_{T,t} + (1-\gamma^*)\,\hat{C}^*_{N^*,t} \qquad \text{(A.156)}$$
$$\hat{C}^U_t = n\hat{C}_t + (1-n)\,\hat{C}^*_t \qquad \text{(A.157)}$$
$$\tilde{C}^U_t = n\tilde{C}_t + (1-n)\,\tilde{C}^*_t \qquad \text{(A.158)}$$
$$\pi^U_t = n\pi_t + (1-n)\,\pi^*_t \qquad \text{(A.159)}$$

A.8.2 Expectational Equations

Euler equations for overall consumption are

$$\hat{C}_t = \mathcal{E}_t\hat{C}_{t+1} - \frac{1}{\rho}(\hat{\imath}_t - \mathcal{E}_t\pi_{t+1}) \qquad \text{(A.160)}$$
$$\hat{C}^*_t = \mathcal{E}_t\hat{C}^*_{t+1} - \frac{1}{\rho}(\hat{\imath}_t - \mathcal{E}_t\pi^*_{F,t+1}) \qquad \text{(A.161)}$$
$$\tilde{C}^U_t = \mathcal{E}_t\tilde{C}^U_{t+1} - \left(n\frac{1}{\rho} + (1-n)\frac{1}{\rho^*}\right)\tilde{\imath}_t \qquad \text{(A.162)}$$

Hybrid sectoral Phillips-curves are given by

$$\pi_{H,t} = \lambda^b_H \pi_{H,t-1} + \frac{\lambda^{mc}_H}{\kappa^{mc}_H}\widehat{MC}_{H,t} + \lambda^f_H \mathcal{E}_t \pi_{H,t+1} \qquad \text{(A.163)}$$
$$\pi_{N,t} = \lambda^b_N \pi_{N,t-1} + \frac{\lambda^{mc}_N}{\kappa^{mc}_N}\widehat{MC}_{N,t} + \lambda^f_N \mathcal{E}_t \pi_{N,t+1} \qquad \text{(A.164)}$$
$$\pi_{F,t} = \lambda^b_F \pi_{F,t-1} + \frac{\lambda^{mc}_F}{\kappa^{mc}_F}\widehat{MC}_{F,t} + \lambda^f_F \mathcal{E}_t \pi_{F,t+1} \qquad \text{(A.165)}$$
$$\pi_{N^*,t} = \lambda^b_{N^*} \pi_{N^*,t-1} + \frac{\lambda^{mc}_{N^*}}{\kappa^{mc}_{N^*}}\widehat{MC}_{N^*,t} + \lambda^f_{N^*} \mathcal{E}_t \pi_{N^*,t+1} \qquad \text{(A.166)}$$

A.8.3 Exogenous Stochastic Processes

Sectoral supply shocks evolve according to

$$S_{H,t+1} = \rho_{S_H} S_{H,t} + \epsilon_{S_H,t+1} \qquad \text{(A.167)}$$
$$S_{N,t+1} = \rho_{S_N} S_{N,t} + \epsilon_{S_N,t+1} \qquad \text{(A.168)}$$
$$S_{F,t+1} = \rho_{S_F} S_{F,t} + \epsilon_{S_F,t+1} \qquad \text{(A.169)}$$
$$S_{N^*,t+1} = \rho_{S_{N^*}} S_{N^*,t} + \epsilon_{S_{N^*},t+1} \qquad \text{(A.170)}$$

and the common industry supply shock is

$$S_{T,t+1} = \rho_{S_T} S_{T,t} + \epsilon_{S_T,t+1}$$

Fiscal rules (demand shocks) obey

$$g_{H,t+1} = \rho_{g_H} g_{H,t} + \epsilon_{g_H,t+1} \tag{A.171}$$
$$g_{N,t+1} = \rho_{g_N} g_{N,t} + \epsilon_{g_N,t+1} \tag{A.172}$$
$$g_{F,t+1} = \rho_{g_F} g_{F,t} + \epsilon_{g_F,t+1} \tag{A.173}$$
$$g_{N^*,t+1} = \rho_{g_{N^*}} g_{N^*,t} + \epsilon_{g_{N^*},t+1} \tag{A.174}$$

Terms of trade follow

$$\tilde{T}_t = \frac{\eta}{1+\eta}\left(G_t^R - S_t^R\right) \tag{A.175}$$

Appendix B

Appendix to Chapter 3

B.1 Additional Tables

Optimal policy					Maastricht constrained optimal policy					
Weighted loss contribution				$\mathcal{E}\left[L_t^o\right]$	Weighted loss contribution					$\mathcal{E}\left[L_t^o\right]$
	$\hat{Y}_H^o$	$\hat{Y}_N^o$	$\pi_t^o - \pi_t^{*,o}$	$(\%)^2$	$\hat{Y}_H^o$	$\hat{Y}_N^o$	$\pi_t^o - \pi_t^{*,o}$	$i_t^o - \tilde{i}_t^{*,o}$	$\hat{S}_t^o$	$(\%)^2$
Visegrad States										
CZ	1.6	0.2	0.4	2.3	1.6	0.2	1.5	0.2	0.1	3.5
HU	1.9	0.2	0.6	2.7	1.9	0.2	0.9	0.1	0.0	3.0
PL	1.6	0.1	0.0	1.7	1.5	0.2	0.0	1.1	0.0	2.8
SK	1.9	0.1	0.0	2.0	1.9	0.1	0.1	1.0	198.4	201.5
Baltic States										
EE	1.0	0.2	0.5	1.6	1.0	0.2	0.6	0.2	0.0	1.9
LV	1.1	0.1	0.5	1.7	1.1	0.1	0.6	0.1	0.0	2.0
LT	1.0	0.1	0.7	1.8	1.0	0.1	0.8	0.1	0.0	2.0
2007 Entrants										
BG	0.3	0.1	0.6	0.9	0.3	0.1	1.0	0.1	0.0	1.4
RO	0.2	0.1	0.7	1.0	0.2	0.1	0.9	0.1	0.0	1.3

Table B.1: Contribution to stabilisation costs by policy objectives. Weight of sectoral output corresponds to weight in GDP, $\mu_J = \frac{Y_J}{Y}$. Contributions derived from unconditional (analytical) variance of each objective under the loss-minimising monetary policy.

Volatility	Historical (1999q1-2007q1)					Estimated DSGE					
	Y_H	π_H	Y_N	π_N	Y	π	Y_H	π_H	Y_N	π_N	Y	π
Visegrad States												
CZ	3.8	5.0	0.3	2.8	1.1	0.8	2.1	2.0	1.2	1.5	1.2	1.2
HU	2.0	2.7	0.2	2.1	0.4	1.0	2.5	4.1	0.8	3.7	1.1	2.8
PL	3.0	4.1	0.2	1.9	1.2	1.0	1.0	4.8	0.8	4.7	0.6	13.7
SK	4.9	3.6	0.8	2.9	2.3	1.8	2.3	2.2	0.6	1.5	1.0	1.2
Baltic States												
EE	3.3	1.4	0.3	1.4	1.7	0.7	2.0	1.5	0.9	0.9	0.8	0.7
LV	1.3	2.7	0.4	1.2	1.7	0.9	2.5	1.6	0.8	0.9	0.9	0.7
LT	2.9	3.4	0.3	1.7	1.6	0.8	1.8	1.4	0.9	0.9	0.8	0.8
2007 Entrants												
BG	-	-	-	-	-	1.9	1.1	1.0	0.8	0.9	0.7	0.8
RO	-	-	-	-	-	3.5	1.1	1.1	0.8	1.0	0.7	0.8

Persistence	Historical (1999q1-2007q1)					Estimated DSGE					
	Y_H	π_H	Y_N	π_N	Y	π	Y_H	π_H	Y_N	π_N	Y	π
Visegrad States												
CZ	0.71	0.12	0.24	0.04	0.91	-0.13	0.71	0.67	0.57	0.79	0.57	0.79
HU	0.74	0.00	0.71	-0.13	0.78	0.31	0.79	0.88	0.78	0.94	0.74	0.91
PL	0.85	-0.05	0.19	-0.22	0.63	0.51	0.69	0.97	0.83	0.99	0.79	0.18
SK	0.50	-0.09	0.43	-0.23	0.41	-0.05	0.76	0.71	0.67	0.89	0.74	0.82
Baltic States												
EE	0.49	0.10	0.55	-0.25	0.54	0.16	0.73	0.63	0.91	0.90	0.84	0.67
LV	0.29	0.30	0.80	0.31	0.78	0.29	0.87	0.61	0.90	0.89	0.88	0.63
LT	0.38	-0.02	0.53	0.30	0.64	0.21	0.74	0.64	0.91	0.89	0.83	0.73
2007 Entrants												
BG	-	-	-	-	-	0.09	0.81	0.84	0.88	0.92	0.87	0.85
RO	-	-	-	-	-	0.81	0.86	0.86	0.89	0.91	0.89	0.70

Cross-correlation	Historical (1999q1-2007q1)			Estimated DSGE		
	$\triangle Y_H,\pi_H$	$\triangle Y_N$, π_N	$\triangle Y,\pi$	$\triangle Y_H,\pi_H$	$\triangle Y_N$, π_N	$\triangle Y$, π
Visegrad States						
CZ	0.07	0.13	0.04	-0.34	-0.27	-0.35
HU	-0.31	-0.10	-0.15	-0.26	-0.07	-0.12
PL	-0.25	0.06	0.04	0.06	0.06	-0.16
SK	0.44	0.18	-0.02	-0.36	-0.02	-0.18
Baltic States						
EE	0.28	-0.18	0.01	-0.30	-0.04	-0.20
LV	-0.05	0.05	0.08	-0.39	-0.02	-0.17
LT	0.02	0.28	-0.03	-0.25	-0.05	-0.17
2007 Entrants						
BG	-	-	-	-0.24	-0.09	-0.18
RO				-0.27	-0.11	-0.17

Table B.2: Comparing historical and model based volatility in output and producer prices. Standard deviation is in percentage points. Inflation is quarter-on-quarter. Persistence measured by first order autocorrelation, quarter on quarter. Cross-correlation is contemporaneous correlation between quantities and prices

B.2 The Steady State

The steady state is the general equilibrium of the economy where stochastic disturbances are zero

$$\left\{\varepsilon_{A_H}, \varepsilon_{A_N}, \varepsilon_{G_H}, \varepsilon_{G_N}, \epsilon_{C_{H^*}}, \varepsilon_{\tilde{i}^*}, \varepsilon_{P^*}, \varepsilon_{ZR}, \varepsilon_i\right\} = 0_{1\times 9}$$

Prices are fully flexible such that nominal rigidities are absent

$$\theta_J = \varpi_J = 0$$

All real variables in levels therefore grow along a balanced growth path with constant rate (typically the growth rate of steady state technology). As we log-linearise about a zero inflation steady state, the log-deviation of the inflation rate about its equilibrium value equals the inflation rate itself

$$\hat{\pi}_t \equiv \hat{P}_t - \hat{P}_{t-1} = \ln P_t - \ln P_t^{ss} - \left(\ln P_{t-1} - \ln P_{t-1}^{ss}\right) = \pi_t - \pi_t^{ss} = \pi_t \tag{B.1}$$

We obtain from the Euler equation determined under (4.17) in chapter 4 that subjective and market discounting are linked by $i = \frac{1-\beta}{\beta}$ which implies an annualised rate of return for holding nominal bonds for three month of $(1+i)^4 - 1 = (1/\beta)^4 - 1 \simeq 4.1\%$. The quarterly steady state net return on physical capital R^J (the net return on renting out physical capital for one quarter), before real depreciation δ is taken into account, is identical in both sectors which arises as an arbitrage condition. It equals the return on the nominal bond i adjusted for δ

$$R^J = \frac{1-\beta(1-\delta)}{\beta} = \frac{1}{\beta} - (1-\delta) = i + \delta \tag{B.2}$$

Given a constant price level in steady state, the physical return on capital (the natural rate of interest in the economy or the gross Euler rate) equals the nominal return on bonds after depreciation. Consequently, the household as investor is indifferent when choosing between buying bonds or renting out capital to the firm.[1] By the factor demand for physical capital, we get steady state capital productivity in the industry sector

$$\frac{P_H Y_H}{P_H K^H} = \frac{1}{\alpha_H} \frac{1-\beta(1-\delta)}{\beta} \frac{\rho_H}{\rho_H - 1} \tag{B.3}$$

after using (C.2) and the equation for steady state real marginal cost, $MC_J = \left(\frac{\rho_H}{\rho_H - 1}\right)^{-1}$. From labour demand, we obtain sectoral labour productivity (sectoral output by hours worked, measured by gross value added $P_J Y_J$ over sectoral overall labour compensation[2] $W_J L_J$)

$$\frac{P_H Y_H}{W_H L_H} = \frac{1}{1-\alpha_H} \frac{\rho_H}{\rho_H - 1}, \qquad \frac{P_N Y_N}{W_N L^N} = \frac{1}{1-\alpha_N} \frac{\rho_N}{\rho_N - 1} \tag{B.4}$$

It follows that only under fully competitive markets, where the price elasticities of demand are very elastic, $\rho_J \to \infty$, labour income shares $\left(\frac{P_J Y_J}{W_J L_J}\right)^{-1}$ are equal to $1 - \alpha_J$.[3]

[1] For $\beta = 0.99$ and $\delta = 0.025$, the values chosen later on in the calibration, we obtain that this 'natural rate of interest' is $R^J = (1/0.99) - (1 - 0.025) \simeq 3.5\%$ or 14.8% at an annualised rate.

[2] Hours worked is a flow variable, whereas if we would calculate labour productivity by manpower employed, we would have to take into account that the latter is a stock.

[3] Details on the derivation of $\frac{P_H C_H}{P_H Y_H}$ and $\frac{P_N C_N}{P_N Y_N}$ as well as the derivation of the current account and the balance of payments can be found in the steady state section in chapter 4.

B.3 National Account and Related Items

For the given market equilibrium, we can infer the national account items. GDP_t (total gross value added at market prices) is given by the value of industry goods (tradables) $Y_{H,t}$ and services (non-tradables) $Y_{N,t}$ evaluated at market prices $P_{H,t}$, $P_{N,t}$ within the period under report, which is one quarter. Therefore

$$GDP_t^S \equiv P_{H,t}Y_{H,t} + P_{N,t}Y_{N,t} \tag{B.5}$$

Note that $P_{H,t}$ and $P_{N,t}$ are producer price indices. From the expenditure side, GDP is the sum of final consumption expenditures of households, expenditures on gross fixed investment, net exports and government expenditures

$$\begin{aligned} GDP_t^D &\equiv \underbrace{(P_{H,t}C_{H,t} + P_{N,t}C_{N,t})}_{C_t^{nom}} + \underbrace{(P_{H,t}I_{H,t} + P_{N,t}I_{N,t})}_{I_t^{nom}} \\ &\quad + \underbrace{\left(P_{H,t}C^*_{H,t} - P_{F,t}C_{F,t}\right)}_{EX_t^{nom} - IM_t^{nom}} + \underbrace{(P_{H,t}G_{H,t} + P_{N,t}G_{N,t})}_{G_t^{nom}} \\ &\equiv C_t^{nom} + I_t^{nom} + EX_t^{nom} - IM_t^{nom} + G_t^{nom} \end{aligned}$$

Expenditures on imports of the tradable good have to be subtracted from the right hand side, as they are already included in domestic private consumption. From the identity $GDP_t^S \equiv GDP_t^D$ one obtains

$$P_{H,t}Y_{H,t} + P_{N,t}Y_{N,t} \equiv C_t^{nom} + I_t^{nom} + EX_t^{nom} - IM_t^{nom} + G_t^{nom} \tag{B.6}$$

In order to get real GDP, we need to calculate the GDP deflator, which is a Paasche index. The GDP deflator is given by

$$defl_t \equiv \frac{P_{H,t}Y_{H,t} + P_{N,t}Y_{N,t}}{P_H Y_{H,t} + P_N Y_{N,t}} \tag{B.7}$$

where the base year in the denominator reflects the price levels in the initial steady state.[4] We obtain real GDP (the volume of GDP) by measuring in constant prices $GDP_t^r \equiv P_H Y_{H,t} + P_N Y_{N,t} \equiv Y_t$ where P_J denotes the steady state price level in sector J.[5]

Furthermore, sectoral deflators from the *production side of GDP* (producer price deflators) are given by

$$defl_{Y_H,t} = \frac{P_{H,t}Y_{H,t}}{P_H Y_{H,t}}, \qquad defl_{Y_N,t} = \frac{P_{N,t}Y_{N,t}}{P_N Y_{N,t}} \tag{B.8}$$

Hence, the log change in the sectoral deflators equals the sectoral producer price inflation rates

$$\ln defl_{Y_J,t} - \ln defl_{Y_J,t-1} = \pi_{J,t} = \hat{\pi}_{J,t}$$

where the latter relationship derives from the fact that we log-linearise about a zero inflation steady state.[6]

[4] Accordingly, deflators for the *expenditure side of GDP* can be derived. The deflator for private consumption expenditures is given by $defl_{C,t} = \frac{P_{T,t}C_{T,t}+P_{N,t}C_{N,t}}{P_T C_{T,t}+P_N C_{N,t}}$. Similar deflators can be derived in case of expenditures on gross fixed capital formation, net exports and government spending.

[5] In the absence of a two-sector production, output equals GDP only if the constant price level is normalised to one. Studying quantity effects (changes in Y_J) and price effects on a sectoral level requires a more realistic setup for the production side of GDP like the one presented here. For aggregate output, we can only study real GDP changes.

[6] The model inflation rates denote the change in price indices that result from minimising expenditures for a given quantity, i.e. from a (producer) price index formulation (Laspeyres concept). As the composition of

B.4 Further Details on the Log-Linearisation of the Model

In this section we spell out parts of the log-linearisation of the model in some detail. Most of the log-linearisations can be found in the appendix to chapter 4 of the dissertation. Log-linearising nominal GDP we get

$$\widehat{GDP}_t \equiv \frac{P_H Y_H}{GDP}\left(\hat{P}_{H,t} + \hat{Y}_{H,t}\right) + \frac{P_N Y_N}{GDP}\left(\hat{P}_{N,t} + \hat{Y}_{N,t}\right) \tag{B.9}$$

where $\hat{P}_{J,t}$ denote log-deviations from the initial price level P_J, i.e. $\hat{P}_{J,t} = \ln\left(P_{J,t}/P_J\right)$. As all goods produced at home are either tradables or non-tradables, we can write $\frac{P_N Y_N}{GDP} = 1 - \frac{P_H Y_H}{GDP}$. Note that P_H and P_N refer to price levels in the initial steady state. As real GDP is expressed in initial steady state price levels that by definition are time invariant, price levels drop out when calculating real GDP. We obtain that real GDP equals aggregate output

$$\widehat{GDP}^r_t = \frac{P_H Y_H}{P_H Y_H + P_N Y_H}\hat{Y}_{H,t} + \frac{P_N Y_N}{P_H Y_H + P_N Y_H}\hat{Y}_{N,t} = \frac{P_H Y_H}{GDP}\hat{Y}_{H,t} + \left(1 - \frac{P_H Y_H}{GDP}\right)\hat{Y}_{N,t} \tag{B.10}$$

Note that we obtain the growth rates of nominal and real GDP by first differencing $\widehat{GDP}_t$ and $\widehat{GDP}^r_t$. For the deflator we obtain

$$\frac{\ln defl_t}{\ln defl} \equiv \ln\left(\frac{GDP_t}{GDP^r_t} / \frac{GDP}{GDP^r}\right) = \widehat{GDP}_t - \widehat{GDP}^r_t \tag{B.11}$$

For government spending, we obtain

$$\hat{G}_t = \frac{P_H G_H}{G}\hat{G}_{H,t} + \frac{P_N G_N}{G}\hat{G}_{N,t}$$

where

$$\frac{P_H G_H}{G} = \frac{P_H G_H/GDP}{G/GDP}, \qquad \frac{P_N G_N}{G} = \frac{P_N G_N/GDP}{G/GDP}$$

$P_H G_H/GDP = \frac{P_H G_H}{P_H Y_H}\frac{P_H Y_H}{GDP}$ and $P_N G_N/GDP = G/GDP - P_H G_H/GDP = \frac{P_N G_N}{P_N Y_N}\frac{P_N Y_N}{GDP}$, as becomes clear by using above equilibrium condition (3.24). Log-linearising the real net foreign asset position, defining real variables $\frac{Z_t}{P_t} = Z^r_t$ and using steady state items $\frac{F}{P} = F^r$, $\frac{Z}{P} = Z^r$, one gets

$$\hat{F}^r_t = \frac{I}{1+\pi}\hat{\imath}_{t-1} + \frac{I}{1+\pi}\hat{F}^r_{t-1} - \frac{I}{1+\pi}\pi_t + \frac{NX^r}{F^r}(\widehat{NX}_t - \hat{P}_t) - \frac{Z^r}{F^r}(\hat{Z}^r_t - \hat{Z}^r_{t-1} + \pi_t)$$

We make use of the steady state conditions $\pi = 0$ and $I = 1 + i = \frac{1}{\beta}$ and the stationarity condition $\frac{NX^r}{F^r} = \frac{NX/P}{F/P} = \frac{NX}{F} = \frac{\beta - 1}{\beta}$. We also employ that real and nominal steady state shares are equal when deflated by the same price index $\frac{Z^r}{F^r} = \frac{Z/P}{F/P} = \frac{Z}{F}$. Then

$$\hat{F}^r_t = \frac{1}{\beta}\hat{\imath}_{t-1} + \frac{1}{\beta}\hat{F}^r_{t-1} - \frac{1}{\beta}\pi_t - \frac{1-\beta}{\beta}(\widehat{NX}_t - \hat{P}_t) - \frac{Z^r}{F^r}(\hat{Z}^r_t - \hat{Z}^r_{t-1} + \pi_t) \tag{B.12}$$

$\hat{\imath}_{t-1} = \ln\frac{1+i_t}{1+i} = i_t - i$ denotes log differences in the nominal interest rate from steady state. We assume that the nominal growth rate of reserves $\hat{Z}^r_t - \hat{Z}^r_{t-1} + \pi_t = \hat{Z}_t - \hat{Z}_{t-1} \equiv dz_t$ follows

firms does not change (there is no firm entry), the concept coincides with the Paasche concept. Consequently, in case we confront the model variables with the data, we correctly equalise the empirical Paasche indices (sectoral deflators) with the model-based sectoral price levels.

an exogenous AR(1) process

$$dz_t = \rho_Z dz_{t-1} + \varepsilon_{Z_t}$$

B.5 Data-Based Calibration of Steady State Shares

In this section we discuss the derivation of steady state shares as presented in section 3.5.2 in table 3.4 in the text. $NX/P_H Y_H$ measures openness of the home economy and can therefore be considered as a measure of globalisation.[7] In the model, this denotes net exports of goods, however empirically there is only data on net trade in both goods and services (there is some tradable component in services empirically).[8] The export content in home industry production, $\frac{P_H C_H^*}{P_H Y_H}$, is obtained from the identity

$$\frac{P_H C_H^*}{P_H Y_H} = \frac{P_H C_H^*}{NX} \frac{NX}{P_H Y_H}$$

where $\frac{P_H C_H^*}{NX}$ are exports as share of net exports and $\frac{NX}{P_H Y_H}$ is calculated from above. We then would obtain values greater than one, which would not make sense in the modelling framework as there are no imported intermediate goods used in production of Y_H, hence always $P_H C_H^* \leq P_H Y_H$. As the same logic applies to $NX/P_H Y_H$, we proxy both measures by the shares of net exports NX and exports $P_H C_H^*$ in relation to GDP. Hence we overstate the role of industry produced goods. As there is no quarterly data available in case of Hungary and Bulgaria on exports as share of GDP, we calculate the ratio by dividing exports by overall gross value added from the national accounts in these cases.[9]

Consumption weights on home tradables v and tradables γ can be taken from the HICP series and choosing item weights according to the COICOP classification. For industry goods we choose IGOODS and use this as a proxy for γ similar to chapter 2. We average the weights that are available at yearly frequency. For services we could pick SERV which is the overall services index excluding goods. However, in our case the measure is already given by $1-\gamma$. v can be obtained residually from the weight of imported industry goods in the price index for tradables. This weight is by definition equal to $1 - v = 1 - \frac{P_H C_H}{P_T C_T}$ which follows from log-linearising total expenditures on tradables around the constant domestic currency price level.[10] Due to the lack of empirical data for this measure, we estimate the parameter. Note that most imported goods are actually energy goods (especially oil) and raw materials. It would therefore be quite misleading to proxy this quantity by some (final goods) import measure. We further estimate the output elasticities α_H and α_N. Data on government sectoral purchases is not available. Overall government expenditures based on final consumption aggregates, $\frac{P_H G_H}{GDP} + \frac{P_N G_N}{GDP}$, are obtained by averaging yearly data. We then assume that purchases fall on each sector in equal amounts.

[7] We use the series for net trade in current prices and exchange rates and divide by total industry gross value added at current prices and exchange rates. Accordingly for exports as share of home gross value added.

[8] Empirically, exports include intermediate goods used in production whereas GDP is the sum of sectoral gross value added.

[9] This calculation would yield a value above 100 per cent for Hungary. We hence pick a value in the upper range of the empirical values. In case of Bulgaria, the resulting C_H/Y_H ratio would be negative - for given government consumption G_H/Y_H and investment I_H/Y_H, we hence limit $\frac{P_H C_H^*}{P_H Y_H}$ to 60%.

[10] Using the Cobb-Douglas specification for the consumption basket and the price index for tradables that results from minimising expenditures, we obtain one unit of C_T

$$\begin{aligned} P_{T,t} C_{T,t} &= P_{H,t} C_{H,t} + P_{F,t} C_{F,t} \\ \hat{P}_{T,t} + \hat{C}_{T,t} &= \frac{P_H C_H}{P_T C_T} \left(\hat{P}_{H,t} + \hat{C}_{H,t} \right) + \frac{P_F C_F}{P_T C_T} \left(\hat{P}_{F,t} + \hat{C}_{F,t} \right) = \nu \left(\hat{P}_{H,t} + \hat{C}_{H,t} \right) + (1-v) \left(\hat{P}_{F,t} + \hat{C}_{F,t} \right) \end{aligned}$$

Accordingly, we obtain $\gamma = \frac{P_T C_T}{PC}$.

We get total hours worked in industry and services from sectoral annual national accounts data.[11] Hours worked is in thousands hours worked per sector as share of overall hours worked. For countries where hours are not available (Latvia, Poland and Romania and the euro area 13) we use sectoral employment as percentage of total. These two measures are very similar empirically.[12] We calculate averages first and then calculate the shares L_J/L of the averaged data. Note that in order to be in accordance with the model structure, we again assume that all hours worked in the non-industry sector are non-tradables hours, i.e. $L_N/L = 1 - L_H/L$.

For the foreign reserves to net foreign assets ratio $\frac{Z}{F}$, there are stationarity restrictions imposed by the model structure to ensure current account sustainability in order to rule out explosive debt paths. As

$$\frac{NX}{P_H Y_H} = \frac{\beta - 1}{\beta} \frac{F}{P_H Y_H}$$

we get that

$$\frac{Z}{F} = \frac{\beta - 1}{\beta} \frac{Z}{NX}$$

We use the series ‘Total reserves including gold’ (TOTRESING) from the monetary and other financial statistics database in millions of national currency at annual frequencies. We express Z as share of external balance of goods and services in millions of national currency. We then average the ratio and therefore assume that the ratio is stationary over time. Note that as Z_t is only available at current prices, we use data on NX_t at current prices for calculating the ratio and then average the annual ratios to get the steady state value.[13]

Given assumptions about elasticities α_J, one can calculate the sector specific markups for given historical capital to output ratios. Hence for historical series on capital services $R_J K_J$ and sectoral gross value added Y_J, one can calculate the capital to output ratio as

$$\frac{P_J K_J}{P_J Y_J} = \frac{R^J P_J K_J}{P_J Y_J} \frac{1}{R^J} = \frac{R^J P_J K_J}{P_J Y_J} \left(\frac{1 - \beta(1 - \delta)}{\beta} \right)^{-1}$$

[11] Sectoral hours worked L_J can be obtained from sectoral labour productivity measured by sectoral output by hours worked $\left(\frac{L_H}{Y_H}\right)^{-1}$. We first calculate capital input elasticities α_J by using that

$$\alpha_{J,i} = \frac{P^{J,i} K^{J,i}}{P_{J,i} Y_{J,i}} \frac{1 - \beta(1 - \delta)}{\beta} \frac{\rho_{N,i}}{\rho_{N,i} - 1}$$

where $P^{J,i} K^{J,i}$ is sectoral gross fixed capital formation in sector J of country i in millions of national currency in constant 1995 prices. Data is taken from annual national accounts. However, gross fixed capital formation can so far not be disentangled sufficiently. Therefore, we set $\alpha_{H,i} = 0.67$ and $\alpha_{N,i} = 0.33$ as Natalucci and Ravenna (2008) propose. We thus argue that industry production has higher capital input. With α_J at hand, we can calculate the implied sectoral labour productivity $\frac{Y_J}{L_J} = \frac{1}{1-\alpha_J} \frac{\rho_J}{\rho_J - 1}$. We then get the share of hours worked in sector J from

$$\frac{L}{L_J} = \frac{Y_J}{L_J} \frac{L}{Y_J} = \frac{1}{1 - \alpha_J} \frac{\rho_J}{\rho_J - 1} \frac{L}{Y_J} = \frac{1}{1 - \alpha_J} \frac{\rho_J}{\rho_J - 1} \frac{L_H + L_N}{Y_J}$$

where $L = L_H + L_N$ and therefore $\frac{L_N}{L} = 1 - \frac{L_H}{L}$. Hence, in order to calculate the ratio, we need total hours worked as well as sectoral output (in actual units Y_J and not in constant prices $P_J Y_J$). We can use $\frac{W_s}{P_{H,s}} = MC_s^H (1 - \alpha_H) \frac{y_{H,s}(z)}{L_{H,s}(z)}$. $\frac{W_s}{P_{H,s}} L_H$ is total compensation per employee and MC_s^H is a measure of real unit labour cost. So far we proxy the shares by sectoral employment over total employment, for which data is easily available.

[12] Note however that hours is a flow variable whereas employment is a stock.

[13] NX_t is taken from the annual national accounts data, from the table ‘Exports and Imports by Member States of the EU/third countries - Current prices’.

using (C.2). From (B.3), the sectoral markup in sector J is obtained as

$$\frac{\rho_J}{\rho_J - 1} = \left(\frac{P_J K_J}{P_J Y_J}\right)^{-1} \alpha_J \frac{\beta}{1-\beta(1-\delta)} = \frac{\alpha_J P_J Y_J}{R^J P_J K_J}\left(\frac{1-\beta(1-\delta)}{\beta}\right)\frac{\beta}{1-\beta(1-\delta)} = \alpha_J \frac{P_J Y_J}{R^J P_J K_J} \tag{B.13}$$

Hence under fully competitive markets, the elasticity of capital factor input α_J times output Y_J equals factor income on capital, $\alpha_J Y_J = R^J K_J$. The data for calculating $\frac{R^J P_J K_J}{P_J Y_J}$ is obtained from the sectoral database EU KLEMS (edition 2007) where we average yearly ratios from 2000 to 2004. We split gross value added where INDUSTRY refers to H. All other items (agriculture, construction, wholesale and retail trade, hotels restaurants, financial intermediation and business services, government services and social security) refer to N. For Poland we use data on Total Manufacturing instead of Total Industry excluding Construction. We explicitly include government services, as otherwise we would have no justification for G_J in the model.

In our approach, sectoral private consumption as share of sectoral cross value added is determined residually. It becomes clear that for given I_H/Y_H and G_H/Y_H and our assumption about $\left(\frac{P_H C_H^*}{P_H Y_H}\right)$ this could produce negative consumption to output ratios for some countries. We therefore set $\left(\frac{P_H C_H^*}{P_H Y_H}\right) = 0.6$ for countries where the value implied by the data is higher than 0.6 and would therefore imply a negative C_H/Y_H ratio. Hence we understate the role of sectoral exports to sectoral output for these countries and therefore the quantitative impact of fluctuations along these lines.[14]

[14] In case of Poland, we set 0.3 instead of the data value of 0.31 as otherwise the rank condition would not be fulfilled.

Appendix C

Appendix to Chapter 4

C.1 Calibration

Description	**Parameter**	**Value**
Home consumption to output ratio H sector	$\frac{C_H}{Y_H}$	0.49324
Home consumption to output ratio N sector	$\frac{C_N}{Y_N}$	0.70414
Investment to output ratio H sector	$\frac{I_H}{Y_H}$	0.39766
Investment to output ratio N sector	$\frac{I_N}{Y_N}$	0.19586
Steady state markup H sector	$(\frac{\rho_H - 1}{\rho_H})^{-1}$	1.2
Steady state markup N sector	$(\frac{\rho_N - 1}{\rho_N})^{-1}$	1.2
Capital to output ratio H sector	$\frac{K_H}{Y_H}$	15.906
Capital to output ratio N sector	$\frac{K_N}{Y_N}$	7.8345
Exports share of H goods	$\frac{C_H^*}{Y_H}$	0.0091

Table C.1: Implied steady state values in the DSGE model for Bulgaria for quarterly data.

Description	Param.	Value	Source
Risk premium	ξ	0.05	Assumed
Share of home-tradables in trad. basket	ν	0.52	Natalucci and Ravenna (2005)
Share of trad in consumption basket	γ	0.5915	Valev (2005)
Inverse of elasticity of labour supply	κ	2	Natalucci and Ravenna (2005)
Depreciation rate of physical capital	δ	0.025	Standard
Demand elasticity of H consumption	ρ_H	6	Dobrinsky et al. (2004)
Demand elasticity of N consumption	ρ_N	6	Dobrinsky et al. (2004)
Household's discount factor	β	0.99	Standard
Share of capital in home trad production	α_H	0.67	Natalucci and Ravenna (2005)
Share of capital in non-trad production	α_N	0.33	Natalucci and Ravenna (2005)
Elasticity of investment-capital ratio	η	0.5	Natalucci and Ravenna (2005)
Share of backw.-looking firms in H sector	ϖ_H	0.4	Lendvai (2005)
Share of backw.-looking firms in N sector	ϖ_N	0.4	Lendvai (2005)
Price resetting probability in H sector	$1-\theta_H$	0.45	Lendvai (2005)
Price resetting probability in N sector	$1-\theta_N$	0.45	Lendvai (2005)
Utility scale parameter real money balances	χ	0.005	Henriksson (2005)
Steady state ratio reserves to assets	$\frac{Z}{F}$	-0.9	Bulgarian National Bank (2006)
Steady st. share of labour supply H sector	$\frac{L_H}{L}$	0.5	Assumed
Steady st. share govt. H consumption	$\frac{G_H}{Y_H}$	0.1	Natalucci and Ravenna (2005)
Steady st share govt. N consumption	$\frac{G_N}{Y_N}$	0.1	Natalucci and Ravenna (2005)
Steady st share exports to net trade balance	$\frac{C_H^*}{NX}$	3/2	Assumed
Coefficient H productivity process	ρ_{Y_H}	$\{0.85, 1\}$	Natalucci and Ravenna (2005)
Coefficient N productivity process	ρ_{Y_N}	$\{0.8, 1\}$	Natalucci and Ravenna (2005)
Coefficient H tradables govt. consumption	ρ_{G_H}	$\{0.43, 1\}$	Natalucci and Ravenna (2005)
Coefficient N non-trad govt. consumption	ρ_{G_N}	$\{0.43, 1\}$	Natalucci and Ravenna (2005)
Coefficient external demand process	$\rho_{C_H^*}$	$\{0.3, 1\}$	Assumed
Coefficient euro area interest rate process	$\rho_{\tilde{\imath}^*}$	$\{0.9, 1\}$	Natalucci and Ravenna (2005)

Table C.2: Calibration of the DSGE model for Bulgaria for quarterly data.

C.2 The Log-Linearised Model

The log-linearised model is described by the following set of equations. The model is solved with the toolkit described in Uhlig (1999). Log-linearisations are carried out in section C.9.

Non-Expectational Equations

$$
\begin{aligned}
0 &= c_t - \gamma c_{T,t} - (1-\gamma)c_{N,t} \\
0 &= c_{T,t} - c_{N,t} - \frac{1}{1-\gamma}i_t + \frac{1}{1-\gamma}c_t - \frac{1}{1-\gamma}m_t + (v + \frac{\gamma v}{1-\gamma})c_{F,t} - (v + \frac{\gamma v}{1-\gamma})c_{H,t} \\
0 &= y_{H,t} - a_t^H - \alpha_H k_{t-1}^H - (1-\alpha_H)l_t^H \\
0 &= y_{N,t} - a_t^N - \alpha_N k_{t-1}^N - (1-\alpha_N)l_t^N \\
0 &= y_{H,t} - \frac{C_H}{Y_H}c_{H,t} - \frac{I_H}{Y_H}i_{H,t} - \frac{G_H}{Y_H}g_{H,t} - \frac{C_H^*}{Y_H}c_{H,t}^* \\
0 &= y_{N,t} - \frac{C_N}{Y_N}c_{N,t} - \frac{I_N}{Y_N}i_{N,t} - \frac{G_N}{Y_N}g_{N,t} \\
0 &= l_t - \frac{L^N}{L}l_t^N - \frac{L^H}{L}l_t^H \\
0 &= w_t - i_t - m_t - (\kappa - 1)l_t \\
0 &= w_t^r - w_t + i_t - c_t + m_t \\
0 &= k_t^H - \delta i_t^H - (1-\delta)k_{t-1}^H \\
0 &= k_t^N - \delta i_t^N - (1-\delta)k_{t-1}^N \\
0 &= q_t^H - \eta i_t^H + \eta k_{t-1}^H \\
0 &= q_t^N - \eta i_t^N + \eta k_{t-1}^N \\
0 &= \hat{\lambda}_t^C + \pi_{H,t} - \pi_t + (1-\beta(1-\delta))r_t^H + \beta q_t^H - \hat{\lambda}_{t-1}^C - q_{t-1}^H \\
0 &= \hat{\lambda}_t^C + \pi_{N,t} - \pi_t + (1-\beta(1-\delta))r_t^N + \beta q_t^N - \hat{\lambda}_{t-1}^C - q_{t-1}^N \\
0 &= \hat{\lambda}_t^C + c_t \\
0 &= r_t^H - mc_t^H - y_t^H + k_{t-1}^H \\
0 &= r_t^N - mc_t^N - y_t^N + k_{t-1}^N \\
0 &= \pi_t - \gamma\pi_{T,t} - (1-\gamma)\pi_{N,t} \\
0 &= \pi_{T,t} - v\pi_{H,t} \\
0 &= \hat{i}_t^* - \hat{i}_t \\
0 &= \hat{i}^* - \widehat{\hat{i}^*} + \xi f_t^r \\
0 &= f_t^r - \frac{1}{\beta}f_{t-1}^r - \frac{1}{\beta}\hat{i}_{t-1} + (\frac{1}{\beta} + \frac{Z}{F})\pi_t + \frac{1-\beta}{\beta}nx_t - \frac{(1-\beta)(\gamma-1)}{\beta}q_t + \frac{(1-\beta)v}{\beta}t_t + \frac{Z}{F}z_t^r - \frac{Z}{F}z_{t-1}^r \\
0 &= f_t - f_t^r - i_t + c_t - m_t \\
0 &= nx_t - \frac{C_H^*}{NX}c_{H^*,t} + \frac{C_H^*}{NX}c_{H,t} + (\frac{C_F}{NX} - \frac{C_H^*}{NX})c_{F,t} \\
0 &= nx_t^r - nx_t - t_t \\
0 &= q_t + c_{T,t} - c_{N,t} \\
0 &= t_t + c_{F,t} - c_{H,t} \\
0 &= m_t^r + i_t - c_t \\
0 &= m_t - z_t
\end{aligned}
$$

Expectational equations

$$
\begin{aligned}
0 &= \mathcal{E}_t c_{t+1} - c_t - \hat{\imath}_t + \mathcal{E}_t \pi_{t+1} \\
0 &= \mathcal{E}_t c_{H,t+1} - c_{H,t} - \hat{\imath}_t + \mathcal{E}_t \pi_{H,t+1} \\
0 &= \mathcal{E}_t c_{N,t+1} - c_{N,t} - \hat{\imath}_t + \mathcal{E}_t \pi_{N,t+1} \\
0 &= -\lambda_H^f \mathcal{E}_t \pi_{H,t+1} + \pi_{H,t} - \lambda_H^{mc} mc_{H,t} - \lambda_H^b \pi_{H,t-1} \\
0 &= -\lambda_N^f \mathcal{E}_t \pi_{N,t+1} + \pi_{N,t} - \lambda_N^{mc} mc_{N,t} - \lambda_N^b \pi_{N,t-1}
\end{aligned}
$$

Exogenous stochastic processes

$$
\begin{aligned}
\mathcal{E}_t a_{H,t+1} &= \rho_{Y_H} a_{H,t} + \mathcal{E}_t \epsilon_{Y_H,t+1} \\
\mathcal{E}_t a_{N,t+1} &= \rho_{Y_N} a_{N,t} + \mathcal{E}_t \epsilon_{Y_N,t+1} \\
\mathcal{E}_t g_{H,t+1} &= \rho_{G_H} g_{H,t} + \mathcal{E}_t \epsilon_{G_H,t+1} \\
\mathcal{E}_t g_{N,t+1} &= \rho_{G_N} g_{N,t} + \mathcal{E}_t \epsilon_{G_N,t+1} \\
\mathcal{E}_t c^*_{H,t+1} &= \rho_{C^*_H} c^*_{H,t} + \mathcal{E}_t \epsilon_{C^*_H,t+1} \\
\mathcal{E}_t \tilde{\imath}^*_{t+1} &= \rho_{\tilde{\imath}^*} \tilde{\imath}^*_t + \mathcal{E}_t \epsilon_{i^*,t+1}
\end{aligned}
$$

C.3 Household's Optimality Conditions

Household j maximises expected utility (4.1) with respect to consumption $\{C^j,\ C^j_{T,s},\ C^j_{H,s},\ C^j_{N,s}\}$, investments $\{I^{N,j}_s, I^{H,j}_s\}$, capital $\{K^{N,j}_s, K^{H,j}_s\}$, labour supply $\{L^{N,j}_s, L^{H,j}_s\}$, real money balances $\frac{M^j_s}{P_s}$, and real bond holdings $\{\frac{B_{H,t}}{P_t}, \frac{B_{F,t}}{P_t}\}$ subject to the resource constraints (4.25), (4.13), (4.14), consumption indices (2.4), (2.6), (4.8), and price composites (4.9), (4.10), (4.11).

Intratemporal Consumption Allocations

As $C_{T,s} = \lambda_s \gamma \frac{C_s}{P_{T,s}}$ and $C_{N,s} = \lambda_s (1-\gamma) \frac{C_s}{P_{N,s}}$ as derived below, we immediately get

$$\frac{C_{T,s}}{C_{N,s}} = \frac{\gamma}{1-\gamma} \frac{P_{N,s}}{P_{T,s}}$$

Allocation between the home produced tradables basket and the imported basket derives from minimising

$$\mathcal{L}_s = P_{H,s} C^j_{H,s} + S_t P^*_{F,s} C^j_{F,s} - \lambda_s \left(\frac{(C^j_{H,t})^\nu (C^j_{F,t})^{1-v}}{\nu^\nu (1-\nu)^{1-\nu}} - 1 \right)$$

where $P^*_{F,s} C^j_{F,s}$ denote nominal expenditures on the imported goods basket expressed in foreign currency. As we assume that the law of one price holds for foreign produced tradables $P_{F,s} = S_t P^*_{F,s}$, where $P_{F,s}$ is the price level of imported tradables expressed in home currency. For convenience, we assume that the foreign price level is non-stochastic and we set $P^*_{F,s} = 1$ as in Bokil (2005). Then

$$\frac{C_{H,s}}{C_{F,s}} = \frac{v}{1-v} \frac{S_t P^*_{F,s}}{P_{H,s}} = \frac{v}{1-v} \frac{P_{F,s}}{P_{H,s}} = \frac{v}{1-v} T_t$$

which is equation (4.16) in the main text.

Intertemporal Consumption/Savings Allocations

To obtain the intertemporal consumption/savings decision, we rewrite the household's budget constraint in real terms, i.e. in units of home total consumption

$$\begin{aligned} & \frac{B^j_{H,s-1}}{P_s}\frac{P_{s-1}}{P_{s-1}}+S_s\frac{B^j_{F,s-1}}{P_s}\frac{P_{s-1}}{P_{s-1}}+\frac{Q^j_s}{P_s}+\frac{M^j_{s-1}}{P_s}\frac{P_{s-1}}{P_{s-1}}+\frac{\int_0^1 \Pi^j_{N,s}(z)dz}{P_s}+\frac{\int_0^1 \Pi^j_{H,s}(z)dz}{P_s} \\ & +\frac{W^j_{H,s}}{P_s}L^j_{H,s}+\frac{W^j_{N,s}}{P_s}L^j_{N,s}+\frac{P_{N,s}}{P_s}R^{N,j}_s K^{N,j}_{t-s}+\frac{P_{H,s}}{P_s}R^{H,j}_s K^{H,j}_{s-1} \\ \geq \; & C^j_s+\frac{T^j_s}{P_s}+\frac{M^j_s}{P_s}+\frac{P^{I,H}_s}{P_s}I^{H,j}_s+\frac{P^{I,N}_s}{P_s}I^{N,j}_s+\frac{B^j_{H,s}}{P_s(1+i_s)}+\frac{S_s B^j_{F,s}}{P_s(1+i^*_s)} \end{aligned} \tag{C.1}$$

Maximising expected utility (4.1) with respect to $\{C^j_s, \frac{B_{H,s}}{P_s}\}$ yields

$$\begin{aligned} \mathcal{L}_{C^j_s} &= \mathcal{E}_t[\beta^{(s-t)}(C^j_s)^{-1}] = \mathcal{E}_t\lambda^C_s \\ \mathcal{L}_{\frac{B^j_{H,s}}{P_s}} &= \mathcal{E}_t[\beta^{(s-t)}\lambda^C_s\frac{1}{(1+i_s)}] = \beta^{(s+1-t)}_t\mathcal{E}_t[\lambda^C_{s+1}\frac{P_s}{P_{s+1}}] \end{aligned}$$

Imposing $s=t$ yields the Euler equation

$$\lambda^C_t = \beta\mathcal{E}_t[\lambda^C_{t+1}(1+i_t)\frac{P_t}{P_{t+1}}]$$

where $\lambda^C_t = (C^j_t)^{-1}$ is the marginal utility obtained by increasing total consumption by one unit.

Intratemporal Labour/Consumption Choice

We obtain the labour/leisure choice by maximising (4.1) with respect to $\{C^j_s, L_{H,s}, L_{N,s}\}$ where

$$\begin{aligned} \mathcal{L}_{C^j_s} &= \mathcal{E}_t\beta^{(s-t)}(C^j_s)^{-1} = \mathcal{E}_t\lambda^C_s \\ \mathcal{L}_{L^j_{H,s}} &= \mathcal{E}_t\beta^{(s-t)}\eta(L^{S,j}_s)^{\kappa-1} = \mathcal{E}_t\lambda^C_s\frac{W_{H,s}}{P_s} \\ \mathcal{L}_{L^j_{N,s}} &= \mathcal{E}_t\beta^{(s-t)}\eta(L^{S,j}_s)^{\kappa-1} = \mathcal{E}_t\lambda^C_s\frac{W_{N,s}}{P_s} \end{aligned}$$

We immediately obtain that $W_{H,s} = W_{N,s} \equiv W_s$, i.e. nominal wage equalisation between the sectors (labour is mobile across the sectors and taxes $T^{\#}_t$ are lump-sum). Further with $s=t$ we obtain total labour supply

$$\eta(L^j_{H,t}+L^j_{N,t})^{\kappa-1}C^j_t = \frac{W_t}{P_t}$$

which states that the household provides labour up to the point where the marginal rate of substitution of consumption for leisure equals the consumption based real wage.

Intratemporal Real Money Holdings/Consumption Choice

We obtain the real money balance/consumption trade-off by maximising (4.1) with respect to $\{C_s^j, \frac{M_s^j}{P_s}\}$ where

$$
\begin{aligned}
\mathcal{L}_{C_s^j} &= \mathcal{E}_t \beta^{(s-t)} (C_s^j)^{-1} = \mathcal{E}_t \lambda_s^C \\
\mathcal{L}_{\frac{M_s^j}{P_s}} &= \mathcal{E}_t \beta^{(s-t)} \chi (\frac{M_s^j}{P_s})^{-1} = \mathcal{E}_t \lambda_s^C - \beta_t^{(s+1-t)} \mathcal{E}_t [\lambda_{s+1}^C \frac{P_s}{P_{s+1}}]
\end{aligned}
$$

With $s = t$ we obtain

$$
\chi (\frac{M_t^j}{P_t})^{-1} = \lambda_t^C - \beta \mathcal{E}_t [\lambda_{t+1}^C \frac{P_t}{P_{t+1}}]
$$

Using the Euler equation we obtain the money demand equation

$$
\begin{aligned}
\chi (\frac{M_t^j}{P_t})^{-1} &= \lambda_t^C - \beta \mathcal{E}_t [\lambda_{t+1}^C \frac{P_t}{P_{t+1}}] = \lambda_t^C - \frac{\lambda_t^C}{1 + i_t} \\
\chi \frac{(\frac{M_t^j}{P_t})^{-1}}{\lambda_t^C} &= 1 - \frac{1}{1 + i_t} = \frac{i_t}{1 + i_t}
\end{aligned}
$$

which states that real money holdings increase in consumption and decrease in the nominal interest rate. With rising i_t, opportunity costs of holding real money balances (foregone earnings from bond holdings) increase, making money less attractive to use as mean to carry value in the future.

Intertemporal Investment/Capital Spending Choice for the H and N Sector

The household maximises utility (4.1) with respect to the constraints (C.1), (4.13), and (4.14). The Lagrangian for the non-traded sector is

$$
\begin{aligned}
\mathcal{L} &= \mathcal{E}_t [... + \beta^{(s-t)} U(C_s^j, \frac{M_s^j}{P_s}, L_s^{S,j}) - \lambda_s^C \{... - \frac{P_s^{I,N}}{P_s} I_s^{N,j} + \frac{P_{N,s}}{P_s} Z_s^{N,j} K_{s-1}^{N,j} - ...\} \\
&\quad - \lambda_s^{I,N} \{K_s^N - \Phi[\frac{I_s^N}{K_{s-1}^N}] K_{s-1}^N - (1 - \delta) K_{s-1}^N\} + ...]
\end{aligned}
$$

where $\lambda_s^{I,N}$ is the Lagrange multiplier attached to the capital accumulation constraint in the non-traded goods sector. Therefore

$$
\begin{aligned}
\mathcal{L}_{I_s^N} &= \beta^{(s-t)} \mathcal{E}_t [\lambda_s^C \frac{P_s^{I,N}}{P_s}] - \beta_t^{(s-t)} \mathcal{E}_t [\lambda_s^{I,N} \Phi' [\frac{I_s^N}{K_{s-1}^N}]] = 0 \\
\mathcal{L}_{K_s^N} &= \beta^{(s-t)} \mathcal{E}_t \lambda_s^{I,N} - \beta^{(s+1-t)} \mathcal{E}_t [\lambda_{s+1}^C \frac{P_{N,s+1}}{P_{s+1}} R_{s+1}^N] = 0
\end{aligned}
$$

With $s = t$

$$
\lambda_t^{I,N} - \beta \mathcal{E}_t [\lambda_{t+1}^C \frac{P_{N,t+1}}{P_{t+1}} R_{t+1}^N] = \beta \mathcal{E}_t [\lambda_{t+1}^{I,N} \{\Phi' [\frac{I_{t+1}^N}{K_t^N}] (-\frac{I_{t+1}^N}{(K_t^N)^2}) K_t^N + \Phi [\frac{I_{t+1}^N}{K_t^N}] + (1 - \delta)\}]
$$

and the shadow price of real investment in the N sector is $\lambda_t^{I,N} = \lambda_t^C \frac{P_t^{I,N}}{P_t}(\Phi'[\frac{I_t^N}{K_{t-1}^N}])^{-1} = \lambda_t^C \frac{P_t^{I,N}}{P_t} Q_t^N$. Finally,

$$
\begin{aligned}
& \lambda_t^C \frac{P_t^{I,N}}{P_t} Q_t^N \\
= \; & \beta \mathcal{E}_t[\lambda_{t+1}^C \frac{P_{N,t+1}}{P_{t+1}} R_{t+1}^N] + \beta \mathcal{E}_t[\lambda_{t+1}^C \frac{P_{t+1}^{I,N}}{P_{t+1}} Q_{t+1}^N \{[\Phi[\frac{I_{t+1}^N}{K_t^N}] - \Phi'[\frac{I_{t+1}^N}{K_t^N}](\frac{I_{t+1}^N}{K_t^N}) + (1-\delta)\}]
\end{aligned}
$$

where ${Q_t}^N$ is Tobin's Q defined as the market value of capital over its replacement cost. In other words, Q_t^N is the (real) shadow value of capital in place. Analogously in the H sector. Capital compared to labour is immobile between the sectors within the country, i.e. ex-post rental rates of equipment in the N and H sector given by R_s^N and R_s^H can differ. Hence there is no ex-post rental price equalisation along the business cycle.

C.4 Production

Factor Demands

Each firm $z \in [0,1]$ in the tradable sector combines physical capital $K_{s-1}^H(z)$ and labour $L_{H,s}(z)$ according to the Cobb-Douglas production technology

$$y_{H,s}(z) = A_s^H (K_{s-1}^H(z))^{\alpha_H} (L_s(z))^{1-\alpha_H}$$

where $K_{s-1}^H(z) = \int_0^1 K_{s-1}^{H,j}(z)dj$, and $L_s(z) = \int_0^1 L_s^{H,j}(z)dj$. A_s^H is total factor productivity in the tradable sector to be equal for all firms given by the exogenous stochastic process (C.3) defined below. Cost minimisation implies that the firm wants to minimise total expenditures on capital and labour to obtain a certain quantity of the produced good $y_{H,s}(z)$

$$\min_{\{K_{s-1}^H(z), L_s(z)\}} \frac{W_s}{P_{H,s}} L_s(z) + R_s K_{s-1}^H(z) \text{ s.t. } y_{H,s}(z) = A_s^H (K_{s-1}^H(z))^{\alpha_H} (L_s(z))^{1-\alpha_H}$$

where firms unlike households calculate real wages as factor costs by deflating by the sector-specific price level $P_{H,s}$ whereas households calculate real wages by deflating by the CPI price index P_s. Then

$$\mathcal{L} = \frac{W_s}{P_{H,s}} L_s(z) + R_s^H K_{s-1}^H(z) - MC_s^H(z)\{y_{H,s}(z) - A_s^H (K_{s-1}^H(z))^{\alpha_H} (L_s(z))^{1-\alpha_H}\}$$

where $MC_s^H(z)$ is the shadow price of the increase in total cost to produce one more unit of good z. This is just the definition of real marginal cost. We obtain the factor demands in the H sector

$$
\begin{aligned}
\mathcal{L}_{L_s(z)} &= \frac{W_s}{P_{H,s}} = MC_s^H(z)(1-\alpha_H)\frac{y_{H,s}(z)}{L_s(z)} \\
\mathcal{L}_{K_{s-1}^H(z)} &= R_s^H = MC_s^H(z)\alpha_H \frac{y_{H,s}(z)}{K_{s-1}^H(z)}
\end{aligned}
$$

Analogously for the non-traded sector.

Market Demand for the Final Good

Household j wants to minimise expenditures for obtaining one unit of the home produced tradables basket $C^j_{H,t} = [\int_0^1 (c^j_{H,s}(z))^{\frac{\rho_H - 1}{\rho_H}} dz]^{\frac{\rho_H}{\rho_H - 1}} = 1$ which yields the demand function $c^j_{H,s}(z) = (\frac{p_{H,s}(z)}{\lambda_s})^{-\rho_H} C^j_{H,t}$. And further

$$[\int_0^1 (p_{H,s}(z))^{1-\rho_H} dz]^{\frac{1}{1-\rho_H}} = \lambda_s$$

λ_s is the shadow price (the increase in total expenditure) of obtaining one more unit of good H which is by definition a price index, hence $\lambda_s C_{H,s} = P_{H,s} C_{H,s} = P_{H,s}$.

Price Dynamics in the H and N sector

The decision problem of the traded sector firm is

$$\begin{aligned} & \max_{\{p_{H,t}(z)\}} \mathcal{E}_t \sum_{s=0H}^{\infty} (\theta_H)^s \Delta_{s,t+s} \{ \frac{p_{H,t}(z)}{P_{H,t+s}} y_{H,t+s}(z) - MC^H_{t+s}(z) y_{H,t+s}(z) \} \\ \text{s.t. } y_{H,t+s}(z) &= (\frac{p_{H,t}(z)}{P_{H,t+s}})^{-\rho_H} Y_{H,t+s} \\ \text{and } P_{H,t}{}^{(1-\rho_H)} &= (1-\theta_H)(p^o_{H,t})^{(1-\rho_H)} + \theta_H P^{(1-\rho_H)}_{H,t-1} \end{aligned}$$

where $E_t \Delta_{s,t+s} = E_t[\beta^s (\frac{C_{t+s}}{C_t})^{-1}]$ is the real stochastic discount factor used by firms to evaluate expected future profit streams. The stochastic discount factor is derived from iterating forward (4.18). Plugging in the market demand function (4.32) yields

$$\max_{\{p_{H,t}(z)\}} \mathcal{E}_t \sum_{s=0}^{\infty} (\theta_H)^s \Delta_{s,t+s} \{ (\frac{p_{H,t}(z)}{P_{H,t+s}})^{1-\rho_H} Y_{H,t+s} - MC^H_{t+s}(z) (\frac{p_{H,t}(z)}{P_{H,t+s}})^{-\rho_H} Y_{H,t+s} \}$$

The first order condition reads

$$\begin{aligned} & \mathcal{E}_t \sum_{s=0}^{\infty} (\theta_H)^s \Delta_{s,t+s} \{ (1-\rho_H) (\frac{p^o_{H,t}(z)}{P_{H,t+s}})^{-\rho_H} \frac{Y_{H,t+s}}{P_{H,t+s}} + \rho_H MC^H_{t+s}(z) (\frac{p^o_{H,t}(z)}{P_{H,t}})^{-\rho_H - 1} Y_{H,t+s} \} \\ = \; & 0 \end{aligned}$$

from which we obtain as a first intermediate result

$$\frac{p^o_{H,t}(z)}{P_{H,t}} = \frac{\rho_H}{\rho_H - 1} \frac{\mathcal{E}_t \sum_{s=0H}^{\infty} (\theta_H)^s \Delta_{s,t+s} MC^H_{t+s}(z) (\frac{P_{H,t+s}}{P_{H,t}})^{\rho_H} Y_{H,t+s}}{\mathcal{E}_t \sum_{s=0}^{\infty} (\theta_H)^s \Delta_{s,t+s} (\frac{P_{H,t+s}}{P_{H,t}})^{\rho_H - 1} Y_{H,t+s}}$$

Plugging in the stochastic discount factor yields

$$\frac{p^o_{H,t}(z)}{P_{H,t}} = \frac{\rho_H}{\rho_H - 1} \frac{\mathcal{E}_t \sum_{s=0}^{\infty} (\theta_H \beta)^s (C_{t+s})^{-1} MC^H_{t+s}(z) (\frac{P_{H,t+s}}{P_{H,t}})^{\rho_H} Y_{H,t+s}}{\mathcal{E}_t \sum_{s=0}^{\infty} (\theta_H \beta)^s (C_{t+s})^{-1} (\frac{P_{H,t+s}}{P_{H,t}})^{\rho_H - 1} Y_{H,t+s}}$$

We see that the firm considers the elasticity of market demand ρ_H, current and expected future real marginal costs, current and expected future price levels $P_{H,t+j}$ as well as current and future levels of total consumption $C_{H,t+j}$ when setting its price. This equation links real activity i.e. current and future real marginal costs to price fluctuations.

C.5 Composite Price Indices and Deflators

The consumption based price index P_s results from minimising household's expenditures for obtaining 1 unit of the consumption index C_s^j over the arguments $C_{N,s}^j$ and $C_{T,s}^j$

$$\begin{aligned} \min_{\{C_{N,s}^j, C_{H,s}^j\}} P_s C_s^j &= P_{N,s} C_{N,s}^j + P_{T,s} C_{T,s}^j \\ \text{s.t. } C_s^j &= 1 \end{aligned}$$

The Lagrangian is

$$\mathcal{L}_s = P_{N,s} C_{N,s}^j + P_{T,s} C_{T,s}^j - \lambda_s \left(\frac{(C_{T,s}^j)^\gamma (C_{N,s}^j)^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}} - 1 \right)$$

As $C_{T,s}^j = \lambda_s \gamma \frac{C_s^j}{P_{T,s}}$ and $C_{N,s}^j = \lambda_s (1-\gamma) \frac{C_s^j}{P_{N,s}}$ we get

$$\frac{(\lambda_s \gamma \frac{C_s^j}{P_{T,s}})^\gamma (\lambda_s (1-\gamma) \frac{C_s^j}{P_{N,s}})^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}} = 1$$

and hence $(P_T)^\gamma (P_N)^{1-\gamma} = \lambda_s C_s^j$. λ_s is the shadow price (the increase in total expenditure) of obtaining one more unit of the consumption good which is by definition the price index, hence $\lambda_s C_s^j = P_s C_s^j = P_s$ and

$$P_s = (P_{T,s})^\gamma (P_{N,s})^{1-\gamma}$$

By the same procedure, we obtain the tradables price index

$$P_{T,s} = (P_{H,s})^v (P_{F,s})^{1-v}$$

The assumed form for overall utility given by (4.2) implies that utility from consumption is additively separable in H, N, F consumption. We then obtain that the Euler equations for non-tradable and home-produced tradables consumption are derived from the following Lagrangian

$$\mathcal{L} = \mathcal{E}_t[\beta^{s-t} \ln \frac{(C_{T,s}^j)^\gamma (C_{N,s}^j)^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}} - \lambda_s^C \{ ... \frac{P_s^N}{P_s} C_{N,s}^j + \frac{P_s^H}{P_S} C_{H,s}^j + \frac{P_s^F}{P_s} C_{F,s}^j - ... \} + ...]$$

$$\mathcal{L}_{C_{N,s}} = \mathcal{E}_t[\beta^{s-t} \frac{1}{C_s^j} (1-\gamma) \frac{C_s^j}{C_{N,s}^j}] = \mathcal{E}_t[\lambda_s^C \frac{P_s^N}{P_s}]$$

with $s = t$ and using $\lambda_s^C = (C_s^j)^{-1}$

$$\begin{aligned} \lambda_t^C &= (1-\gamma) \frac{P_s}{C_{N,t}^j P_t^N} \\ \frac{P_t}{C_{N,t}^j P_t^N} &= \beta \mathcal{E}_t[\frac{P_{t+1}}{C_{N,t+1}^j P_{t+1}^N} \frac{P_t}{P_{t+1}} (1+i_t)] \\ \frac{1}{C_{N,t}^j} &= \beta \mathcal{E}_t[\frac{1}{C_{N,t+1}^j} \frac{P_t^N}{P_{t+1}^N} (1+i_t)] \end{aligned}$$

Analogously $\frac{1}{C_{H,t}^j} = \beta \mathcal{E}_t[\frac{1}{C_{H,t+1}^j} \frac{P_t^H}{P_{t+1}^H} (1+i_t)]$.

C.6 The Foreign Sector

Household Optimal Portfolio Choice between Home and Foreign Bonds

Maximising expected utility (4.1) with respect to $\{B_{H,s}, B_{F,s}\}$

$$
\begin{aligned}
\mathcal{L}_{B^j_{H,s}} &= \beta^{(s-t)}\mathcal{E}_t[\lambda^C_s \frac{1}{P_s(1+i_s)}] = \beta^{(s+1-t)}\mathcal{E}_t[\lambda^C_{s+1}\frac{1}{P_{s+1}}] \\
\mathcal{L}_{B^j_{H,s}} &= \beta^{(s-t)}\mathcal{E}_t[\lambda^C_s \frac{S_s}{P_s(1+i^*_s)}] = \beta^{(s+1-t)}\mathcal{E}_t[\lambda^C_{s+1}\frac{S_{s+1}}{P_{s+1}}]
\end{aligned}
$$

or with $s = t$

$$
\lambda^C_t = \beta\mathcal{E}_t[\lambda^C_{t+1}\frac{P_t}{P_{t+1}}(1+i_t)] = \beta\mathcal{E}_t[\lambda^C_{t+1}\frac{P_t}{P_{t+1}}(1+i^*_t)\frac{S_{t+1}}{S_t}]
$$

which yields the uncovered interest parity condition

$$
\mathcal{E}_t[\lambda^C_{t+1}\frac{P_t}{P_{t+1}}\{(1+i_t)-(1+i^*_t)\frac{S_{t+1}}{S_t}\}] = 0
$$

Note that the assumption, that i^*_s equals the exogenous world interest rate $\tilde{i}^*_s$ adjusted for a risk premium leads to the incompleteness of asset markets and no perfect risk-sharing. The risk premium decreases in the real value of net foreign assets $\frac{F_t}{P_t}$ / increases in real value of net foreign liabilities $-\frac{F_t}{P_t}$

$$
(1+i^*_t) = (1+\tilde{i}^*_t)\phi[-\frac{F_t}{P_t}]
$$

The intertemporal budget constraint of the economy can be obtained by iterating forward (4.55)

$$
\begin{aligned}
F^r_t &= \frac{1+i_{t-1}}{1+\pi_t}F^r_{t-1} + NX^r_t - (Z^r_t - \frac{Z^r_{t-1}}{1+\pi_t}) \\
F^r_t &= a_t^{-1}F^r_{t-1} + NX^r_t - x_t \\
F^r_{t-1} &= a_tF^r_t - a_tNX^r_t + a_tx_t
\end{aligned}
$$

where we have defined $a_t^{-1} = \frac{1+i_{t-1}}{1+\pi_t}$ as well as $x_t = Z^r_t - \frac{Z^r_{t-1}}{1+\pi_t}$. Ex post, all values have realised and we can omit the expectations operator $F^r_t = a_{t+1}F^r_{t+1} - a_{t+1}NX^r_{t+1} + a_{t+1}x_{t+1}$. Plug into above

$$
F^r_{t-1} = a_t\{a_{t+1}F^r_{t+1} - a_{t+1}NX^r_{t+1} + a_{t+1}x_{t+1}\} - a_tNX^r_t + a_tx_t
$$

Therefore

$$
\begin{aligned}
F^r_{t-1} &= \prod_{s=t}^{T} a_sF^r_T - \sum_{s=t}^{T}\prod_{j=t}^{s} a_jNX^r_s + \sum_{s=t}^{T}\prod_{j=t}^{s} a_j(Z^r_s - \frac{Z^r_{s-1}}{1+\pi_s}) \\
F^r_{t-1} &= F^r_T\prod_{s=t}^{T}(\frac{1+i_{s-1}}{1+\pi_s})^{-1} - \sum_{s=t}^{T}(\prod_{j=t}^{s}(\frac{1+i_{j-1}}{1+\pi_j})^{-1})\{NX^r_s - (Z^r_s - \frac{Z^r_{s-1}}{1+\pi_s})\}
\end{aligned}
$$

Ruling out Ponzi schemes requires $\lim_{T\to\infty}(\prod_{s=t}^{T}(\frac{1+i_{s-1}}{1+\pi_s})^{-1})F^r_T = 0$. Finally

$$
\begin{aligned}
F^r_{t-1} &= -\sum_{s=t}^{\infty}(\prod_{j=t}^{s}(\frac{1+i_{j-1}}{1+\pi_j})^{-1}\{NX^r_s - (Z^r_s - \frac{Z^r_{s-1}}{1+\pi_s})\} \\
F^r_{t-1} &= \sum_{s=t}^{\infty}(\prod_{j=t}^{s}(\frac{1+i_{j-1}}{1+\pi_j})^{-1}\{Z^r_s - \frac{Z^r_{s-1}}{1+\pi_s} - NX^r_s\}
\end{aligned}
$$

Price Level

We can express the price level given by (4.9) in terms of the internal real exchange rate (4.46) and the terms of trade (4.47)

$$P_t = (P_{T,t})^{\gamma}(P_{N,t})^{1-\gamma} = (\frac{P_{T,t}}{P_{N,t}})^{\gamma} P_{N,t} = (Q_t)^{\gamma} P_{N,t}$$

As $P_{N,t} = P_{T,t}(Q_t)^{-1}$ and from (4.10)

$$P_{T,t} = (P_{H,t})^{v}(P_{F,t})^{1-v} = (P_{H,t})^{v}(S_t)^{1-v} = (\frac{P_{H,t}}{S_t})^{v} S_t = T_t^{-v} S_t$$

where we have used that the LOP holds for tradables $P_{F,t} = S_t P^*_{F,t} = S_t$ and that by the terms of trade $(T_t)^{-1} = (\frac{S_t}{P_{H,t}})^{-1}$. Therefore $P_{N,t} = S_t T_t^{-v}(Q_t)^{-1}$ and eventually $P_t = S_t T_t^{-v}(Q_t)^{\gamma-1}$. The price level ratio $\frac{P_{N,t}}{P_{H,t}}$ can be written as

$$\frac{P_{N,t}}{P_{H,t}} = \frac{S_t T_t^{-v}(Q_t)^{-1}}{S_t/T_t} = \frac{S_t T_t^{1-v}}{Q_t}$$

C.7 The Steady State

Steady State Nominal and Real Interest Rate

In steady state, we obtain from the Euler equation given by (4.17) that subjective and market discounting are linked by

$$1 = \beta(1+i)$$

Use the law of motion for capital $K_t^N = \Phi(\frac{I_t^N}{K_{t-1}^N})K_{t-1}^N + (1-\delta)K_{t-1}^N$ to obtain

$$\begin{aligned} K^N &= \Phi(\frac{I^N}{K^N})K^N + (1-\delta)K^N \\ \delta &= \Phi(\frac{I^N}{K^N}) \end{aligned}$$

The same logic applies to the traded goods sector H. Use that then Tobin's Q in steady state $Q^N = (\Phi'[\frac{I^N}{K^N}])^{-1} = 1$. The intertemporal investment equation for home tradables given by (4.24) in steady state reads

$$\begin{aligned} 1 &= \frac{1}{1}\frac{1}{(1+i)}R^H + \beta\{[\Phi[\frac{I^H}{K^H}] - 1(\frac{I^H}{K^H}) + (1-\delta)\} \\ 1 &= \frac{1}{(1+i)}R^H + \beta\{[\delta - \delta + (1-\delta)\} \\ R^H &= (1-\beta(1-\delta))(1+i) = \frac{1}{\beta} - 1 + \delta = i + \delta \end{aligned} \tag{C.2}$$

by using that $\frac{1}{\beta} = 1 + i$. Hence nominal and real rates of return are linked by

$$1 + i = \frac{1}{1-\beta(1-\delta)}R^H = \frac{1}{1-\beta(1-\delta)}R^N$$

which is equation (4.60) in the main text. We see that when both sectors face the same depreciation rate δ of real capital, real returns on physical capital have to equalise in steady state. Hence steady state investment/capital ratios will turn out to be equal in both sectors.

Steady State Resource Constraints

We want to substitute out steady state ratios from the resource constraints. From the capital accumulation equations we obtained that $\frac{I^N}{K^N} = \delta = \frac{I^H}{K^H}$. From the resource constraint in the tradable sector H

$$\begin{aligned} Y_H &= C_H + I_H + G_H + C_H^* \\ 1 &= \frac{C_H}{Y_H} + \delta \frac{K_H}{Y_H} + \frac{G_H}{Y_H} + \frac{C_H^*}{Y_H} \end{aligned}$$

By the factor demand for physical capital

$$\begin{aligned} R^H &= MC^H \alpha_H \frac{y_H(z)}{K^H(z)} = \frac{\rho_H - 1}{\rho_H} \alpha_H \frac{Y_H}{K^H} \\ \frac{Y_H}{K^H} &= \frac{1}{\alpha_H} \frac{1-\beta(1-\delta)}{\beta} \frac{\rho_H}{\rho_H - 1} \end{aligned}$$

we obtain the inverse of the capital-output ratio in steady state where we have used (C.2) and that firms real marginal cost in steady state is $\frac{\rho_H - 1}{\rho_H}$. From the labour demands (4.28) and (4.30) follows

$$\begin{aligned} \frac{W}{P_H} &= MC^H (1-\alpha_H) \frac{Y_H}{L^H} \\ 1 &= MC^H (1-\alpha_H) \frac{Y_H}{L^H} \\ \frac{Y_H}{L_H} &= \frac{1}{1-\alpha_H} \frac{\rho_H}{\rho_H - 1} \\ \frac{Y_N}{L^N} &= \frac{1}{1-\alpha_N} \frac{\rho_N}{\rho_N - 1} \end{aligned}$$

From aggregate supply in the economy we obtain the capital-labour ratio in the traded goods sector

$$\begin{aligned} Y_H &= (K^H)^{\alpha_H} (L^H)^{1-\alpha_H} \\ \frac{Y_H}{L^H} &= (\frac{K^H}{L^H})^{\alpha_H} \\ \frac{K^H}{L^H} &= (\frac{Y_H}{L^H})^{\frac{1}{\alpha_H}} = (\frac{1}{1-\alpha_H} \frac{\rho_H}{\rho_H - 1})^{\frac{1}{\alpha_H}} \end{aligned}$$

We have aggregated over the average firm's production and used that by assumption $A^H = 1$. As the share of real exports $\frac{C_H^*}{Y_H}$ is truly exogenous as home cannot influence foreign economic outcomes and as home government consumption is not guided by optimising behaviour, the steady state consumption share is uniquely determined by exogenous variables

$$\frac{C_H}{Y_H} = 1 - \delta(\frac{1}{\alpha_H} \frac{1-\beta(1-\delta)}{\beta} \frac{\rho_H}{\rho_H - 1})^{-1} - \frac{G_H}{Y_H} - \frac{C_H^*}{Y_H}$$

Applying the same logic in the non-tradables sector we obtain

$$\frac{C_N}{Y_N} = 1 - \delta(\frac{1}{\alpha_N} \frac{1-\beta(1-\delta)}{\beta} \frac{\rho_N}{\rho_N - 1})^{-1} - \frac{G_N}{Y_N}$$

Current Account and Balance of Payments

To obtain the current account, we add (4.25), (4.42), (4.39) and take into account that aggregate equilibrium profits in both sectors are

$$\begin{aligned}\Pi_{H,t} &= P_{H,t}Y_{H,t} - W_{H,t}L_{H,t} - P_{H,t}R_t^H K_{t-1}^H \\ \Pi_{N,t} &= P_{N,t}Y_{N,t} - W_{N,t}L_{N,t} - P_{N,t}R_t^N K_{t-1}^N\end{aligned}$$

Hence

$$\begin{aligned} & B_{H,t-1}+S_tB_{F,t-1}+M_{t-1} + T_t^{\#} + v_t+Q_t^{\#} + S_tB_{F,t-1}^C + M_t - M_{t-1} \\ & +P_{H,t}Y_{H,t} + P_{N,t}Y_{N,t} \\ = \; & P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + P_{H,t}I_t^H + P_{H,t}G_{H,t} + P_{N,t}C_{N,t} + P_{N,t}I_t^N + P_{N,t}G_{N,t} + T_t^{\#} \\ & +v_t + Q_t^{\#} + M_t + \frac{B_{H,t}}{1+i_t} + \frac{S_tB_{F,t}}{1+i_t^*} + \frac{S_tB_{F,t}^C}{1+i_t^*} + Z_t - Z_{t-1}\end{aligned}$$

We eventually obtain

$$B_{H,t-1}+S_tB_{F,t-1}+S_tB_{F,t-1}^C+P_{H,t}C_{H,t}^*=P_{F,t}C_{F,t}+\frac{B_{H,t}}{1+i_t}+\frac{S_tB_{F,t}}{1+i_t^*}+\frac{S_tB_{F,t}^C}{1+i_t^*}+Z_t-Z_{t-1}$$

which yields (4.52). In (4.53) we obtained the balance of payments

$$B_{H,t-1}+S_tB_{F,t-1}+S_tB_{F,t-1}^C+NX_t+(-\frac{B_{H,t}}{1+i_t}-\frac{S_t(B_{F,t}+B_{F,t}^C)}{1+i_t^*}) - (Z_t-Z_{t-1}) \equiv 0$$

where CA_t denotes the current account, FA_t denotes the financial account and net exports are given by $NX_t = P_{H,t}C_{H,t}^* - P_{F,t}C_{F,t}$. Omitting time subscripts

$$\begin{aligned} B_H+SB_F+SB_F^C+NX + (-\frac{B_H}{1+i}-\frac{S(B_F+B_F^C)}{1+i^*}) - (Z-Z) &\equiv 0 \\ \frac{i}{1+i}B_H + \frac{i^*}{1+i^*}(B_F+B_F^C) &= -NX\end{aligned}$$

Remembering the risk premium equation

$$(1+i_t^*) = (1+\tilde{i}_t^*)\phi[-\frac{F_t}{P_t}]$$

where F_t denote net nominal foreign financial assets given in our model by

$$F_t = \frac{B_{H,t}}{1+i_t}+\frac{S_t(B_{F,t}+B_{F,t}^C)}{1+i_t^*}$$

In steady state $\phi[-F] = 1$. Using the uncovered interest rate parity condition

$$\begin{aligned} E_t[\lambda_{t+1}^C\frac{P_t}{P_{t+1}}\{(1+i_t) - (1+i_t^*)\frac{S_{t+1}}{S_t}\}] &= 0 \\ (1+i) &= (1+i^*)\end{aligned}$$

yields the result that households can borrow at the world interest rate $i^* = \tilde{\imath}^*$ in steady state. Hence $i = i^* = \tilde{\imath}^*$. Steady state indebtedness can then be written as

$$\frac{\tilde{\imath}^*}{1+\tilde{\imath}^*}(B_H + B_F + B_F^C) = -NX$$

The nominal/real trade balance in steady state is

$$NX = C_H^* - C_F$$

Real net financial assets equal nominal net financial assets in steady state and can be obtained by omitting all time subscripts from (4.55)

$$\begin{aligned} F &= IF + NX \\ NX &= (1-I)F \\ NX &= (1-(1+i))F \\ \frac{NX}{F} &= \frac{\beta-1}{\beta} < 0 \end{aligned}$$

which implies that if steady state financial assets are negative, they have to be offset by a positive net trade balance in order to yield a balanced net financial asset account.[1] In other words, the current account is balanced in steady state as $NX + iF = 0$. Otherwise we would have indeterminacy where temporary shocks might cause permanent deviations from steady state. If we assume that $F < 0$ (as we do in the calibration) we need $NX > 0$. This implies that $\frac{Z}{F} < 0$ as well.

C.8 Exogenous Stochastic Processes

For sector-specific total factor productivity (TFP) in sector J we assume

$$A_{J,t} = A_J \exp\left[a_{J,s}\right] = A_J \exp[\rho_{Y_J} a_{J,t-1} + \epsilon_{Y_J,t}] \tag{C.3}$$

where $a_{J,s} \equiv \ln \frac{A_{J,s}}{A_J} \times 100\%$ is the percentage deviation of the level of technology about its steady state value. Fluctuations are triggered by structural innovations $\exp\left[\epsilon_{Y_J,t}\right]$ that are described by a log-normal distribution $\ln\left[\exp\left[\epsilon_{Y_J,t}\right]\right] = \epsilon_{Y_J,t} \sim n.i.d.(0, \sigma_{Y_J}^2)$. Government spending follows

$$G_{J,t} = G_J \exp[\rho_{G_J} g_{J,t-1} + \epsilon_{G_J,t}]$$

And external demand follows

$$C_{H,t}^* = C_H^* \exp[\rho_{C_H^*} c_{H,t-1}^* + \epsilon_{C_H^*,t}]$$

where same distributional assumptions apply.

C.9 The Log-Linear Approximation

We denote first order linear approximations by lower case letters if not stated otherwise. In general, $x \equiv \ln \frac{X_s}{X} \times 100\%$ denotes the (logarithmic) percentage deviation of X_s about its steady state X.

[1] We could have also imposed long-run equilibrium on (4.56) to obtain the same steady state relationship.

Consumption, Prices and Inflation Rates

Log-linearising total consumption given by (2.4) yields

$$C(1+c_t) = \frac{1}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}(C_T)^{\gamma}(C_N)^{1-\gamma}(1+\gamma c_{T,t}+(1-\gamma)c_{N,t})$$

Making use of the steady state eqpression $C = \frac{1}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}(C_T)^{\gamma}(C_N)^{1-\gamma}$ we obtain

$$c_t = \gamma c_{T,t} + (1-\gamma)c_{N,t}$$

Analogously for (2.6)

$$c_{T,t} = \nu c_{H,t} + (1-\nu)c_{F,t}$$

The sharing rules are

$$c_{T,t} - c_{N,t} = p_{N,t} - p_{T,t}$$

and analogously for the choice between the home produced and the foreign produced tradables basket

$$c_{H,t} - c_{F,t} = s_t - p_t^H = -p_t^H$$

We have used that under the currency board $s_t = 0$. The consumption based price index P_s given by (4.9) becomes

$$p_t = \gamma p_{T,t} + (1-\gamma)p_{N,t}$$

and

$$p_{T,t} = \nu p_{H,t} + (1-\nu)p_{F,t}$$

We can simplify the latter by making use of the LOP so that $P_{F,t} = S_t P^*_{F,t}$. Then

$$p_{T,t} = \nu p_{H,t} + (1-\nu)s_t + (1-v)p^*_{F,t} = \nu p_{H,t}$$

where in our case $s_t = p^*_{F,t} = 0$. We can use the price indices to obtain inflation rates. By

$$\begin{aligned}
1+\pi_{H,t} &\equiv \frac{P_{H,t}}{P_{H,t-1}} \\
(1+\pi)(1+\widehat{1+\pi_{H,t}}) &= \frac{P_H}{P_H}(1+p_{H,t}-p_{H,t-1})
\end{aligned}$$

We obtain

$$\begin{aligned}
\widehat{1+\pi_{H,t}} &= p_{H,t} - p_{H,t-1} \\
\frac{1+\pi_{H,t}-(1+\pi_H)}{(1+\pi_H)} &= p_{H,t} - p_{H,t-1} \\
\pi_{H,t} &= p_{H,t} - p_{H,t-1}
\end{aligned}$$

and therefore analogously

$$\begin{aligned}
\pi_{N,t} &= p_{N,t} - p_{N,t-1}, \qquad \pi_{T,t} = p_{T,t} - p_{T,t-1} \\
\pi_{T,t} &= v p_{H,t} - v p_{H,t-1} = v\pi_{H,t}
\end{aligned}$$

and CPI inflation

$$\begin{aligned}
p_t - p_{t-1} &= \gamma p_{T,t} - \gamma p_{T,t-1} + (1-\gamma)p_{N,t} - (1-\gamma)p_{N,t-1} \\
\pi_t &= \gamma\pi_{T,t} + (1-\gamma)\pi_{N,t}
\end{aligned}$$

Investment, Capital Accumulation and Production

We just go through the derivation for the H sector, solutions for the N sector are obtained similarly. For the production function we had in aggregate that

$$Y_{H,t} = A_t^H (K_{t-1}^H)^{\alpha_H} (L_t^H)^{1-\alpha_H}$$

which yields

$$y_{H,t} = a_t^H + \alpha_H k_{t-1}^H + (1-\alpha_H) l_t^H$$

Log-linearising Tobin's Q

$$Q^H(1+q_t^H) = [\Phi'(\frac{I^H}{K^H})]^{-1}(1-\widehat{\Phi'(\frac{I_t^H}{K_{t-1}^H})})$$

which yields that in steady state $Q^H = 1$. As $\Phi(\frac{I^H}{K^H}) = \delta$ and further $\frac{I^H}{K^H} = \delta$ we get $\Phi'(\frac{I^H}{K^H}) = 1$. Then

$$\begin{aligned}
q_t^H &= -\widehat{\Phi'(\frac{I_t^H}{K_{t-1}^H})} \\
q_t^H &= -\frac{\Phi''}{\Phi'}\frac{I^H}{K^H}(i_t^H - k_{t-1}^H) \\
q_t^H &= \eta(i_t^H - k_{t-1}^H)
\end{aligned}$$

where $\eta = -\frac{\Phi''}{\Phi'}\frac{I^H}{K^H}$ denotes the elasticity of the capital-investment ratio. Capital accumulation can be written as

$$K^H(1+k_t^H) = \Phi[\frac{I^H}{K^H}]K^H(1+\widehat{\Phi(\frac{I_t^H}{K_{t-1}^H})} + k_{t-1}) + (1-\delta)K^H(1+k_{t-1}^H)$$

In steady state

$$\begin{aligned}
K^H &= \Phi[\frac{I^H}{K^H}]K^H + (1-\delta)K^H \\
\Phi[\frac{I^H}{K^H}] &= \delta
\end{aligned}$$

Then

$$\begin{aligned}
(1+k_t^H) &= \delta(1+\widehat{\Phi(\frac{I_t^H}{K_{t-1}^H})} + k_{t-1}) + (1-\delta)(1+k_{t-1}^H) \\
k_t^H &= \delta\widehat{\Phi(\frac{I_t^H}{K_{t-1}^H})} + k_{t-1}^H = \delta(\frac{\Phi'[\frac{I^H}{K^H}]}{\Phi[\frac{I^H}{K^H}]}\frac{I^H}{K^H}(i_t^H - k_{t-1}^H)) + k_{t-1}^H \\
k_t^H &= \delta(\frac{1}{\delta}\delta(i_t^H - k_{t-1}^H)) + k_{t-1}^H = \delta i_t^H + (1-\delta)k_{t-1}^H
\end{aligned}$$

The intertemporal investment decision was

$$\begin{aligned}
&\lambda_t^C \frac{P_t^{I,H}}{P_t} Q_t^H \\
= \quad & \beta\mathcal{E}_t[\lambda_{t+1}^C \frac{P_{H,t+1}}{P_{t+1}} R_{s+1}^H] + \beta\mathcal{E}_t[\lambda_{t+1}^C \frac{P_{t+1}^{I,H}}{P_{t+1}} Q_{t+1}^H]\{[\Phi[\frac{I_{t+1}^H}{K_t^H}] - \Phi'[\frac{I_{t+1}^H}{K_t^H}](\frac{I_{t+1}^H}{K_t^H}) + (1-\delta)\}
\end{aligned}$$

We assume that the price level of investment goods is same as the price level of the respective final produced good in that sector. Hence we get

$$\begin{aligned}
& \lambda_t^C \frac{P_{H,t}}{P_t} Q_t^H \\
= \quad & \beta \mathcal{E}_t[\lambda_{t+1}^C \frac{P_{H,t+1}}{P_{t+1}} R_{s+1}^H] + \beta \mathcal{E}_t[\lambda_{t+1}^C \frac{P_{t+1}^H}{P_{t+1}} Q_{t+1}^H]\{[\Phi[\frac{I_{t+1}^H}{K_t^H}] - \Phi'[\frac{I_{t+1}^H}{K_t^H}](\frac{I_{t+1}^H}{K_t^H}) + (1-\delta)\}
\end{aligned}$$

which log-linearises to

$$\hat{\lambda}_t^C + p_{H,t} - p_t + q_t^H = \mathcal{E}_t[\hat{\lambda}_{t+1}^C + p_{H,t+1} - p_{t+1}] + (1 - \beta(1-\delta))\mathcal{E}_t[r_{t+1}^H] + \beta \mathcal{E}_t[q_{t+1}^H]$$

and analogously for the N sector.

Factor Supply and Factor Demand

For total labour available in the economy we obtain

$$l_t = \frac{L^N}{L} l_t^N + \frac{L^H}{L} l_t^H$$

Labour supply is guided by $\eta(L_t)^{\kappa-1} C_t = \frac{W_t}{P_t}$ which yields

$$(\kappa - 1) l_t + c_t = w_t - p_t$$

For labour demand of firms we obtain, for each sector

$$\begin{aligned}
w_t - p_t^H &= mc_t^H + y_t^H - l_t^H \\
w_t - p_t^N &= mc_t^N + y_t^N - l_t^N
\end{aligned}$$

Sector-specific capital rental price is determined by

$$\begin{aligned}
r_t^H &= mc_t^H + y_t^H - k_{t-1}^H \\
r_t^N &= mc_t^N + y_t^N - k_{t-1}^N
\end{aligned}$$

Euler Equation and Money Demand

The intertemporal consumption/savings decision is guided by $\lambda_t^C = \beta \mathcal{E}_t[\lambda_{t+1}^C (1+i_t) \frac{P_t}{P_{t+1}}]$ which becomes in steady state $1 = \beta(1+i)$, therefore

$$\hat{\lambda}_t^C = \mathcal{E}_t[\hat{\lambda}_{t+1}^C - (p_{t+1} - p_t)] + \hat{\imath}_t$$

Further $\lambda_t^C = (C_t)^{-1}$ from which we directly obtain that

$$\begin{aligned}
-c_t &= \mathcal{E}_t[-c_{t+1} - (p_{t+1} - p_t)] + \hat{\imath}_t \\
\mathcal{E}_t[c_{t+1}] &= c_t + \mathcal{E}_t[\hat{\imath}_t - (p_{t+1} - p_t)]
\end{aligned}$$

Further we obtained that $\frac{1}{C_{H,t}} = \beta \mathcal{E}_t[\frac{1}{C_{H,t+1}} \frac{P_t^H}{P_{t+1}^H}(1+i_t)]$ which becomes

$$\begin{aligned}
-c_{H,t} &= \mathcal{E}_t[-c_{H,t+1} - (p_{H,t+1} - p_{H,t})] + \hat{\imath}_t \\
\mathcal{E}_t[c_{H,t+1}] &= c_{H,t} + E_t[\hat{\imath}_t - (p_{H,t+1} - p_{H,t})]
\end{aligned}$$

and

$$\mathcal{E}_t[c_{N,t+1}] = c_{N,t} + \mathcal{E}_t[\hat{\imath}_t - (p_{N,t+1} - p_{N,t})]$$

Money demand by the households is guided by $\chi \frac{(\frac{M_t}{P_t})^{-1}}{\lambda_t^C} = \frac{i_t}{1+i_t}$ which becomes

$$\begin{aligned}
\chi \frac{P}{M} C(1 + c_t + p_t - m_t) &= \frac{i}{1+i}(1 + \widehat{\frac{i_t}{1+i_t}}) \\
c_t + p_t - m_t &= \frac{\widehat{i_t}}{1+i} = \hat{\imath}_t \\
m_t - p_t &= c_t - \hat{\imath}_t
\end{aligned}$$

Resource Constraints, Balance of Payments and Risk Premium

Log-linearising the resource constraint for home tradables and non-tradables

$$\begin{aligned}
y_{H,t} &= \frac{C_H}{Y_H} c_{H,t} + \frac{I_H}{Y_H} i_{H,t} + \frac{G_H}{Y_H} g_{H,t} + \frac{C_H^*}{Y_H} c_{H^*,t} \\
y_{N,t} &= \frac{C_N}{Y_N} c_{N,t} + \frac{I_N}{Y_N} i_{N,t} + \frac{G_N}{Y_N} g_{N,t}
\end{aligned}$$

Balance of payments in nominal terms was given by (4.53) and we obtained that

$$\begin{aligned}
F_t &= (1 + i_{t-1}^*) \frac{S_t}{S_{t-1}} F_{t-1} + NX_t - (Z_t - Z_{t-1}) \\
F_t &= (1 + i_{t-1}) F_{t-1} + NX_t - (Z_t - Z_{t-1})
\end{aligned}$$

Log-linearising the later and as $I_{t-1} \equiv 1 + i_{t-1}$

$$F(1 + f_t) = IF(1 + \hat{I}_{t-1} + f_{t-1}) + NX(1 + nx_t) - Z(1 + z_t) + Z(1 + z_{t-1})$$

Hence, in steady state $\frac{NX}{F} = \frac{\beta - 1}{\beta} < 0$ as derived earlier. Further

$$\begin{aligned}
F f_t &= IF(\hat{I}_{t-1} + f_{t-1}) + NX nx_t - Z(z_t - z_{t-1}) \\
f_t &= I(\hat{I}_{t-1} + f_{t-1}) + \frac{NX}{F} nx_t - \frac{Z}{F}(z_t - z_{t-1})
\end{aligned}$$

Note that as is usual in this literature, we denote $\hat{\imath}_t$ as the percentage deviation of the net nominal rate of return from steady state in levels, whereas all other linearised variables denote log-linear deviations from steady state, see e.g. Walsh (2003). Hence $\hat{I}_{t-1} \equiv d \ln \frac{I_t}{I} = i_{t-1} - i \equiv \hat{\imath}_{t-1}$ and therefore

$$f_t = I\hat{\imath}_{t-1} + I f_{t-1} + \frac{NX}{F} nx_t - \frac{Z}{F}(z_t - z_{t-1}) = \frac{1}{\beta}\hat{\imath}_{t-1} + \frac{1}{\beta} f_{t-1} - \frac{1-\beta}{\beta} nx_t - \frac{Z}{F}(z_t - z_{t-1})$$

where we have used that $I = (1 + i) = \frac{1}{\beta}$. The real net foreign asset position deflated by the CPI index is

$$\begin{aligned}
\frac{F_t}{P_t} &= (1 + i_{t-1}) \frac{F_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} + \frac{NX_t}{P_t} - (\frac{Z_t}{P_t} - \frac{Z_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t}) \\
F_t^r &= \frac{1 + i_{t-1}}{1 + \pi_t} F_{t-1}^r + \frac{NX_t}{P_t} - (Z_t^r - \frac{Z_{t-1}^r}{1 + \pi_t})
\end{aligned}$$

Log-linearising, defining real variables $\frac{Z_t}{P_t} = Z_t^r$ and using steady state items $\frac{F}{P} = F^r$, $\frac{Z}{P} = Z^r$ one obtains

$$f_t^r = \frac{I}{1+\pi}\hat{\imath}_{t-1} + \frac{I}{1+\pi}f_{t-1}^r - \frac{I}{1+\pi}\pi_t + \frac{NX^r}{F^r}(nx_t - p_t) - \frac{Z^r}{F^r}(z_t^r - z_{t-1}^r + \pi_t)$$

Using that in steady state $\pi = 0$ and $I = 1 + i = \frac{1}{\beta}$ and by the stationarity condition $\frac{NX^r}{F^r} = \frac{NX/P}{F/P} = \frac{NX}{F} = \frac{\beta-1}{\beta}$ and as $\frac{Z^r}{F^r} = \frac{Z/P}{F/P} = \frac{Z}{F}$ we obtain that

$$\begin{aligned}
f_t^r &= \frac{1}{\beta}\hat{\imath}_{t-1} + \frac{1}{\beta}f_{t-1}^r - \frac{1}{\beta}\pi_t - \frac{1-\beta}{\beta}(nx_t - p_t) - \frac{Z^r}{F^r}(z_t^r - z_{t-1}^r + \pi_t) \\
f_t^r &= \frac{1}{\beta}f_{t-1}^r + \frac{1}{\beta}\hat{\imath}_{t-1} - (\frac{1}{\beta} + \frac{Z}{F})\pi_t - \frac{1-\beta}{\beta}nx_t + \frac{(1-\beta)(\gamma-1)}{\beta}q_t \\
&\quad - \frac{(1-\beta)v}{\beta}t_t - \frac{Z}{F}z_t^r + \frac{Z}{F}z_{t-1}^r
\end{aligned}$$

where we used that $F = F^r$ in a zero inflation steady state and that $P = 1$. Furthermore $p_t = s_t + (\gamma - 1)q_t - vt_t$ from log-linearising (4.48) where $s_t = 0$. The nominal net trade balance (in current units of national currency), where exports are invoiced in units of the domestic currency, reads

$$\begin{aligned}
NX_t &= EX_t - IM_t = P_{H,t}C_{H,t}^* - P_{F,t}C_{F,t} \\
NX(1 + nx_t) &= P_H C_H^*(1 + c_{H^*,t} + p_{H,t}) - P_F C_F(1 + c_{F,t} + p_{F,t}) \\
nx_t &= \frac{P_H C_H^*}{NX}(c_{H^*,t} + p_{H,t}) - \frac{P_F C_F}{NX}(c_{F,t} + p_{F,t})
\end{aligned} \tag{C.4}$$

where we have used that due to the LOP $p_{F,t} = s_t = 0$. As $NX = P_H C_H^* - P_F C_F$, we can write $\frac{P_F C_F}{NX} = \frac{P_H C_H^*}{NX} - 1$. The real trade balance in terms of the home produced tradable is defined as

$$\begin{aligned}
NX_t^r &= \frac{NX_t}{P_{H,t}} = C_{H,t}^* - \frac{P_{F,t}}{P_{H,t}}C_{F,t} = C_{H,t}^* - T_t C_{F,t} \\
nx_t^r &= \frac{C_H^*}{NX^r}c_{H^*,t} - T\frac{C_F}{NX^r}(c_{F,t} + t_t) = \frac{P_H C_H^*}{NX}c_{H^*,t} - T\left(\frac{P_H C_H^*}{NX} - 1\right)(c_{F,t} + t_t)
\end{aligned} \tag{C.5}$$

Market interest rates at which home households can borrow/lend internationally and the world interest rate were linked by

$$(1 + i_t^*) = (1 + \tilde{\imath}_t^*)\phi[-\frac{F_t}{P_t}]$$

We assume that in steady state $\phi[-\frac{F}{P}] = 1$. Using that $P = 1$

$$\begin{aligned}
(1 + i^*) &= (1 + \tilde{\imath}^*)\phi[-F] \\
i^* &= \tilde{\imath}^*
\end{aligned}$$

Log-linearising

$$\begin{aligned}
I(1 + (\widehat{1 + i_t^*})) &= I\phi[-F](1 + (\widehat{1 + \tilde{\imath}^*}) + \widehat{\phi[-\frac{F_t}{P_t}]}) \\
\hat{\imath}^* &= \widehat{\tilde{\imath}^*} - \frac{\phi'}{\phi}F(f_t - p_t) = \widehat{\tilde{\imath}^*} - \xi(f_t - p_t)
\end{aligned}$$

where $\xi = \frac{\phi'}{\phi}F$ is our risk premium. We see that $\hat{\imath}^*$ increases in net real financial liabilities / decreases in net real financial assets which ensures the stability of the model. Log-linearising the uncovered interest-parity condition given by $(1+i_t) = (1+i_t^*)\mathcal{E}_t\frac{S_{t+1}}{S_t}$ yields $\hat{\imath}_t = \hat{\imath}_t^*$ where we have made use of the currency board condition that $s_t = 0$ at all dates t. In the currency board mechanism, money supply and foreign reserves are linked by $Z_t = M_t$, hence $z_t = m_t$. For the real exchange rate we obtain

$$\begin{aligned} Q(1+q_t) &= \frac{P_T}{P_N}(1+p_{T,t}-p_{N,t}) \\ q_t &= p_{T,t}-p_{N,t} \end{aligned}$$

from which follows that $Q = 1$. Terms of trade given by the relative price of imported goods in terms of home produced tradables $T_t = \frac{P_{F,t}}{P_{H,t}} = \frac{S_t P_{F,t}^*}{P_{H,t}} = \frac{S_t}{P_{H,t}}$ directly yields $T = 1$ and

$$t_t = s_t - p_{H,t} = -p_{H,t}$$

Hybrid New Keynesian Phillips Curve

Forward-Looking Price Setters

We follow Walsh (2003, pp. 263-265) for the derivation of the forward-looking part of the Phillips curve. (4.33) is the starting point for obtaining the so-called forward-looking New Keynesian Phillips curve. We go through the derivation for the home tradables sector H. The solution for the N sector is obtained similarly. The overall price level in period t is given by

$$\begin{aligned} P_{H,t}^{1-\rho_H} &= (1-\theta_H)(p_{H,t}^o(z))^{(1-\rho_H)} + \theta_H P_{H,t-1}^{(1-\rho_H)} \\ 1 &= (1-\theta_H)V_t^{(1-\rho_H)} + \theta_H(\frac{P_{H,t-1}}{P_{H,t}})^{(1-\rho_H)} \end{aligned}$$

where we have defined $V_t = \frac{p_{H,t}^o(z)}{P_{H,t}}$. Log-linearising around a zero inflation steady state yields

$$1 = (1-\theta_H)V^{(1-\rho_H)}(1+(1-\rho_H)\hat{V}_t) + \theta_H(\frac{P_H}{P_H})^{(1-\rho_H)}(1-(1-\rho_H)(p_{H,t}-p_{H,t-1}))$$

where lower case variables indicate log-linear deviations from steady state. In steady state

$$\begin{aligned} 1 &= (1-\theta_H)V^{(1-\rho_H)} + \theta_H \\ V &= 1 \end{aligned}$$

Then $0 = (1-\theta_H)\hat{V}_t - \theta_H(p_{H,t}-p_{H,t-1})$ and as by definition, the inflation rate in the home tradable sector is $\pi_{H,t} = \frac{P_{H,t}-P_{H,t-1}}{P_{H,t-1}}$. Log-linearising $\pi_{H,t}$

$$\begin{aligned} 1+\pi_{H,t} &= \frac{P_{H,t}}{P_{H,t-1}} \\ (1+\pi)(1+\widehat{1+\pi_{H,t}}) &= \frac{P_H}{P_H}(1+p_{H,t}-p_{H,t-1}) \end{aligned}$$

we obtain $\pi_{H,t} = p_{H,t}-p_{H,t-1}$ and hence $\hat{V}_t = \frac{\theta_H}{1-\theta_H}\pi_{H,t}$. Price setting of firms given by (4.33) under flexible prices (i.e. firms can reset prices each period) becomes

$$\frac{p_{H,t}^o(z)}{P_{H,t}} = \frac{\rho_H}{\rho_H-1}MC_t^H(z)$$

where optimal set price $p^o_{H,t}(z)$ equals real marginal costs times a positive markup which indicates market power of firm z. $\frac{\rho_H}{\rho_H-1} \equiv \mu$ is the Lerner-index of industry concentration. Further, from the law of motion for the home-tradables price level under flexible prices

$$\begin{aligned} 1 &= 1V_t^{(1-\rho_H)} + 0(\frac{P_{H,t-1}}{P_{H,t}})^{(1-\rho_H)} \\ V_t^{flex} &= 1 = \frac{p^o_{H,t}(z)}{P_{H,t}} \end{aligned}$$

We use these results to rewrite (4.33) under sticky prices ($\theta_H > 0$)

$$V_t\mathcal{E}_t\sum_{s=0}^{\infty}(\theta_H\beta)^s(C_{t+s})^{-1}(\frac{P_{H,t+s}}{P_{H,t}})^{\rho_H-1}Y_{H,t+s} = \mu\mathcal{E}_t\sum_{s=0}^{\infty}(\theta_H\beta)^s(C_{t+s})^{-1}MC^H_{t+s}(\frac{P_{H,t+s}}{P_{H,t}})^{\rho_H}Y_{H,t+s}$$

Log-linearising the left hand side

$$\begin{aligned} & V(1+\hat{V}_t) \\ & \mathcal{E}_t\sum_{s=0}^{\infty}(\theta_H\beta)^s(C_{t+s})^{-1}(\frac{P_H}{P_H})^{\rho_H-1}Y_H[1-c_{t+s}+(\rho_H-1)(p_{H,t+s}-p_{H,t})+y_{H,t+s}] \\ \Leftrightarrow\ & \frac{C^{-1}Y_H}{1-\theta_H\beta} + \frac{C^{-1}Y_H\hat{V}_t}{1-\theta_H\beta} + C^{-1}Y_H\mathcal{E}_t\sum_{s=0}^{\infty}(\theta_H\beta)^s[-c_{t+s}+(\rho_H-1)(p_{H,t+s}-p_{H,t})+y_{H,t+s}] \end{aligned}$$

with $\hat{V}_tc_{t+s} = \hat{V}_tp_{H,t+s} = \hat{V}_tp_{H,t} = \hat{V}_ty_{H,t+s} \simeq 0$. Log-linearising the right hand side

$$\begin{aligned} & \mu\mathcal{E}_t\sum_{s=0H}^{\infty}(\theta_H\beta)^s(C_{t+s})^{-1}MC^H_{t+s}(\frac{P_{H,t+s}}{P_{H,t}})^{\rho_H}Y_{H,t+s} \\ \Leftrightarrow\ & \frac{C^{-1}Y_H}{1-\theta_H\beta} + C^{-1}Y_H\mathcal{E}_t\sum_{s=0}^{\infty}(\theta_H\beta)^s[-c_{t+s}+mc^H_{t+s}+\rho_H(p_{H,t+s}-p_{H,t})+y_{H,t+s}] \end{aligned}$$

where in the last step we made use of $\mu MC^H = 1$. Then the price setting equation becomes, by dividing through with $C^{-1}Y_H$ and dropping terms,

$$\begin{aligned} & \frac{\hat{V}_t}{1-\theta_H\beta} + \mathcal{E}_t\sum_{s=0}^{\infty}(\theta_H\beta)^s[-c_{t+s}+(\rho_H-1)(p_{H,t+s}-p_{H,t})+y_{H,t+s}] \\ = \ & \mathcal{E}_t\sum_{s=0}^{\infty}(\theta_H\beta)^s[-c_{t+s}+mc^H_{t+s}+\rho_H(p_{H,t+s}-p_{H,t})+y_{H,t+s}] \end{aligned}$$

Remembering the definition of $\hat{V}_t$ and the definition of the inflation rate yields eventually

$$\begin{aligned} \frac{\theta_H}{1-\theta_H}\pi_{H,t} &= (1-\theta_H\beta)mc^H_t + \theta_H\beta\mathcal{E}_t[\pi_{H,t+1} + \frac{\theta_H}{1-\theta_H}\pi_{H,t+1}] \\ \pi_{H,t} &= \frac{(1-\theta_H)(1-\theta_H\beta)}{\theta_H}mc^H_t + \beta\mathcal{E}_t\pi_{H,t+1} \end{aligned}$$

For the N sector we proceed in the same way.

Backward Looking Price Setters

Backward-looking price setters take the last period's sectoral gross inflation rate $\frac{P_{J,t-1}}{P_{J,t-2}}$ and the index of optimally set prices $P^{\#}_{J,t-1}$ as informational sources when setting prices. For derivation of their pricing schemes as well as the overall inflation dynamics, please refer to the appendix of chapter 2 of this dissertation under A.6.3 on page 272.

Log-linear Exogenous Stochastic Processes

Log-linearising (C.3) directly yields

$$\ln \frac{A_{J,t}}{A_J} = a_{J,t} = \rho_{Y_J} a_{J,t-1} + \epsilon_{Y_J,t}$$

The log-linear law of motion for the other exogenous processes is derived accordingly

$$\ln \frac{G_{J,t}}{G_J} = g_{J,t} = \rho_{G_J} g_{J,t-1} + \epsilon_{G_J,t}, \qquad \ln \frac{C^*_{H,t}}{C^*_H} = c^*_{H,t} = \rho_{C^*_H} c^*_{H,t-1} + \epsilon_{C^*_H,t}$$

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