Chapter 7

Appendices

Appendix A

Notations

Notations

∇	Nabla operator.
Δ	Dilatation
Θ	Phase angle
Σ	Encasing surface of interconnected pore space.
Ψ	Outer surface of rock sample.
Ω	Bulk rock electrical resistivity.
Ω_{fl}	Fluid resistivity.
Ω_{st}	Fluid resistivity of hypothetical rock at $\mathrm{P}_{\mathrm{eff}} = 0$ with $\phi_{\mathrm{s}} = \phi_{\mathrm{c}} = 0$.
α	Biot coefficient.
γ	Thomsen's anisotropy parameter.
$\gamma^{(1)}$	Tsvankin's VTI parameter γ in the $[x_2,x_3]$-plane.
$\gamma^{(2)}$	Tsvankin's VTI parameter γ in the $[x_1,x_3]$-plane.
δ_{ij}	Kronecker delta
δ	Thomsen's anisotropy parameter.
$\delta^{(1)}$	Tsvankin's VTI parameter δ in the $[x_2,x_3]$-plane.
$\delta^{(2)}$	Tsvankin's VTI parameter δ in the $[x_1,x_3]$-plane.
$\delta^{(3)}$	Tsvankin's VTI parameter δ in the $[x_1,x_2]$-plane.
ϵ	Thomsen's anisotropy parameter ϵ
$\epsilon^{(1)}$	Tsvankin's VTI parameter ϵ in the $[x_2,x_3]$-plane.
$\epsilon^{(2)}$	Tsvankin's VTI parameter ϵ in the $[x_1,x_3]$-plane.
ϵ_{ij}	Strain tensor
ζ_{ij}	Inner surface tensor.
η_{ij}	Outer surface tensor
θ^{c}_{ijklmn}	Tensor of stress sensitivity.
θ_c	Tensor of stress sensitivity, isotropic case.
$\theta^{c\Lambda}_{ijklmn}$	First derivative of property Λ with respect to changes of compliant part of generalized porosity.
$\theta^{s\Lambda}_{ijklmn}$	First derivative of property Λ with respect to changes of stiff part of generalized porosity.
κ	Permeability.
κ_M	Matrix permeability.
κ_F	Fracture permeability.
λ	Lamé constant.
μ_0	Shear modulus of matrix forming mineral.
μ_{sat}	Shear modulus of saturated rock.
μ_{dry}	Shear modulus of dry rock matrix.
μ_{dryS}	Hypothetical shear modulus of the dry rock matrix at $\sigma^{\mathrm{e}} = 0$ and $\phi_{\mathrm{c}} = \phi_{\mathrm{s}} = 0$.
ν	Poisson's ratio.

χ^2	Sum of squared model deviations from observations where the deviations are normalized by the standard deviation of the observations.
υ	Viscosity.
ρ	Density
ρ_{dry}	Dry rock density
ρ_0	Mineral density
ρ_{fl}	Fluid density
ρ_{sat}	Saturated rock density
σ_1	Maximum principal stress.
σ_2	Intermediate principal stress.
σ_3	Minimum principal stress.
σ_v	Vertical stress.
σ_{hor}	Horizontal stress.
σ_{h}	Minimum horizontal stress.
σ_{H}	Maximum horizontal stress.
σ_{ij}	Stress tensor
$\sigma_{\mathrm{ij}}^{\mathrm{c}}$	Confining stress tensor.
$\sigma_{\mathrm{ij}}^{\mathrm{d}}$	Differential stress tensor.
$\sigma_{\mathrm{ij}}^{\mathrm{e}}$	Effective stress tensor.
$\sigma_{\mathrm{ij}}^{\mathrm{f}}$	Stress tensor of pore filling material.
ϕ	Porosity.
ϕ_{ij}	Generalized porosity.
ϕ_{ij}^{s0}	Stress independent stiff part of generalized porosity.
ϕ_{ij}^{s}	Stress dependent stiff part of generalized porosity.
ϕ_{ij}^{c}	Compliant part of generalized porosity.
ϕ_{fr}	Fracture porosity.
ϕ_m	Matrix porosity.
ϕ_{c}	Crack porosity.
ϕ_{c0}	Initial crack porosity at $\mathrm{P_{eff}} = 0$.
ϕ_{s}	Stress dependent part of stiff porosity ($\mathrm{P_{eff}} \neq 0$).
ϕ_{s0}	Stress independent part of stiff porosity.
A	Cross sectional area.
A_X	Best fit parameter A with respect to considered property X.
B_X	Best fit parameter B with respect to considered property X.
C	Compressibility
C_{IJ}	Stiffness tensor in Voigt notation.
C_{IJ}^{VTI}	Stiffness tensor of VTI media in Voigt notation.
C_{IJ}^{Ortho}	Stiffness tensor of orthorhombic media in Voigt notation.
C_{dry}	Compressibility of dry rock matrix.
C_{dryS}	Hypothetical compressibility of dry rock matrix at $\sigma^{\mathrm{e}} = 0$ and $\phi_{\mathrm{c}} = \phi_{\mathrm{s}} = 0$.
C_{ijkl}	Stiffness tensor.
D	Hydraulic diffusivity.
D_X	Best fit parameter D with respect to considered property X.
E	Young's modulus.

F	Electrical formation factor.
G_{ij}	Christoffel matrix.
L_y	Fracture width normal to direction of fluid flow.
K	Bulk modulus.
K_0	Bulk modulus of the matrix forming mineral.
K_ϕ	Porosity bulk modulus.
K_{dry}	Bulk modulus of dry rock matrix.
K_{dryS}	Hypothetical bulk modulus of the dry rock matrix at $\sigma^{\mathrm{e}} = 0$ and $\phi_{\mathrm{c}} = \phi_{\mathrm{s}} = 0$.
K_{fl}	Bulk modulus of pore fluid.
K_{sat}	Saturated rock bulk modulus.
K_{satS}	Hypothetical bulk modulus of the saturated rock at $\sigma^{\mathrm{e}} = 0$ and $\phi_{\mathrm{c}} = \phi_{\mathrm{s}} = 0$.
K_X	Best fit parameter K with respect to considered property X.
M	Reservoir thickness
P	Pressure.
$\mathrm{P_c}$	Confining pressure.
$\mathrm{P_{diff}}$	Differential pressure.
$\mathrm{P_{eff}}$	Effective pressure.
$\mathrm{P_{fl}}$	Pore fluid pressure.
Q	Volumetric flow rate.
R^2	Coefficient of determination.
S^{dry}	Compliance tensor of dry rock matrix.
S^{mt}	Compliance tensor matrix forming mineral.
S^{p}	Compliance tensor of pore space.
S	Storage coefficient.
S_s	Specific storage coefficient.
T	Transmissivity.
V_{P}	P-wave velocity
V_{P0}	Vertical P-wave velocity in weak anisotropic orthorhombic media.
V_{Psat}	P-wave velocity of saturated rock.
V_{Pdry}	P-wave velocity of dry rock.
V_{PdryS}	P-wave velocity of hypothetical dry rock at $\mathrm{P_{eff}}\bar{0}$ with $\phi_{\mathrm{c}} = \phi_{\mathrm{s}} = 0$.
V_{S}	S-wave velocity
V_{S0}	Velocity of the vertically traveling S-wave polarized
$\mathrm{V_{SH}}$	Transversely (out-off-plain polarized) S-wave.
$\mathrm{V_{SV}}$	In-plane polarized S-wave.
V_{Ssat}	S-wave velocity of saturated rock
V_{Sdry}	S-wave velocity of dry rock.
V_{SdryS}	S-wave velocity of hypothetical dry rock at $\mathrm{P_{eff}}\bar{0}$ with $\phi_{\mathrm{c}} = \phi_{\mathrm{s}} = 0$.
V_{ij}	A seismic velocity propagating in the i direction with polarization in the j direction.
V_{Gi}	Group velocity vector.
b	Fracture aperture.
g	Acceleration due to gravity.
i_i	Unit coordinate vector.

k_f Hydraulic conductivity.
k_i Phase vector.
m Archie's cementation factor.
n Effective stress coefficient.
n_i Direction vector of wave propagation.
t Time.
u_i Displacement vector.

Appendix B

Anisotropic stress sensitivity

B.1 Deformation of pore space

This part of the Appendix describes the theoretical background of the stress sensitivity approach in detail. It represents the paper of Shapiro & Kaselow (2003).

In order to quantify the deformation of the pore space geometry due to an applied load it is necessary to define quantitatively (a) the bulk and the pore space volume V_b and V_p, respectively, and (b) to introduce several compliances of the porous system which describe the deformation of the system due to an applied load. This will be done following and extending the approach of Brown & Korringa (1975).

The geometry of the sample bulk volume can be described in terms of a surface Σ covering the sample as shown in Fig. (B.1). The surface normal is defined positive in the outward direction. In the same way, a second surface Ψ is defined representing the inner surface of the rock, i.e., it covers the interconnected pore space. The permeability of the interconnected pore space is sufficient in order to equillibrate deformation induced pore pressure gradients within the sample. Per definition, the positive normal direction of the inner surface is pointing into the sample. Where the outer surface of the sample cuts a pore it coincides with the inner surface and simultaneously seals the pore. However, the normals point in opposite directions. In this way, it is possible to represent the

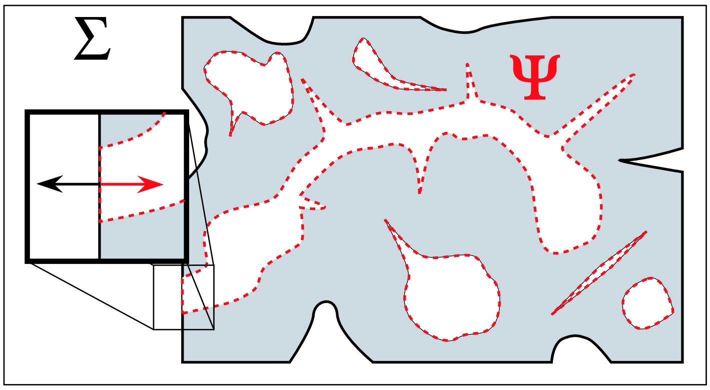

Figure B.1: Sketch of bulk and pore space geometry. Both volumes are described in terms of covering surfaces Σ and Ψ, respectively. The positive normal direction of Σ points outward and in the case of Ψ into the sample. In 3D all pores are interconnected and effective for fluid flow.

bulk and pore space volume of the rock sample in terms of the encasing surfaces Σ and Ψ, respectively. Thus, it is possible to describe changes of both volumes by the displacement of points on the surfaces Σ and Ψ.

The rock may be subjected to two different load components, an externally applied confining stress $\sigma^{\mathrm{c}}_{\mathrm{ij}}$ and an internally applied stress σ^{f}_{ij}, where i and j can be 1, 2, and 3. Here, the latter load component is denoted as pore stress. In most realistic situations this stress is a pressure, i.e., the pore pressure. Assume that the confining stress and/or pore stress have changed from an initial state of stress $(\sigma_{ij}^{c0}, \sigma_{ij}^{f0})$ to the current state $(\sigma_{ij}^{c}, \sigma_{ij}^{f})$. As a result the bulk and pore space volume will be deformed. The deformation of both volumes can be described by the displacement of the corresponding surface points. All displacements are assumed to be very small in comparison to the size of the rock volume under consideration. This can be observed in many laboratory experiments where the deformations are usually in the order of 10^{-3} or even smaller.

The points of the external surface may have been displaced by $u_i(\hat{x})$, where $\hat{x}$ is a surface point. Following Brown & Korringa (1975) it is possible to introduce a symmetric tensor:

$$\eta_{ij} = \int_{\Sigma} \frac{1}{2}(u_i n_j + u_j n_i) d^2\hat{x}. \tag{B.1}$$

Here, n is the surface normal at point $\hat{x}$. Since η_{ij} is related to the outer surface of the sample it will be denoted as the *outer surface tensor*. In the case of a continuous elastic body replacing the porous rock (i.e., a differentiable displacement is given at all its points) Gauss' theorem can be used to relate the integral over surface Σ to an integral over the corresponding volume.

$$\eta_{ij} = \int_{V_b} \frac{1}{2}(\partial_j u_i + \partial_i u_j) d^3x. \tag{B.2}$$

The integrand here is the strain tensor and V_b is the sample bulk volume. Thus, $\epsilon_{ij} = \eta_{ij}/V_b$ is the volume averaged strain.

In this way the outer surface tensor η_{ij} can be related to the deformation of the rock sample. In the same way, it is possible to define a second symmetric tensor related to the deformation of the pore space.

$$\zeta_{ij} = \int_{\Psi} \frac{1}{2}(u_i n_j + u_j n_i) d^2\hat{x}, \tag{B.3}$$

Here, $\hat{x}$ is a point of surface Ψ, u_i is a component of the displacement of points $\hat{x}$, and n_i is a component of the outward normal of Ψ. In analogy to the outer surface tensor η_{ij}, the tensorial quantity ζ_{ij} will be denoted as the *inner surface tensor*.

If the pore space is completely filled with some material (e.g., a fluid or clay or cement) then, in analogy with the outer surface tensor η_{ij}, the pore volume averaged inner surface tensor ζ_{ij}/V_p will denote the volume averaged strain of this material, where V_p denotes the volume of the pore filling material. Moreover, using the summation convention, $-\zeta_{ii}$ denotes a volume change of the pore filling material.

Up to this point two tensorial quantities η_{ij} and ζ_{ij} were introduced which describe the bulk and pore space volume, respectively, in terms of encasing surfaces and the deformation of these volumes through the displacement of the corresponding surface points. However, these deformations result from the application of two stress fields. These stress fields will be explained in more detail in the following.

As common in rock mechanics, stress acting compressional with respect to the solid phase is defined negative. Assume that the stress σ_{ij}^{f} acting on the pore space surface Ψ is isostatic. In this case, the diagonal elements of the stress tensor are identical and shear stress is absent. This described the most realistic situation, namely, that the pore space is filled with a fluid. In the literature, this state of stress is, in general, denoted as hydrostatic, because it is typical for fluids. Since a more general situation is considered here, where the pore space can be filled with an arbitrary material, this state of stress should is denoted as isostatic. In the case of an isostatic stress σ^{f}_{ij} it is possible to define a scalar pressure P_p acting on the pore space surface: $\sigma^{f}_{ij} = -\delta_{\mathrm{ij}} P_p$.

As common in rock mechanics and already mentioned in section (3.1) both load components are usually combined. Here, the difference between the external confining stress and internal pore stress is defined as the effective stress σ^{e}:

$$\sigma^{\mathrm{e}}_{\mathrm{ij}} = \sigma^{\mathrm{c}}_{\mathrm{ij}} - \sigma^{f}_{ij}. \tag{B.4}$$

In the case of an isostatic pore stress, this gives:

$$\sigma^{\mathrm{e}}_{\mathrm{ij}} = \sigma^{\mathrm{c}}_{\mathrm{ij}} + \delta_{\mathrm{ij}} P_p. \tag{B.5}$$

Assuming further a complete isostatic state of stress, i.e., also the external confining stress is isostatic, gives:

$$\sigma^{\mathrm{e}}_{\mathrm{ij}} = \delta_{\mathrm{ij}}\sigma^{\mathrm{c}}_{\mathrm{ij}} + \delta_{\mathrm{ij}} P_p = (-P_c + P_p)\delta_{\mathrm{ij}} = -(P_c - P_p)\delta_{ij}, \tag{B.6}$$

or, completely written in terms of pressure:

$$\mathrm{P}_{\mathrm{eff}} = \mathrm{P}_{\mathrm{c}} - P_p, \tag{B.7}$$

where $\mathrm{P}_{\mathrm{eff}}$ and P_{c} are the effective and confining pressure, respectively.

After considering the load components acting on a porous rock rock compliances will be introduced which relate the acting stress to the deformation of the rock.

In analogy to the paper of Brown & Korringa (1975), three fundamental compliance tensors of an anisotropic porous body can be defined which can be obtained from appropriate laboratory measurement.

$$S^{\mathrm{dry}}_{ijkl} = \frac{1}{V_b}\left(\frac{\partial \eta_{ij}}{\partial \sigma^{e}_{kl}}\right)\Bigg|_{\sigma^f}, \tag{B.8}$$

$$S^{\mathrm{mt}}_{ijkl} = \frac{1}{V_b}\left(\frac{\partial \eta_{ij}}{\partial \sigma^{f}_{kl}}\right)\Bigg|_{\sigma^e}, \tag{B.9}$$

$$S^{\mathrm{p}}_{ijkl} = -\frac{1}{V_p}\left(\frac{\partial \zeta_{ij}}{\partial \sigma^{f}_{kl}}\right)\Bigg|_{\sigma^e}. \tag{B.10}$$

Again, V_b is the bulk volume of the porous body and V_p is the volume of the interconnected pore space. S^{dry} denotes the compliance tensor of the drained rock matrix. It is obtained in an experiment where the rock samples strain (in the sense of eq. B.1) is measured as a function of the effective stress while keeping σ^{f}_{ij} constant. This corresponds to a compressional experiment on a dry sample or a drained experiment where the confining stress is variable and the pore pressure is kept constant by letting the pore fluid freely entering or leaving the sample. S^{mt} and S^{p} are the compliance tensors characterizing the grain material and the pore space, respectively. They are obtained in experiments where the inner stress, i.e., the pore pressure, is changed, but the effective stress as defined above (eq. B.5) is constant. Then, S^{mt}_{ijkl} and S^{p}_{ijkl} are obtained by relating the rock deformation to the bulk and pore volume, respectively.

If the effective stress is constant but the pore stress σ^f can be changed the confining stress must change correspondingly. This means that the same stress variation is applied to the inner and outer surface of the rock. If the material of the rock skeleton (i.e., the grain material) is homogeneous and linear, i.e., if the rock is in the Gassmann limit, then the condition about uniformly distributed stress variations will be equivalent to the replacement of the material in the pores with the grain material. This, in other words, means that the pore volume as well as the bulk volume change due to an applied load, but the porosity stays constant. Moreover, in this case the volume averaged strain is independent of the geometry of considered averaged rock domain. This yields the following modification of the last definition above:

$$S^{\mathrm{p}}_{ijkl} = -\frac{1}{V}(\frac{\partial(\zeta_{ij}/\phi)}{\partial\sigma^{\mathrm{f}}_{kl}})_{\sigma^{\mathrm{e}}} = \frac{1}{V}(\frac{\partial\eta_{ij}}{\partial\sigma^{\mathrm{f}}_{kl}})_{\sigma^{\mathrm{e}}} = \S^{\mathrm{mt}}_{ijkl}. \tag{B.11}$$

Obviously, if the rock skeleton material is homogeneous and linear both compliance tensors S^{p}_{ijkl} and S^{mt}_{ijkl} are equal to the compliance tensor of the rock matrix forming material.

One more but not-independent compliance tensor can be introduced:

$$S'_{ijkl} = -\frac{1}{V}(\frac{\partial\zeta_{ij}}{\partial\sigma^{\mathrm{e}}_{kl}})_{\sigma^{\mathrm{f}}}. \tag{B.12}$$

Using the reciprocity theorem (see, e.g., Amenzade, 1976) analogously to Brown & Korringa (1975) this gives:

$$S'_{ijkl} = S^{\mathrm{dry}}_{klij} - \sigma^{\mathrm{mt}}_{klij}. \tag{B.13}$$

However, an assumed arbitrary change of the applied load will deform the pore space, thus, will change ζ_{ij}. Such an arbitrary change of the applied load can be understood as the sum of two load changes as described above. Thus, the inner surface tensor ζ_{ij} will change due to $\delta\sigma^{\mathrm{e}}$ while keeping a constant pore stress σ^{f} plus an effect of applying $\delta\sigma^{\mathrm{f}}$ from inside and outside while keeping the effective stress σ^{e} constant:

$$\delta\zeta_{ij} = \left(\frac{\partial\zeta_{ij}}{\partial\sigma^{\mathrm{e}}_{kl}}\right)_{\sigma^{\mathrm{f}}}\delta\sigma^{\mathrm{e}}_{kl} + \left(\frac{\partial\zeta_{ij}}{\partial\sigma^{\mathrm{f}}_{kl}}\right)_{\sigma^{\mathrm{e}}}\delta\sigma^{\mathrm{f}}_{kl}. \tag{B.14}$$

In the same way changes of the outer surface tensor η_{ij} due to an arbitrary load change can be expressed using an analogous equation:

$$\delta\eta_{ij} = \left(\frac{\partial\eta_{ij}}{\partial\sigma^{\mathrm{e}}_{kl}}\right)_{\sigma^{\mathrm{f}}}\delta\sigma^{\mathrm{e}}_{kl} + \left(\frac{\partial\eta_{ij}}{\partial\sigma^{\mathrm{f}}_{kl}}\right)_{\sigma^{\mathrm{e}}}\delta\sigma^{\mathrm{f}}_{kl}. \tag{B.15}$$

Taking into account that η_{ij} is related to the deformation of the sample bulk volume it is clear that $\delta V_b = \delta\eta_{ii}$. Hence,

$$\delta V = \left(\frac{\partial\eta_{ii}}{\partial\sigma^{\mathrm{e}}_{kl}}\right)_{\sigma^{\mathrm{f}}}\delta\sigma^{\mathrm{e}}_{kl} + \left(\frac{\partial\eta_{ii}}{\partial\sigma^{\mathrm{f}}_{kl}}\right)_{\sigma^{\mathrm{e}}}\delta\sigma^{\mathrm{f}}_{kl}. \tag{B.16}$$

However, relating the inner surface tensor ζ_{ij} to the bulk volume of the sample can be used to define an additional tensorial quantity ϕ_{ij}:

$$\phi_{ij} = \frac{\zeta_{ij}}{V_b}. \tag{B.17}$$

In analogy to the porosity, which is defined as the ratio of the pore space volume to the bulk volume, ϕ_{ij} will be denoted as *generalized* porosity. Stress induced changes of ϕ_{ij} satisfy the following rule:

$$\delta\phi_{ij} \equiv \delta\left(\frac{\zeta_{ij}}{V_b}\right) = \frac{\delta\zeta_{ij}}{V_b} - \phi_{ij}\frac{\delta V_b}{V_b}. \tag{B.18}$$

Since ζ_{ij} is related to the geometry of the pore space the generalized porosity ϕ_{ij} is, in fact, closely related to the porosity ϕ. Indeed, if V_{p0} is the volume of the interconnected pores in a reference state with $\zeta_{ij} = 0$, the complete change of the porosity due to a change in the applied load is:

$$\begin{aligned} \delta\phi &= \delta\left(\frac{V_p}{V_b}\right) = \delta\left(\frac{V_{p0} - \zeta_{ii}}{V_b}\right) = -\frac{V_{p0}}{V_b}\frac{\delta V_b}{V_b} - \delta\phi_{ii} = -\frac{\delta\zeta_{ii}}{V_b} - \frac{V_{p0} - \zeta_{ii}}{V_b}\frac{\delta V_b}{V_b} \\ &= -\frac{\delta\zeta_{ii}}{V_b} - \phi\frac{\delta V_b}{V_b} = \frac{\delta V_p}{V_b} - \phi\frac{\delta V_b}{V_b}. \end{aligned} \tag{B.19}$$

Thus, using equations (B.13) - (B.16) and definitions (B.8)-(B.10) changes of the generalized porosity ϕ_{ij} can be related to arbitrary load changes using the previously defined compliances:

$$-\delta\phi_{ij} = \left(S^{\mathrm{dry}}_{klij} - S^{\mathrm{mt}}_{klij} + \phi_{ij}S^{\mathrm{dry}}_{mmkl}\right)\delta\sigma^{\mathrm{e}}_{kl} + \left(\phi S^{\mathrm{p}}_{ijkl} + \phi_{ij}S^{\mathrm{mt}}_{mmkl}\right)\delta\sigma^{\mathrm{f}}_{kl}. \tag{B.20}$$

For the porosity this gives:

$$\delta\phi = (S^{\mathrm{dry}}_{klii} - S^{\mathrm{mt}}_{klii} - \phi S^{\mathrm{dry}}_{iikl})\delta\sigma^{\mathrm{e}}_{kl} + \phi(S^{\mathrm{p}}_{iikl} - S^{\mathrm{mt}}_{iikl})\delta\sigma^{\mathrm{f}}_{kl}. \tag{B.21}$$

Equation (B.21) provides a quite general and exact way to describe porosity changes of an arbitrary anisotropic rock as a function of arbitrary load changes. In most realistic situations, this relation can be simplified by taking into account that the pore filling material is a fluid, i.e., water, brine, oil or gas. Therefore, the inner surface of the rock is subjected to a pressure rather than to a stress.

Then, with $\sigma^f_{kl} = -\delta_{kl}\mathrm{P_{fl}}$ this gives:

$$-\delta\phi_{ij} = (S^{\mathrm{dry}}_{klij} - S^{\mathrm{mt}}_{klij} + \phi_{ij}S^{\mathrm{dry}}_{mmkl})\delta\sigma^{\mathrm{e}}_{kl} - (\phi S^{\mathrm{p}}_{ijkk} + \phi_{ij}S^{\mathrm{mt}}_{mmkk})\delta\mathrm{P_{fl}}. \tag{B.22}$$

Note, up to here $\mathrm{P_{fl}}$ was denoted as P_p. This different notation should indicate that P_p describes an isostatic state of stress in an arbitrary pore filling material. Hence, using $\mathrm{P_{fl}}$ instead of P_p should indicate that the pore space is filled with a fluid. However, the following deviations are still valid for an arbitrary pore filling as long the it acts isostatically on the inner surface of the rock. However, for the porosity this gives:

$$\delta\phi = (S^{\mathrm{dry}}_{klii} - S^{\mathrm{mt}}_{klii} - \phi S^{\mathrm{dry}}_{mmkl})\delta\sigma^{\mathrm{e}}_{kl} - \phi(S^{\mathrm{p}}_{iikk} - S^{\mathrm{mt}}_{mmkk})\delta\mathrm{P_{fl}}. \tag{B.23}$$

This equation provides an exact relation describing the dependence of porosity on changes in pore fluid pressure and confining stress. Hence, it formulates quite well the dependence of porosity on a geologically realistic state of stress, although is it limited to elastic, i.e, usually small, deformations. Consequently, it may provide a suitable formalism to estimate porosity reduction with burial depth for already consolidated rocks in real geological environments. In practice, the in situ stress is usually approximated with a confining pressure, identical to the overburden pressure. Thus, taking also an isostatic confining stress into account (i.e., also $StressE_{kl} = -\delta_{kl}\mathrm{P}_{\mathrm{eff}}$) these eq. (B.22) and (B.23) are reduced to:

$$-\delta\phi_{ij} = (S^{\mathrm{dry}}_{llij} - S^{\mathrm{mt}}_{llij} + \phi_{ij}S^{\mathrm{dry}}_{mmll})\delta\mathrm{P}_{\mathrm{eff}} - (\phi S^{\mathrm{p}}_{ijkk} + \phi_{ij}S^{\mathrm{mt}}_{mmkk})\delta\mathrm{P}_{\mathrm{fl}}. \tag{B.24}$$

and

$$\delta\phi = (S^{\mathrm{dry}}_{llii} - S^{\mathrm{mt}}_{llii} - \phi S^{\mathrm{dry}}_{mmll})\delta\mathrm{P}_{\mathrm{eff}} - \phi(S^{\mathrm{p}}_{iikk} - S^{\mathrm{mt}}_{mmkk})\delta\mathrm{P}_{\mathrm{fl}}. \tag{B.25}$$

Results (B.22)-(B.25) show that generally the porosity is a function of both, the effective stress as well as the pore pressure. Remember, effective stress is defined as the pure difference between confining stress and pore pressure. If $S^{mt}_{iikk} = S^{p}_{iikk}$, i.e., the rock matrix is homogenous and/or the interconnected porosity is small then porosity changes depend on the difference between confining stress and pore pressure only. The following considerations are restricted to this case. Under isostatic conditions and considering isotropic rocks only this relation reduces to the porosity dependence on the differential pressure, as it was shown by Zimmerman *et al.* (1986) and Goulty (1998); Detournay & Cheng (1993):

$$\frac{d\phi}{d\mathrm{P}_{\mathrm{eff}}} = C^{\mathrm{mt}} - (1-\phi)C^{\mathrm{dry}}, \tag{B.26}$$

where C^{mt} and C^{dry} are compressibilities of the grain material and of the drained rock matrix, respectively. They are related to the compliance tensors as follows:

$$C^{\mathrm{mt,dry}} = S^{\mathrm{mt,dry}}_{1111} + S^{\mathrm{mt,dry}}_{2222} + S^{\mathrm{mt,dry}}_{3333} + 2(S^{\mathrm{mt,dry}}_{1122} + S^{\mathrm{mt,dry}}_{1133} + S^{\mathrm{mt,dry}}_{2233}) \equiv S^{\mathrm{mt,dry}}_{iikk}. \tag{B.27}$$

Until now the presented derivations provide several precise results describing stress dependencies of the pore space geometry. The compliance tensors S^{mt} and S^{p} are practically independent of effective stress at least up to a few hundred MPa. Thus, in equations (B.22) - (B.26) only two quantities are significantly stress dependent: the porosity ϕ and the dry rock matrix compliance tensor S^{dry}. Since porosity variations depend on the dry rock compliances and the dry rock compliances depend, in turn, on the porosity at least one more equation is required, which would mutually relate them. This equation cannot be obtained exactly, since S^{dry} depends upon the complete geometry of the pore space rather than on the magnitude of the porosity alone. Thus, a further analysis requires to involve some empirical observations and heuristic assumptions.

B.2 Elastic compliances

Typical stress dependencies of elastic moduli, hence, seismic velocities, look like shown in Fig. (B.2). Increasing effective stress leads first to a rapid non-linear increase of seismic velocities. Then, for higher stresses, the velocity stress dependence tapers into a flat linear relation. Occasionally, the linear part of the velocity-stress relation does not show any further increase of velocities with increasing stress, at least up to some hundred MPa (approx. 200-400 MPa, depending on the rock) effective stress. Although it is a quite intuitive assumption that this velocity dependence upon stress results from the closure of the porosity, many porosity measurements show that porosity does not change at all or even very slightly while velocities change remarkably (e.g., Khaksar *et al.*, 1999). Thus, it is a common interpretation that the rapid increase of velocities at low stresses results from the closure of cracks and grain contact vicinities. This part of the porosity, denoted as the *compliant porosity* usually represents only a very small fraction of the total porosity (< 1% in typical sandstones). Hence, even its complete closure does not change the bulk porosity remarkably. When this easily deformable part of the bulk porosity is closed the velocity increase is caused by the closure of the hardly deformable remaining stiff pores.

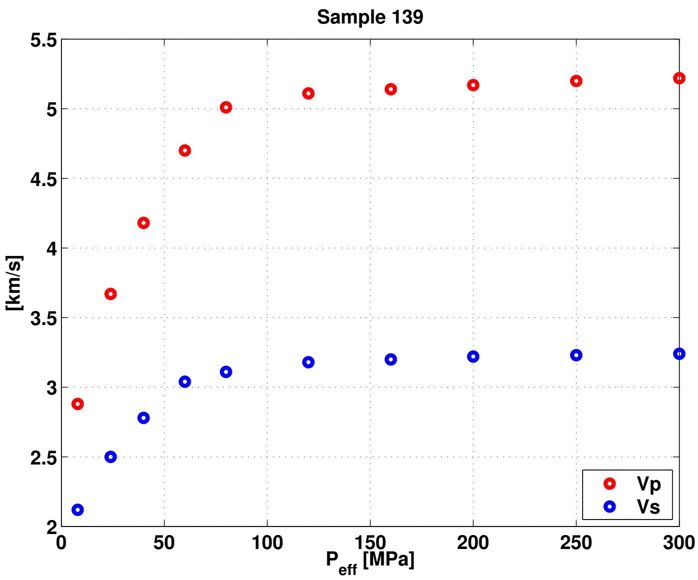

Figure B.2: Typical ultrasonic P- (red circles) and S-wave (blue circles) velocity as a function of isostatic effective stress. This example shows a sandstone from SALZWEDEL drilling site (data from Freund, 1992).

This distinct deformation behavior of stiff and compliant porosity is taken in account by formulating:

$$\phi = \phi_{\mathrm{c}} + [\phi_{\mathrm{s0}} + \phi_{\mathrm{s}}] \,, \tag{B.28}$$

as done by Shapiro (2003). Here, ϕ is the bulk interconnected porosity, ϕ_{c} is the compliant porosity supported by cracks and grain contact vicinities. As a rule of thumb, compliant porosity shows an aspect ratio γ (a relationship between the minimum and maximum dimensions of a pore) less than 0.01 (see Zimmerman *et al.*, 1986). The second part, $[\phi^{s0} + \phi^{s}]$, comprises the stiff porosity supported by more or less isometric or spherical pores (i.e., equidimensional or equant pores, see also Hudson *et al.*, 2001; Thomsen, 1995). The aspect ratio of such pores is typically larger than 0.1. Such a separation of the porosity into a compliant and a stiff part is very similar to the definitions of stiff and soft porosity by Mavko & Jizba (1991) and others.

In turn, stiff porosity is further separated into a stress independent part ϕ_{s0}, which is equal to the stiff porosity in the case of an effective stress $\sigma^{\mathrm{e}} = 0$, and into a part ϕ_{s} which describes an amount of stiff porosity due to a deviation of the effective stress from zero. As mentioned above it is reasonable to assume that the relative changes of the stiff porosity, $\phi_{\mathrm{s}}/\phi_{\mathrm{s0}}$, are small. In contrast, the relative changes of the compliant porosity $(\phi_{\mathrm{c}}\text{- } \phi_{\mathrm{c0}})/\phi_{\mathrm{c0}}$ can be very large, i.e., of the order of 1 where ϕ_{c0} denotes the compliant porosity in the unloaded case $\sigma^{\mathrm{e}} = 0$. In general, the compliant porosity is a very small quantity since it represents only a very small part of the bulk porosity. As a rule of thumb, the amount of compliant porosity is much smaller than the stress independent part of the stiff porosity ϕ^{s0} and even than the absolute value of the stress induced change of stiff porosity ϕ^{s}. Thus, the following inequality is usually valid:

$$\phi_{\mathrm{s0}} \gg |\phi_{\mathrm{s}}| \gg \phi_{\mathrm{c}}. \tag{B.29}$$

In sedimentary rocks the orders of magnitude for these quantities are approximately $\phi_{\mathrm{s0}} = 0.1$, $|\phi_{\mathrm{s}}| = 0.01$ and $\phi_{\mathrm{c}} = 0.001$. Inequality (B.29) may not be valid in low porosity crystalline rocks. For instance, calculating the stress dependence of crack porosity from strain measurements, as introduced by Brace (1965), Kern *et al.* (1991) implicitly assume that the porosity of the KTB rocks consist of compliant porosity only. However, this has no implication on the following considerations since even if a rock shows no stiff porosity eq. (B.28) is still valid.

In analogy to eq. (B.28) generalized porosity is also written as:

$$\phi_{ij} = \phi^{c}_{ij} + \left[\phi^{s0}_{ij} + \phi^{s}_{ij}\right]. \tag{B.30}$$

In the following, an inner surface tensor $\zeta_{ij} = 0$ denotes a state of the rock with a completely closed porosity. Note, closed porosity refers to a pore space volume $V_p = 0$ and not to isolated pores. Then, the initial pore space volume $V_{p0} = 0$ and eq. (B.19) give $\phi = -\phi_{ii}$. This illustrates, that it is reasonable to denote the quantity ϕ_{ij} as a generalized porosity as introduced in section (3.2.1). Quantity ϕ^{s0}_{ij} denotes the stiff part of ϕ_{ij} in the unloaded state and ϕ^{s}_{ij} denotes the stress induced changes of the stiff part of the generalized porosity. Thus, if the load is absent, $\phi^{s}_{ij} = 0$. Further, ϕ^{c}_{ij} denotes the compliant part of the generalized porosity. It can be completely closed under a compressional effective stress of the order of a few hundred megapascal. ϕ^{c0}_{ij} denotes the compliant part of the generalized porosity in the unloaded state. It is clear, that $\phi^{c} = -\phi^{c}_{ii}$, $\phi^{s0} = -\phi^{s0}_{ii}$, and $\phi^{s} = -\phi^{s}_{ii}$.

The definitions above are somehow "asymmetric" definitions of stiff and compliant porosity. This means, the stiff porosity is separated into a stress dependent and a stress dependent part while the crack porosity is treated as a whole. This is justified by their distinct deformation behavior and accounts for the assumption that under moderate loads considered here (approx. 200 - 300 MPa, dependent on the rock under consideration) the stiff porosity suffers small changes only. In contrast to this, the compliant porosity can be significantly changed or even completely closed. Thus, the notation system introduced above is convenient for describing such an "asymmetric" behavior.

Taking into account that both stress dependent parts ϕ^{s}_{ij} and ϕ^{c}_{ij} of the generalized porosity introduced above are very small (of the order of the strain), it is reasonable to assume a first, linear approximations of the skeleton compliances as functions of these quantities. A Taylor expansion gives:

$$S^{\mathrm{dry}}_{ijkl}(\phi^{s0}_{mn} + \phi^{s}_{mn}, \phi^{c}_{mn}) = S^{\mathrm{drys}}_{ijkl} + C^{\mathrm{drys}}\theta^{s}_{ijklmn}\phi^{s}_{mn} + C^{\mathrm{drys}}\theta^{c}_{ijklmn}\phi^{c}_{mn}, \tag{B.31}$$

where S^{drys}_{ijkl} is the drained compliance tensor of a hypothetical rock with a closed compliant porosity (i.e., $\phi_c = 0$) and the stiff porosity equal to ϕ_{s0}. Further,

$$\theta^s_{ijklmn} = \frac{1}{C^{\mathrm{drys}}}\frac{\partial S^{\mathrm{dry}}_{ijkl}}{\partial \phi^s_{mn}}, \tag{B.32}$$

$$\theta^c_{ijklmn} = \frac{1}{C^{\mathrm{drys}}}\frac{\partial S^{\mathrm{dry}}_{ijkl}}{\partial \phi^c_{mn}}, \tag{B.33}$$

where the derivatives are taken in points $\phi_{\mathrm{s}} = 0$ and $\phi_{\mathrm{c}} = 0$, respectively. The θ quantities in eq. (B.32) and (B.33) are individual for a given rock sample. Moreover, same load paths will give the same configuration of ϕ^s_{ij} and ϕ^c_{ij}. This means, only non-hysteresis deformations are considered. Note also that the elements ϕ_{ij} of the generalized porosity do not take pure rotations of the pore space into account. However, such rotations should be small, i.e., in the order of the strain. Moreover, they do not change the geometries of the grain contact zones. Thus, the influence of a possible pure rotation of the pore space on compliances can be neglected.

Approximation (B.31) implies that the products of the form $\theta\phi$ are smaller than 1. Numerous laboratory experiments and practical experience show that the drained compressibilities depend strongly on changes in the compliant porosity, and depend much weaker on changes in the stiff porosity. This empirical observation might be expressed by the restriction:

$$|\theta^s_{ijklmn}\phi^s_{mn}| \ll |\theta^c_{ijklmn}\phi^c_{mn}|. \tag{B.34}$$

If so, approximation (B.31) can be used further in the following simplified form:

$$S^{\mathrm{dry}}_{ijkl}(\phi^{s0}, \phi^c_{ij} + \phi^s_{ij}) = S^{\mathrm{drys}}_{ijkl} + C^{\mathrm{drys}}\theta^c_{ijklmn}\phi^c_{mn}. \tag{B.35}$$

This simplification formulates the observations of the approx. stress independent behavior of seismic moduli and velocities for high stresses, where the cracks are assumed to be completely closed. If this high stress behavior reflects the usually negligible linear dependence upon the closure of stiff porosity, this should especially valid in the low stress regime and thus, only the changes in compliant porosity are significant. A consequence of the different significance of stiff and compliant porosity closure is that the tensor θ^c is the most important property regarding the stress dependence of elastic rock moduli. This tensor is called the *tensor of stress sensitivity*.

Beside the mentioned approximations also the following simplifying assumptions will be used. In direct additive terms the porosity ϕ is neglected in comparison with 1, i.e., only rocks with moderate or small porosity in the order of 0.1 or less are considered. This limiting value is just a thumbnail and this approach may even work sufficiently for porosities up to 20 or 30 percent. However, if desired it is straightforward to modify our derivations. Further, it is assume that S^{mt} and S^p can be neglected in comparison to S^{dry}. This is quite a realistic assumption for rocks at least in the upper Earth's crust. Finally, it is assumed that the magnitude of effective stress changes is not smaller than those of pore pressure changes. This corresponds to realistic geological and technical processes in the underground.

Under the assumptions summarized above it is possible to modify equation (B.22):

$$-\delta\phi_{ij} = (S^{\mathrm{drys}}_{klij} + \theta^c_{klijmn}\phi^c_{mn}C^{\mathrm{drys}} - S^{\mathrm{mt}}_{klij})\delta\sigma^{\mathrm{e}}_{kl}. \tag{B.36}$$

Equations (B.31) and (B.36) together provide the searched for description of stress dependencies of elastic moduli. The last equation shows that the tensor of stress sensitivity θ^c_{klijmn} mainly controls these dependencies. It is analogous to the scalar dimensionless quantity introduced by Shapiro (2003) as the piezosensitivity.

The symmetry of this tensor reflects to the symmetry of the drained matrix compliance tensor of the rock under consideration. For example, the stress sensitivity tensor of a triclinic medium has 56 independent components and 3 in the case of an isotropic medium (see Appendix B.7 for details). The complexity of the stress sensitivity tensor reflects the highly complex variety of possible reactions of elastic moduli of porous systems due to applied stress.

B.3 Stress dependence of porosity

Assume that stress induced changes of stiff and compliant porosity are independent of each other. In this case, if the compliant porosity is closed then $\phi^c_{ij} = 0$ and eq. (B.36) gives:

$$-\delta\phi^s_{ij} = (S^{\mathrm{drys}}_{klij} - S^{\mathrm{mt}}_{klij})\delta\sigma^e_{kl}. \tag{B.37}$$

If this is valid then this relationship will also be valid for an arbitrary and usually small ϕ_{c}. Therefore,

$$-\delta\phi^c_{ij} = C^{\mathrm{drys}}\theta^c_{klijmn}\phi^c_{mn}\delta\sigma^e_{kl}. \tag{B.38}$$

Taking into account that $\phi_{\mathrm{s}} = 0$ if no load is applied yields:

$$\phi^s_{ij} = (-S^{\mathrm{drys}}_{klij} + S^{\mathrm{mt}}_{klij})\sigma^e_{kl}. \tag{B.39}$$

For the further analysis additional simplifying assumptions have to be made in order to analyze eq. (B.38). In the following, orthorhombic media are considered only. Moreover, the principal stress components $\tau_I \equiv \sigma^e_{ii}$ (I is equal to $1, 2$ or 3 denoting one of index combinations 11, 22 or 33 and there is no summation over i here) are assumed to act perpendicular to the symmetry planes of the orthorhombic system. For example, the principal stress component τ_1 is acting on the [2,3] symmetry plane. Moreover, these considerations are restricted to media that stay orthorhombic with the same symmetry plains in the loaded state or to media representing a special case orthorhombic symmetry. This also includes possibilities for the medium to become transversely isotropic, cubic or isotropic. Consequently, θ^c_{klijmn} can differ from zero only if $k = l$ and $i = j$ or $k = j$ and $i = l$ or, finally, $k = i$ and $l = j$ (see eq. B.31). However, in eq. (B.38) $k = l$, because coordinate axes coincide with the principal stress directions. Thus,

$$\frac{\partial\phi^c_{ij}}{\partial\tau_I} = 0, \quad \text{for i} \neq \text{j}. \tag{B.40}$$

Equation (B.40) states that all off-diagonal elements of the tensor ϕ^c_{ij} do not change due to the principal stress components. Since it is clear that all entries of the compliant porosity tensor are zero at high stresses where all cracks are closed, it follows directly that the off-diagonal elements are always identically zero. Moreover,

$$\frac{\partial\phi^c_K}{\partial\tau_I} = -C^{\mathrm{drys}}\theta^c_{IKM}\phi^c_M, \tag{B.41}$$

where all capital indices are equal to $1, 2$ or 3 denoting one of combinations $11, 22$ or 33, respectively. This equation introduces the following restriction on the components of the stress sensitivity tensor (which can be obtained from the existence condition of second partial derivatives for quantities ϕ):

$$\theta^c_{IKM}\theta^c_{JML}\phi^c_L = \theta^c_{JKM}\theta^c_{IML}\phi^c_L. \tag{B.42}$$

In general, this means that the rank of the matrix composed from elements $\theta^c_{IKM}\theta^c_{JML} - \theta^c_{JKM}\theta^c_{IML}$ is less than 3 (one dimension of this matrix is given by index L, another one by all possible combinations of I, K, and L). Consequently, a lot of internal relations exist between elements of the stress sensitivity tensor (note that a similar equation is valid in the very general case of triclinic anisotropy, where all capital indices accept values from 1 to 6 and the rank of the matrix mentioned above must be less than 6). These additional constrains on the tensor of stress sensitivity arise from the fact that the non-linearity of the medium has a known physical reason: the existence of the pore space.

For a further analyzis of equation system (B.38), the most simple case of the stress sensitivity tensor of an isotropic medium is considered. In this case only 3 components of this tensor are independent (see also Hearmon, 1953). These are θ^c_{111111}, θ^c_{111122} and θ^c_{112233}. Moreover, a detailed analysis of equation (B.41) and restriction (B.42) shows that only two types of media can satisfy these conditions. The first one is characterized by $\theta_{111111} = \theta_{112233}$, which implies that only two entries of the stress sensitivity tensor differ from each other. Then,

$$\phi^c_{11} = \phi^c_{22} = \phi^c_{33} = \phi^{c0}_{11} e^{-(\theta_{111111}+2\theta_{111133})(\tau_1+\tau_2+\tau_3)C^{\mathrm{drys}}}. \tag{B.43}$$

In such a situation different stress components (τ_1, τ_2, τ_3) have the same influence on different porosity components. This means, e.g., that the elastic anisotropy of such a medium will not change even in the case of a uniaxial stress. This situation is not realistic for rocks. In contrast, it is well known from many experiments that initially isotropic rocks become anisotropic under uniaxial load due to the preferred closure of suitably oriented cracks. Therefore, such media are not considered further.

The second type of media with a stress sensitivity tensor of an isotropic medium is characterized by $\theta_{111122} = \theta_{112233} = 0$. In other words, only one of the three independent entries of the tensor of stress sensitivity is non-zero. The solutions for porosity components are similar to those given by eq. (B.43). The changes of the principal components ϕ^c_{11}, ϕ^c_{22} and ϕ^c_{33} of the compliant porosity tensor are completely independent of each other. This is equivalent to the situation that there are three non-intersecting components of the compliant porosity independently changing by applying corresponding uniaxial effective stress.

Assuming that changes of ϕ^c_{11}, ϕ^c_{22} and ϕ^c_{33} are completely independent from each other, even for lower symmetries of the stress sensitivity tensor, the consideration will become more general. This assumption along with the symmetry properties of the stress-sensitivity tensor discussed in Appendix (B.7) requires that for capital indices equal to $1, 2$ or 3 quantities θ^c_{IKM} will not vanish only if $I = K = M$. This is also in agreement with equation (B.42). The last assumption formulates the only possible realistic situation if the symmetry of the stress sensitivity tensor corresponds the one of an isotropic medium.

In this approximation the stress dependencies of the diagonal elements of the generalized compliant porosity tensor read:

$$\phi_{11}^{c} = \phi_{11}^{c0} \exp(-\theta_1^c \tau_1 C^{\text{drys}}), \tag{B.44}$$
$$\phi_{22}^{c} = \phi_{22}^{c0} \exp(-\theta_2^c \tau_2 C^{\text{drys}}), \tag{B.45}$$
$$\phi_{33}^{c} = \phi_{33}^{c0} \exp(-\theta_3^c \tau_3 C^{\text{drys}}). \tag{B.46}$$

where a new notation θ_1^c, θ_2^c and θ_3^c for θ_{111111}^c, θ_{222222}^c and θ_{333333}^c, respectively, was introduced.

The basic assumption for the above mentioned considerations is that the dry rock compliance tensor and the pore space are the rock characteristics most sensitive to changes in effective stress. A mutual relation between both quantities was found by taking the distinct deformation behavior of the stiff and compliant porosity into account. Especially the stress induced closure of compliant porosity causes significant stress induced changes of the dry rock compliance tensor. The new introduced rock characteristic which describes the changes of the dry rock compliance tensor with changes of generalized compliant porosity is the tensor of stress sensitivity. Taking the most general case of an arbitrary anisotropic medium under non-isostatic load into account shows that this tensor is of rank six. It is directly related to non-linear elasticity of porous rocks. In order to analyze this tensor the considerations were limited to orthorhombic media subjected to a non-isostatic effective stress with principal stress components oriented normal to the symmetry planes of the medium. However, these considerations include media showing symmetries that could be understood as special cases of orthorhombic symmetry as well, i.e, transversal, cubic, and isotropic symmetry. In the case of a stress sensitivity tensor of an isotropic medium only three independent entries remain. It turned out that the only realistic situation relevant for a rock physical application arises if two of these entries are identically zero and only one independent entry remains. In this case, the stress dependencies of the principal elements of the generalized compliant porosity tensor are independent from each other.

B.4 Stress dependence of elastic moduli

Although numerous simplifying assumptions were made the derived stress dependencies of the dry rock compliances and the (generalized) porosity are still quite general with respect to rocks and the state of stress that can be expected in rock physical practice. Thus, in the following an arbitrary elastic characteristic Λ (e.g., a seismic velocity, a stiffness or a compliance) of a porous drained body is considered.

However, this requires to take only those elastic characteristics into account that dependent on stress via the stress dependence of the pore space. Thus, let us assume that Λ, in a vicinity of the state where ϕ_{s0} and ϕ_{c} as well as the effective stress are identically zero, can be expanded into a Taylor series similar to equation (B.31)) with respect to the porosity (this should be valid for all such characteristics like seismic velocities and elastic moduli):

$$\Lambda(\phi_{ij}^{s0} + \phi_{ij}^{s}, \phi_{ij}^{c}) = \Lambda^{\text{drys}} \left[1 + \theta_{ij}^{s\Lambda} \phi_{ij}^{s} + \theta_{ij}^{c\Lambda} \phi_{ij}^{c}\right], \tag{B.47}$$

where only the linear part of the Taylor expansion was kept. Λ^{drys} is a hypothetical rock characteristics, since it assumes that ϕ_{s0} and P_{eff} equal zero and ϕ_{c} is zero as well.

This is an unrealistic situation for rocks since it implicitly means that the rock has no crack-like porosity at all. Hence, the mentioned formulation of the Taylor expansion is purely a mathematical concept. Here,

$$\theta_{ij}^{s\Lambda} = \frac{1}{\Lambda^{drys}} \frac{\partial \Lambda}{\partial \phi_{ij}^{s}}, \tag{B.48}$$

$$\theta_{ij}^{c\Lambda} = \frac{1}{\Lambda^{drys}} \frac{\partial \Lambda}{\partial \phi_{ij}^{c}} \tag{B.49}$$

and the derivatives are taken at $\phi^s = 0$ and $\phi^c = 0$, respectively. Substituting the corresponding stress dependencies of the generalized porosity (equations B.39 and B.44) into eq. (B.47) gives:

$$\Lambda(\tau) = \Lambda^{drys} \left[1 + \theta_{ij}^{s\Lambda} \left(S_{ijK}^{mt} - S_{ijK}^{drys} \right) \tau_K + \theta_I^{c\Lambda} \phi_I^{c0} \exp\left(-\theta_I^c \tau_I C^{drys}\right) \right], \tag{B.50}$$

where K and I can assume one of values $1, 2$ or 3 denoting $11, 22$ and 33, respectively. In the exponent there is no summation over repeating indices. Formulating this general relation in terms of the dry rock compliances gives:

$$\begin{aligned} S_{ijkl}^{dry} = \; & S_{ijkl}^{drys} \left[1 + \theta_{ijklmn}^{s} (S_{mnK}^{drs} - S_{mnK}^{mt}) \tau_K + \right. \\ & \left. \theta_{ijklM}^{c} \phi_M^{c0} \exp\left(-\theta_M^c \tau_M C^{drys}\right) \right]. \end{aligned} \tag{B.51}$$

Again, summation over M in the exponent is not allowed. Since only the diagonal elements of the stress sensitivity tensor differ from zero, also the diagonal entries of the orthorhombic compliance matrix suffer significant changes only (i.e., at least from changes due to the variations in compliant porosity).

A comparison of all these results with equation (1.1) shows a surprising but striking result. All mentioned stress dependencies have the same form $A + KP - B \exp(-PD)$. Thus, the theoretically derived stress dependencies of all mentioned elastic rock characteristics have the same form as the empirically found best fit equation. Especially the physical meaning of parameter D is important. If the medium is isotropic or the stress sensitivity tensor corresponds to one of an isotropic medium the fit parameter D reads

$$D = \theta^c C^{\text{drys}}. \tag{B.52}$$

Thus, it is independent from the property under consideration, in other words, it is a universal quantity for the rock compliances.

If Λ is understood in terms of seismic velocities this a rough approximation since the use of a Taylor expansion implies small changes of velocities with stress. In the case of significant velocity variations due to an applied stress this approximation might become erroneous. However, it was found that the error introduced by using an equation of the form $A + KP - B \exp(-PD)$ for seismic velocities is usually negligible, as will be shown in section (3.3).

However, if eq. (B.51) is valid it is reasonable to expect that in the case of a uniaxial stress τ_N of a given direction N all coefficients D are identic to a single exponent D_N characterizing all elastic quantities changing by such a load, at least in the first approximation. Hence, the result of an arbitrary three axial load is equivalent to a simple sum of changes due to corresponding uniaxial stress.

B.5 Stress dependence of elastic anisotropy

Since a quite general relation of the stress dependent dry rock compliances has already been derived it is only consequent to analyze the corresponding stress dependence of elastic anisotropy. Since significant changes of elastic rock characteristics are produced by changes of compliant porosities only it is reasonable to neglect the contributions of the stiff porosity to stress dependences of elastic moduli in the following. Remember that orthorhombic symmetry is considered. However, the limitation to situations where the principal axes of the effective stress tensor are oriented normal to the symmetry planes is strongly restrictive and my only be satisfied in carefully conducted laboratory experiments.

In the case of a uniaxial stress τ_N applied in a given direction N the compliances that undergo changes due to such a load are: S_{NN}^{dry} (no summation over N here), $S_{44}^{dry}, S_{55}^{dr}$ and S_{66}^{dry}. In the case of a three-axial effective stress eq. (B.51) gives the following stress dependencies of the dry rock matrix compliances, whereby, from this point on, all capital indices can accept one of the values $1, 2, 3, 4, 5$ or 6 and standard contracted notation is used:

$$
\begin{aligned}
S_{11}^{dry} &= S_{11}^{drys}(1 + B_{111}E_1), && \text{(B.53)}\\
S_{22}^{dry} &= S_{11}^{drys}(1 + B_{222}E_2), && \text{(B.54)}\\
S_{33}^{dry} &= S_{33}^{drys}(1 + B_{333}E_3), && \text{(B.55)}\\
S_{44}^{dry} &= S_{44}^{drys}(1 + B_{441}E_1 + B_{442}E_2 + B_{443}E_3), && \text{(B.56)}\\
S_{55}^{dry} &= S_{55}^{drys}(1 + B_{551}E_1 + B_{552}E_2 + B_{553}E_3), && \text{(B.57)}\\
S_{66}^{dry} &= S_{66}^{drys}(1 + B_{661}E_1 + B_{662}E_2 + B_{663}E_3), && \text{(B.58)}
\end{aligned}
$$

where

$$B_{NNM} = C^{drys}\theta_{NNM}^{c}\phi_{M}^{c0}/S_{NN}^{drs} \tag{B.59}$$

and

$$E_i = \exp\left(-\theta_i^c \tau_i C^{drys}\right). \tag{B.60}$$

In eq. (B.59) no summation over N, M, as well as no summation over i in equation (B.60) is applied. Note that in the contracted notation components θ_{NKM}^{c} as well as S_{NK}^{drs} and S_{NK} are 2^n times the corresponding tensor coefficient, where n is the number of appearances of $4, 5$ or 6 as a subscript.

In the discussion on stress dependence of anisotropy Tsvankin's parameters (Tsvankin, 1997) for orthorhombic media under drained conditions are considered (in the following the index *dry* is omitted), which have been introduced in more detail in section (2.1.4). Usually these parameters are given in terms of stiffnesses (see, e.g., Sarkar *et al.*, 2003). In terms of compliances they are given in Appendix (E).

Here, only weak anisotropic media are considered. In this case, the dry rock compliance tensor S_{ijkl}^{drs} of the medium is weakly anisotropic only or may even be approximated as isotropic. In addition, the stress sensitivity tensor and ϕ_{ij}^{c0} are assumed to show an even weaker anisotropy. This means that the stress sensitivity tensor corresponds effectively to one of an isotropic medium. This last assumption leads to the following equivalences for non vanishing θ_{NNM}^{c}: $\theta_{111}^{c} = \theta_{222}^{c} = \theta_{333}^{c} = \theta_{442}^{c} = \theta_{443}^{c} = \theta_{551}^{c} = \theta_{553}^{c} = \theta_{661}^{c} = \theta_{662}^{c} = -\theta^{c}$, and also $\phi_1^{c0} = \phi_2^{c0} = \phi_3^{c0} = -\phi^{c0}/3$. In this approximation

Tsvankin's parameters simplify to:

$$\epsilon^{(1)} = \epsilon_0^{(1)} + A_1\theta^c\phi^{c0}(E_2 - E_3), \tag{B.61}$$
$$\epsilon^{(2)} = \epsilon_0^{(2)} + A_1\theta^c\phi^{c0}(E_1 - E_3), \tag{B.62}$$
$$\delta^{(1)} = \delta_0^{(1)} + A_1\theta^c\phi^{c0}(E_2 - E_3), \tag{B.63}$$
$$\delta^{(2)} = \delta_0^{(2)} + A_1\theta^c\phi^{c0}(E_1 - E_3), \tag{B.64}$$
$$\delta^{(3)} = \delta_0^{(3)} + A_1\theta^c\phi^{c0}(E_2 - E_1), \tag{B.65}$$
$$\gamma^{(1)} = \gamma_0^1 + A_2\theta^c\phi^{c0}(E_3 - E_2), \tag{B.66}$$
$$\gamma^{(2)} = \gamma_0^2 + A_2\theta^c\phi^{c0}(E_3 - E_1), \tag{B.67}$$

where ϵ_0, δ_0, and γ_0 refer to these parameters in the unstressed state. Moreover,

$$A_1 = \frac{2}{3}\frac{S_{11}}{S_{44}(S_{44} - 4S_{11})} \quad \text{and} \quad A_2 = \frac{1}{6}\frac{1}{S_{44}}.$$

If stress changes are small the exponential functions E_i in these equations can be expanded in Taylor series. Then, in the linear approximation with respect to stress the resulting formulas will be completely analogous to equations (19)-(23) of Sarkar *et al.* (2003). However, in contrast to their results the equations above are also valid for moderate stress changes.

In the special case of a completely isostatic load all E_i terms become equal, i.e.,

$$E_1 = E_2 = E_3 = E = \exp(-\theta_c C^{drys}\mathrm{P}_{\mathrm{eff}}) \tag{B.68}$$

Hence, in equations (B.61) to (B.67) the right hand sides reduce to the initial values of the anisotropy parameters in the unstressed state. This means, if the rock is weakly anisotropic and the stress sensitivity corresponds effectively to one of an isotropic medium the initial elastic anisotropy of the rock will not change under an isostatic stress.

Considering initially isotropic rocks the situation reduces to the one considered in Shapiro (2003). Quantity θ^c is then the piezosensitivity introduced in this paper. Moreover, even in the case of initially anisotropic rocks only one single quantity $D = \theta_c$ controls the exponential parts of the stress dependence of any compliance, of any stiffness and of any elastic wave velocity.

At this point it is necessary to consider that the assumptions made above might be to restrictive for real rocks. However, it will be shown in section (4.3) that there are anisotropic sedimentary and as well as metamorphic rocks which show this interesting phenomenom.

Exactly as found by Sarkar *et al.* (2003) equation (B.61) - (B.64) also describe elliptical changes of anisotropy due to non-isostatic stress only, because $\epsilon^{(1)} - \epsilon_0^{(1)} = \delta^{(1)} - \delta_0^{(1)}$, $\epsilon^{(2)} - \epsilon_0^{(2)} = \delta^{(2)} - \delta_0^{(2)}$ and $\delta^{(1)} - \delta_0^{(1)} = (\epsilon^{(1)} - \epsilon_0^{(1)}) - (\epsilon^{(2)} - \epsilon_0^{(2)})$.

However, stress induced anisotropy changes are not elliptical anymore in more general situations of a tensor of stress sensitivity that does not correspond to one of an isotropic medium. For example, assume that the compliances tensor S_{ijkl}^{drs} is isotropic or weakly anisotropic only, and the tensor of stress sensitivity has a cubic symmetry along with the condition $\phi_1^{c0} = \phi_2^{c0} = \phi_3^{c0} = -\phi^{c0}/3$ (i.e., complete equivalence of the symmetry planes). This assumption leads to the following equivalences for non vanishing θ^c_{NNM}: $\theta^c_{111} = \theta^c_{222} = \theta^c_{333} = -U$. $\theta^c_{441} = \theta^c_{552} = \theta^c_{663} = -V$ and $\theta^c_{442} = \theta^c_{443} = \theta^c_{551} = \theta^c_{553} = \theta^c_{661} = \theta^c_{662} = -W$. Thus,

$$\epsilon^{(1)} = \epsilon_0^{(1)} + A_1 U \phi^{c0}(E_2 - E_3), \quad \text{(B.69)}$$
$$\epsilon^{(2)} = \epsilon_0^{(2)} + A_1 U \phi^{c0}(E_1 - E_3), \quad \text{(B.70)}$$
$$\delta^{(1)} = \delta_0^{(1)} - A_1 \phi^{c0} F(E_3, E_2, E_1), \quad \text{(B.71)}$$
$$\delta^{(2)} = \delta_0^{(2)} - A_1 \phi^{c0} F(E_3, E_1, E_2), \quad \text{(B.72)}$$
$$\delta^{(3)} = \delta_0^{(3)} - A_1 \phi^{c0} F(E_1, E_2, E_3), \quad \text{(B.73)}$$
$$\gamma^{(1)} = \gamma_0^1 + A_2 \phi^{c0}(W - V)(E_3 - E_2), \quad \text{(B.74)}$$
$$\gamma^{(2)} = \gamma_0^2 + A_2 \phi^{c0}(W - V)(E_3 - E_1), \quad \text{(B.75)}$$

with

$$F(X,Y,Z) = [2(2X+Y)U - 3VZ - 3(X+Y)W] + [(X+Y)(W-U) + VZ]\,\frac{S_{44}}{S_{11}}. \quad \text{(B.76)}$$

An anellipticity remains non vanishing also in the case of uniaxial stress and even in the case of an isostatic load. Indeed, in the last case δ-parameters are changing only:

$$\delta^{(1)} - \delta_0^{(1)} = \delta^{(2)} - \delta_0^{(2)} = \delta^{(3)} - \delta_0^{(3)} = A_1 \phi^{c0}(2U - V - 2W)\frac{(S_{44} - 3S_{11})}{S_{11}} E. \quad \text{(B.77)}$$

Then, the anisotropy changes are cubic.

B.6 Stress sensitivity versus third-order elastic constants

When dealing with elastic characteristics of rocks as a function of an applied effective stress it might become necessary to take non-linear elasticity into account since the applied stress usually leads to finite strain much larger than the strain induced by a passing seismic wave. The magnitude of the stress induced strain might become to large to be sufficiently described with a linear stress-strain relation. However, in the following, only simple non-linearity is considered, i.e., only the stress level is taken into account and not its history. This means, hysteresis is assumed to be negligible.

A general third-order non-linear elasticity-theory based consideration provides the following equation for compliances (they can be obtained from the second order Taylor expansion of the strain tensor with respect to the stress tensor using definitions (9.11) and (9.12) of Thurston (1974) using additionally Thurston equations (9.15) and (9.16) one arrives to equation (17) of Sarkar *et al.* (2003); note that in this equation a degree of ambiguity is present: to avoid it the pair of indices kl must be renamed to, e.g., uv by keeping lm unchanged):

$$S^{dry}_{ijpq} = S^0_{ijpq} + B^0_{ijpqtm}\tau_{tm}, \quad \text{(B.78)}$$

where B^0_{ijpqtm} are third-order compliances (Thurston, 1974, p. 126), and index 0 denotes the unstressed state. The third-order compliances can be related to the third-order elastic moduli A^0_{klnors} used in Sarkar *et al.* (2003) as follows:

$$A^0_{klnors} = -C^0_{ijkl} C^0_{tmno} C^0_{pqrs} B^0_{ijtmpq}. \quad \text{(B.79)}$$

Expanding in eq. (B.51) the exponential term in the Taylor series by keeping the first two terms and comparing the result with eq. (B.78) gives:

$$B^0_{ijkl\mu\mu} \approx C^{drys}\left[\theta^s_{ijklmn}(S^{drys}_{mn\mu\mu} - S^{mt}_{mn\mu\mu}) - \theta^c_{ijkl\mu\mu}\phi^{c0}_\mu\theta^c_\mu\right], \tag{B.80}$$

where there is no summation over μ.

In the last equation, on the right hand side the second term provides usually a dominant contribution. For the following estimate the first term is thus neglected. Again, an isotropic tensor of stress sensitivity[1] is assumed and the elastic anisotropy of the tensor S^{drys}, i.e., returning back to the medium described before equation (B.70) is neglected. Then, using equations (B.79) and (B.78) gives all three, in general situations independent, coefficients of the isotropic tensor of third-order elastic moduli:

$$\begin{aligned}
A_{111} &= -(C^3_{11} + 2C^3_{12})(C^{drys})^2\theta^2_c\phi^{c0}, && \text{(B.81)}\\
A_{112} &= -(C^2_{11}C_{12} + C^2_{12}C_{11} + C^3_{12})(C^{drys})^2\theta^2_c\phi^{c0}, && \text{(B.82)}\\
A_{123} &= -3(C^2_{12}C_{11})(C^{drys})^2\theta^2_c\phi^{c0}. && \text{(B.83)}
\end{aligned}$$

We see now that in the case of compliant-porosity-related non-linearity these three quantity are not independent any more. Usually C_{11} is larger or even much larger than C_{12}. Then, one can expect that A_{111} has the largest absolute value. This is also observed in reality (see Table 3 of Sarkar *et al.* (2003)).

B.7 Symmetry properties of the stress sensitivity tensor

In the following, the general symmetry properties of the tensor of stress sensitivity are considered (Shapiro & Kaselow, 2003). Therefore, equation (B.31) is considered in more details. Firstly, due to the symmetry of tensor ϕ_{ij} the following symmetry is valid for the tensor of stress sensitivity:

$$\theta^c_{klijmn} = \theta^c_{klijnm}. \tag{B.84}$$

Taking into account that quantities ϕ^s and ϕ^c can assume any values including zero, and also assuming that usual symmetry properties, at least approximately, are valid for the tensor of drained compliances even in the loaded state (note, that according to the underlying concept of physical processes under consideration, the rock transfers from one state of linear elasticity to another one due to changes in the applied stress), this gives

$$\theta^c_{klijmn} \approx \theta^c_{ijklmn} \approx \theta^c_{jiklmn} \approx \theta^c_{ijlkmn}. \tag{B.85}$$

Note also, that under assumptions described above eq. (B.36) approximately gives:

$$\delta\epsilon_{ij} \approx -\delta\phi_{ij}. \tag{B.86}$$

This result follows from a comparison of equations (B.15) and (B.20). Thus,

$$\frac{\partial S^{dry}_{ijkl}}{\partial\phi_{mn}} \approx -\frac{\partial S^{dry}_{ijkl}}{\partial\epsilon_{mn}}. \tag{B.87}$$

[1]Talking about an isotropic tensor of stress sensitivity means that a stress sensitivity tensor is considered with symmetry properties corresponding to those of an isotropic medium.

Moreover,

$$S_{IK}^{dry} C_{KJ}^{dry} = \delta_{IJ}, \tag{B.88}$$

where S_{IK}^{dry} and C_{KJ}^{dry} denote the elements of the compliances and stiffnesses matrices in the contracted notations (see also Auld, 1990) and δ_{IJ} denotes the Kronecker delta. This equation follows from the fact that the stiffness matrix is inverse to the compliance matrix. In addition,

$$\frac{\partial C_{IK}^{dry}}{\partial \epsilon_J} = \frac{\partial C_{IJ}^{dry}}{\partial \epsilon_K}, \tag{B.89}$$

because the compliances are directly proportional to the second strain derivative of the free energy (consider isothermic processes here). Again, ϵ_J denotes components of the strain vector in the contracted notations.

Writing now equation (B.88) using different indices:

$$S_{IJ}^{dry} C_{JK}^{dry} = \delta_{IK}, \tag{B.90}$$

and differentiating both equations (B.88) and (B.90) by ϵ_J and ϵ_K, respectively, gives

$$\frac{\partial S_{IK}^{dry}}{\partial \epsilon_J} = \frac{\partial S_{IJ}^{dry}}{\partial \epsilon_K}. \tag{B.91}$$

Thus, the stress sensitivity tensor shows approximataly the following symmetry:

$$\theta_{klijmn}^{c} \approx \theta_{ijmnkl}^{c} \approx \theta_{klmnij}^{c}. \tag{B.92}$$

Consequently, at least in the first approximation, the stress-sensitivity tensor possess the symmetry properties (B.84), (B.85), and (B.92). These symmetries coincide with the symmetries of the third-order elastic coefficient tensor of non-linear elastic media (see also Hearmon, 1953). Therefore, all medium-symmetry caused mutual relationships between elements of these tensors will coincide. This means, for example, that for a triclinic material the stress-sensitivity tensor has 56 independent components. In monoclinic media there are 32 independent components. In orthorhombic - 20; hexagonal - 10 and isotropic -3. The complexity of the stress-sensitivity tensor reflects a variety of possible reactions of elastic moduli of porous systems on stresses.

Appendix C

Stiffness matrices

Here, the stiffness tensors of anisotropic media are given in Voigt notation, according to Tsvankin (2001).

- Triclinic symmetry, 21 independent entries.

$$C_{IJ} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{pmatrix}$$

- Monoclinic symmetry, 13 independent entries.

$$C_{IJ} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{55} & 0 \\ C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66} \end{pmatrix}$$

- Orthorhombic symmetry, 9 independent entries.

$$C_{IJ} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{pmatrix}$$

- Transversel or hexagonal symmetry, 5 independent entries.
  - Vertical transversal isotropic (VTI) medium. 3-axis is assumed to represent the symmetry axis.

$$C_{IJ}^{VTI} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{pmatrix},$$

with

$$C_{12} = C_{11} - 2C_{66}.$$

– Horizontal transversal isotropic (HTI) medium. 1-axis is assumed to represent the symmetry axis.

$$C_{IJ}^{HTI} = \begin{pmatrix} C_{11} & C_{13} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{33} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{55} \end{pmatrix},$$

with

$$C_{23} = C_{33} - 2C_{44}.$$

- Cubic symmetry, 3 independent entries.

$$C_{IJ} = \begin{pmatrix} C_{33} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{33} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{pmatrix}$$

- Isotropic symmetry, 2 independent entries

$$C_{IJ} = \begin{pmatrix} C_{33} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{33} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{55} \end{pmatrix},$$

with

$$C_{12} = C_{33} - 2C_{55},$$

and

$$C_{55} = \mu \text{ and } C_{33} = K + \frac{4}{3}\mu.$$

Appendix D

Exact velocities in VTI media

When dealing with seismic data from transversely isotropic or even orthorhombic media they are usually assumed to show weak anisotropy, i.e., anisotropy is in the range of 10-20% or even below. And in fact, most rocks relevant in practice can be described in this way. The advantage of the weak anisotropy approach is the remarkably reduced algebraic complexity in comparison to the exact relations. While the weak anisotropy approximation is discussed in detail in the main text, here, the exact equations for velocities in VTI media are given for completeness, as discussed by Thomsen (1986).

The three phase velocities in VTI media as a function of the phase angel Θ are:

$$\begin{aligned} \rho V_P^2(\Theta) &= \frac{1}{2}\left[C_{33} + C_{44} + (C_{11} - C_{33})\sin^2\Theta + D(\Theta)\right], && \text{(D.1)} \\ \rho V_{SV}^2(\Theta) &= \frac{1}{2}\left[C_{33} + C_{44} + (C_{11} - C_{33})\sin^2\Theta - D(\Theta)\right], && \text{(D.2)} \\ \rho V_{SH}^2(\Theta) &= C_{66}\sin^2\Theta + C_{44}\cos^2\Theta,, && \text{(D.3)} \end{aligned}$$

with

$$\begin{aligned} D(\Theta) \equiv\ & \Big((C_{33} - C_{44})^2 \\ & +2\left(2\left(C_{13} + C_{44}\right)^2 - \left(C_{33} - C_{44}\right)\left(C_{11} + C_{33} - 2C_{44}\right)\right)\sin^2\Theta \\ & + \left(\left(C_{11} + C_{33} - 2C_{44}\right)^2 - 4\left(C_{13} + C_{44}\right)^4\right)\sin^4\Theta\Big)^{1/2}. \end{aligned} \tag{D.4}$$

The obvious algebraic complexity is the main obstacle to use eq. (D.4) in practice. To overcome this complexity Thomsen (1986) introduced a set of parameters representing useful combinations of different stiffnesses in eq. (D.1) to (D.4), involving only two elastic velocities and three measures of anisotropy. These are:

$$\begin{aligned} V_{P0} &= \sqrt{\frac{C_{33}}{\rho}}, && \text{(D.5)} \\ V_{S0} &= \sqrt{\frac{C_{44}}{\rho}}, && \text{(D.6)} \\ \epsilon &\equiv \frac{C_{11} - C_{33}}{2C_{33}}, && \text{(D.7)} \\ \gamma &\equiv \frac{C_{66} - C_{44}}{2C_{44}}, && \text{(D.8)} \end{aligned}$$

and

$$\delta^* \equiv \frac{1}{2C_{33}^2}\left(2\left(C_{13} + C_{44}\right)^2 - \left(C_{33} - C_{44}\right)\left(C_{11} + C_{33} - 2C_{44}\right)\right). \tag{D.9}$$

With these definitions eq. (D.1) - (D.3) can exactly be written as

$$\begin{aligned} V_P^2(\Theta) &= V_{P0}^2\left(1 + \epsilon\sin^2\Theta + D^*(\Theta)\right), && \text{(D.10)} \\ V_{SV}^2(\Theta) &= V_{S0}^2\left(1 + \frac{V_{P0}^2}{V_{S0}^2}\epsilon\sin^2\Theta - \frac{V_{P0}^2}{V_{S0}^2}D^*(\Theta)\right), && \text{(D.11)} \\ V_{SH}^2(\Theta) &= V_{S0}^2\left(1 + 2\gamma\sin^2\Theta\right), && \text{(D.12)} \end{aligned}$$

with

$$D^*(\Theta) \equiv \frac{1}{2}\left(1 - \frac{V_{P0}^2}{V_{S0}^2}\right)\left(\left(1 + \frac{4\delta^*}{\left(1 - \frac{V_{S0}^2}{V_{P0}^2}\right)^2}\sin^2\Theta\cos^2\Theta + \frac{4\left(1 - \frac{V_{S0}^2}{V_{P0}^2} + \epsilon\right)\epsilon}{\left(1 - \frac{V_{S0}^2}{V_{P0}^2}\right)^2}\sin^4\Theta\right)^{1/2} - 1\right). \tag{D.13}$$

It is important to note at this point, that the given equations are valid for an arbitrary anisotropy and not weak anisotropy only.

However, from many observations it is known that most rocks, especially sediments, show only a weak-to-moderate anisotropy below 20%. In this case, the anisotropic velocities of a rock can be understood as a linear combination of an isotropic background velocity (represented by V_{P0} and V_{S0}) and small anisotropic perturbations (represented by ϵ, γ, and δ^*). If these perturbations are small the constitutive equations can be approximated by a first order Taylor expansion in the small parameters at fixed Θ, i.e., retaining only linear terms for these parameters. Then D^* can be approximated by:

$$D^* \approx \frac{\delta^*}{1 - \frac{V_{S0}^2}{V_{P0}^2}}\sin^2\Theta\cos^2\Theta + \epsilon\sin^4\Theta. \tag{D.14}$$

Substituting eq. (D.14) into eq. (D.10) and (D.11), and further linearization finally gives a set of equations for the three phase velocities valid in weak anisotropic media:

$$V_P(\Theta) = V_{P0}\left(1 + \delta\sin^2\Theta\cos^2\Theta + \epsilon\sin^4\Theta\right), \tag{D.15}$$

$$V_{SV}(\Theta) = V_{S0}\left(1 + \frac{V_{P0}^2}{V_{S0}^2}(\epsilon - \delta)\sin^2\cos^2\right), \tag{D.16}$$

$$V_{SH}(\Theta) = V_{S0}\left(1 + \gamma\sin^2\Theta\right),. \tag{D.17}$$

We further obtain:

$$\delta \equiv \frac{1}{2}\left(\epsilon + \frac{\delta^*}{1 - \frac{V_{S0}^2}{V_{P0}^2}}\right) = \frac{(C_{13} + C_{44})^2 - (C_{33} - C_{44})^2}{2C_{33}(C_{33} - C_{44})}. \tag{D.18}$$

Appendix E

Orthorhombic anisotropy parameters in terms of compliances

Tsvankins anisotropy parameters for orthorhombic media (Tsvankin, 1997) as given in terms of stiffnesses in section 2.1.4 can be expressed as function of the corresponding compliances. Then they read (Shapiro & Kaselow, 2003):

$$\begin{aligned}
\epsilon^{(1)} &= \frac{a_0 - b_0}{2b_0}, &\text{(E.1)}\\
\epsilon^{(2)} &= \frac{c_0 - b_0}{2b_0}, &\text{(E.2)}\\
\delta^{(1)} &= \frac{a_1^2 - 2a_1 b_1}{2b_1}, &\text{(E.3)}\\
\delta^{(2)} &= \frac{a_2^2 - 2a_2 b_2}{2b_2}, &\text{(E.4)}\\
\delta^{(3)} &= \frac{a_3^2 - 2a_3 b_3}{2b_3}, &\text{(E.5)}\\
\gamma^{(1)} &= \frac{S_{55} - S_{66}}{2S_{66}}, &\text{(E.6)}\\
\gamma^{(2)} &= \frac{S_{44} - S_{66}}{2S_{66}}, &\text{(E.7)}\\
& &\text{(E.8)}
\end{aligned}$$

with

$$\begin{aligned}
a_0 &= S_{11}S_{33} - S_{13}^2, a_1 = 1 + \frac{c_1}{b_0}, a_2 = 1 + \frac{c_2}{b_0}, a_1 = 1 + \frac{c_3}{b_0}, &\text{(E.9)}\\
b_0 &= S_{11}S_{22} - S_{12}^2, b_1 = 1 - \frac{S_{33}b_0 + S_{23}c_1 + S_{13}c_2}{b_0 S_{44}}, &\\
b_2 &= 1 - \frac{S_{33}b_0 + S_{23}c_1 + S_{13}c_2}{b_0 S_{55}}, b_3 = 1 - \frac{S_{11}c_0 + S_{12}c_3 + S_{13}c_2}{c_0 S_{66}}, &\text{(E.10)}\\
c_0 &= S_{33}S_{22} - S_{23}^2, c_1 = S_{12}S_{13} - S_{11}S_{23}, &\\
c_2 &= S_{12}S_{23} - S_{13}S_{22}, c_3 = S_{13}S_{23} - S_{12}S_{33}. &\text{(E.11)}
\end{aligned}$$

Appendix F

Best fit results for sandstones.

Table F.1: Best fit parameter from Eberhart-Phillips *et al.* (1989). Porosity ϕ is dimensionless, indices P and S indicate P- and S-wave. A and B are given in km/s, K is in km/s/MPa and D is in 1/MPa.

ϕ	AP	KP	BP	DP	AS	KS	BS	DS
0.064	5.470	0.002	0.503	0.090	3.440	0.004	0.399	0.110
0.227	3.910	0.003	0.622	0.220	2.310	0.002	0.537	0.190
0.222	3.820	0.003	0.441	0.180	2.320	0.002	0.353	0.120
0.195	4.010	0.002	0.451	0.100	2.440	0.003	0.443	0.130
0.259	3.650	0.002	0.337	0.230	2.040	0.001	0.139	0.150
0.111	4.680	0.001	0.187	0.090	2.900	0.002	0.141	0.130
0.236	3.800	0.003	0.716	0.240	2.280	0.002	0.616	0.170
0.111	4.530	0.005	0.397	0.160	2.690	0.005	0.407	0.180
0.046	5.170	0.001	0.419	0.080	3.060	0.003	0.270	0.150
0.160	4.450	0.004	0.257	0.280	2.730	0.002	0.278	0.180
0.156	4.700	0.003	0.532	0.350	3.000	0.002	0.518	0.300
0.200	4.360	0.003	0.664	0.220	2.780	0.002	0.571	0.190
0.144	3.580	0.004	0.492	0.260	1.850	0.003	0.365	0.270
0.143	3.500	0.003	0.301	0.140	1.920	0.002	0.224	0.140
0.132	3.460	0.005	0.425	0.190	1.930	0.002	0.399	0.160
0.312	3.120	0.002	0.558	0.170	1.650	0.003	0.389	0.180
0.305	3.110	0.002	0.350	0.140	1.730	0.001	0.333	0.130
0.111	3.860	0.003	0.429	0.160	2.140	0.001	0.390	0.130
0.158	3.720	0.007	0.608	0.220	1.960	0.005	0.625	0.210
0.162	3.740	0.006	0.631	0.130	1.990	0.003	0.556	0.130
0.256	3.200	0.004	0.732	0.190	1.800	0.003	0.643	0.130
0.264	3.380	0.004	0.641	0.140	1.870	0.002	0.515	0.110
0.155	3.640	0.003	0.502	0.150	2.000	0.003	0.418	0.120
0.123	3.690	0.004	0.359	0.240	2.050	0.002	0.374	0.190
0.159	3.690	0.004	0.466	0.190	2.000	0.004	0.437	0.200
0.272	3.550	0.003	0.610	0.160	2.090	0.003	0.398	0.140
0.276	3.450	0.004	0.713	0.180	1.980	0.003	0.774	0.170
0.294	3.410	0.004	0.807	0.190	1.940	0.003	0.674	0.170
0.283	3.410	0.004	0.795	0.150	1.960	0.003	0.632	0.130
0.275	3.320	0.005	0.716	0.130	1.810	0.004	0.804	0.210
0.213	3.600	0.007	0.747	0.180	2.070	0.004	0.605	0.120
0.127	4.000	0.006	0.682	0.170	2.380	0.003	0.409	0.060
0.162	4.370	0.006	0.776	0.120	2.410	0.008	0.681	0.180
0.117	4.170	0.006	0.568	0.140	2.370	0.006	0.485	0.120
0.069	4.420	0.004	0.675	0.140	2.570	0.004	1.270	0.280

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ϕ	AP	KP	BP	DP	AS	KS	BS	DS
0.161	3.890	0.005	0.452	0.140	2.220	0.004	0.451	0.160
0.266	3.200	0.004	0.308	0.120	1.920	0.002	0.305	0.100
0.261	3.370	0.005	0.323	0.200	1.910	0.004	0.301	0.250
0.240	3.700	0.000	0.366	0.070	2.120	0.003	0.294	0.160
0.245	3.480	0.005	0.380	0.210	2.040	0.003	0.390	0.200
0.243	3.710	0.005	0.507	0.240	2.260	0.003	0.320	0.190
0.184	4.210	0.003	0.378	0.150	2.540	0.002	0.369	0.130
0.184	4.160	0.004	0.343	0.160	2.470	0.002	0.345	0.170
0.212	3.940	0.002	0.646	0.120	2.250	0.004	0.522	0.170
0.097	4.760	0.003	0.239	0.130	2.930	0.003	0.308	0.150
0.096	4.590	0.003	0.364	0.130	2.770	0.004	0.322	0.200
0.073	4.300	0.002	0.375	0.150	2.550	0.002	0.386	0.210
0.080	4.170	0.002	0.271	0.190	2.380	0.003	0.377	0.310
0.121	3.950	0.004	0.464	0.240	2.230	0.003	0.424	0.270
0.098	4.210	0.001	0.323	0.080	2.440	0.002	0.296	0.130
0.103	4.060	0.003	0.299	0.140	2.310	0.003	0.330	0.210
0.077	4.220	0.003	0.387	0.130	2.450	0.003	0.376	0.150
0.147	4.440	0.002	0.742	0.070	2.550	0.003	0.575	0.110
0.170	4.130	0.005	0.380	0.210	2.370	0.004	0.548	0.300
0.162	4.030	0.006	0.472	0.150	2.360	0.004	0.464	0.120
0.180	4.070	0.004	0.572	0.130	2.310	0.003	0.509	0.110
0.177	4.000	0.005	0.478	0.130	2.260	0.004	0.515	0.170
0.167	3.660	0.004	0.338	0.190	2.010	0.002	0.357	0.190
0.131	3.920	0.004	0.539	0.140	2.110	0.003	0.484	0.170
0.205	4.210	0.002	0.746	0.240	2.580	0.002	0.781	0.200
0.187	4.550	0.003	0.800	0.190	2.830	0.002	0.741	0.160
0.136	4.170	0.001	0.467	0.060	2.220	0.004	0.255	0.130
0.194	4.030	0.004	0.669	0.190	2.390	0.003	0.535	0.150
0.059	4.860	0.002	0.109	0.140	3.050	0.002	0.123	0.170

Table F.2: Best fit parameter from repeated fit of Eberhart-Phillips *et al.* (1989) data as given in Tab. (F.1). Porosity ϕ is dimensionless, indices P and S indicate P- and S-wave. A and B are given in km/s, K is in km/s/MPa and D is in 1/MPa.

ϕ	AP	KP	BP	DP	AS	KS	BS	DS
0.064	5.459	0.002	0.500	0.100	3.447	0.004	0.400	0.100
0.227	3.966	0.003	0.594	0.205	2.354	0.002	0.520	0.205
0.222	3.854	0.003	0.430	0.150	2.331	0.002	0.353	0.150
0.195	4.010	0.002	0.451	0.115	2.459	0.002	0.442	0.115
0.259	3.681	0.002	0.320	0.190	2.048	0.001	0.137	0.190
0.111	4.676	0.001	0.186	0.110	2.906	0.002	0.141	0.110
0.236	3.868	0.003	0.677	0.205	2.325	0.002	0.603	0.205
0.111	4.557	0.005	0.391	0.170	2.721	0.004	0.396	0.170
0.046	5.149	0.002	0.412	0.115	3.076	0.003	0.267	0.115
0.160	4.476	0.004	0.239	0.230	2.751	0.002	0.271	0.230
0.156	4.757	0.002	0.485	0.325	3.053	0.002	0.479	0.325
0.200	4.420	0.002	0.634	0.205	2.826	0.001	0.553	0.205
0.144	3.628	0.003	0.462	0.265	1.886	0.003	0.341	0.265
0.143	3.516	0.003	0.299	0.140	1.932	0.002	0.223	0.140
0.132	3.494	0.005	0.412	0.175	1.957	0.002	0.393	0.175
0.312	3.160	0.002	0.546	0.175	1.680	0.002	0.379	0.175
0.305	3.128	0.002	0.348	0.135	1.744	0.001	0.332	0.135
0.111	3.889	0.003	0.422	0.145	2.157	0.001	0.389	0.145
0.158	3.775	0.007	0.580	0.215	2.015	0.004	0.600	0.215
0.162	3.767	0.006	0.629	0.130	2.014	0.003	0.555	0.130
0.256	3.259	0.003	0.709	0.160	1.828	0.002	0.641	0.160
0.264	3.414	0.004	0.637	0.125	1.879	0.002	0.516	0.125
0.155	3.670	0.003	0.497	0.135	2.013	0.003	0.418	0.135
0.123	3.724	0.003	0.340	0.215	2.080	0.002	0.362	0.215
0.159	3.728	0.003	0.452	0.195	2.037	0.003	0.421	0.195
0.272	3.591	0.003	0.600	0.150	2.111	0.003	0.395	0.150
0.276	3.505	0.004	0.695	0.175	2.036	0.002	0.758	0.175
0.294	3.475	0.003	0.782	0.180	1.989	0.002	0.660	0.180
0.283	3.458	0.004	0.786	0.140	1.987	0.003	0.630	0.140
0.275	3.351	0.004	0.714	0.170	1.880	0.004	0.771	0.170
0.213	3.658	0.006	0.728	0.150	2.089	0.004	0.605	0.150
0.127	4.049	0.006	0.668	0.115	2.324	0.004	0.382	0.115
0.162	4.394	0.006	0.776	0.150	2.462	0.007	0.663	0.150
0.117	4.200	0.006	0.564	0.130	2.385	0.006	0.485	0.130
0.069	4.455	0.004	0.671	0.210	2.698	0.003	1.182	0.210
0.161	3.914	0.004	0.449	0.150	2.250	0.003	0.444	0.150
0.266	3.210	0.003	0.308	0.110	1.920	0.002	0.305	0.110
0.261	3.397	0.004	0.311	0.225	1.939	0.003	0.284	0.225
0.240	3.668	0.001	0.353	0.115	2.140	0.002	0.289	0.115
0.245	3.513	0.005	0.365	0.205	2.073	0.003	0.376	0.205

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ϕ	AP	KP	BP	DP	AS	KS	BS	DS
0.243	3.758	0.005	0.480	0.215	2.286	0.003	0.310	0.215
0.184	4.233	0.003	0.374	0.140	2.556	0.002	0.368	0.140
0.184	4.183	0.003	0.338	0.165	2.495	0.002	0.338	0.165
0.212	3.960	0.002	0.646	0.145	2.288	0.003	0.511	0.145
0.097	4.770	0.003	0.238	0.140	2.949	0.002	0.305	0.140
0.096	4.606	0.003	0.363	0.165	2.797	0.004	0.310	0.165
0.073	4.323	0.002	0.371	0.180	2.584	0.002	0.370	0.180
0.080	4.192	0.002	0.263	0.250	2.419	0.002	0.347	0.250
0.121	3.994	0.003	0.439	0.255	2.272	0.003	0.396	0.255
0.098	4.194	0.002	0.318	0.105	2.453	0.002	0.295	0.105
0.103	4.076	0.003	0.297	0.175	2.339	0.003	0.317	0.175
0.077	4.237	0.003	0.386	0.140	2.473	0.003	0.372	0.140
0.147	4.375	0.003	0.716	0.090	2.560	0.003	0.576	0.090
0.170	4.163	0.005	0.365	0.255	2.426	0.004	0.507	0.255
0.162	4.058	0.006	0.467	0.135	2.375	0.004	0.464	0.135
0.180	4.095	0.004	0.571	0.120	2.319	0.003	0.510	0.120
0.177	4.021	0.005	0.477	0.150	2.297	0.004	0.504	0.150
0.167	3.687	0.004	0.328	0.190	2.039	0.001	0.346	0.190
0.131	3.948	0.003	0.536	0.155	2.145	0.003	0.474	0.155
0.205	4.281	0.001	0.706	0.220	2.646	0.001	0.753	0.220
0.187	4.615	0.002	0.775	0.175	2.879	0.002	0.729	0.175
0.136	4.106	0.002	0.436	0.095	2.231	0.004	0.254	0.095
0.194	4.084	0.004	0.648	0.170	2.422	0.003	0.529	0.170
0.059	4.866	0.002	0.108	0.155	3.059	0.002	0.120	0.155

Table F.3: Best fit parameter for P and S-wave velocities given by Jones (1995). Parameter A is given in [m/s], K in [m/s/MPa], B in [m/s], and D in [1/MPa].

Sample	AP	KP	BP	DP	AS	KS	BS	DS
8VBP(20)	4350.1	2.299	624.539	0.089	2933.0	0.416	537.449	0.065
1DYK(101)	4131.4	3.503	810.914	0.160	2589.1	1.470	477.110	0.080
2MYK(77)	3900.3	3.330	563.386	0.112	2272.1	3.129	830.622	0.186
B79	3644.2	2.592	868.560	0.141	2194.8	1.932	866.219	0.156
5SU(452)	4386.6	2.862	522.414	0.095	2683.0	2.703	574.732	0.112
FD(444)	4557.8	3.283	531.470	0.179	2591.0	2.044	480.020	0.106
HW(448)	5160.9	0.004	323.722	0.063	3035.6	0.000	338.094	0.060
RS(443)	4273.3	3.113	734.308	0.133	2552.8	1.444	458.371	0.095
TS(451)	4395.8	3.121	333.616	0.177	2665.8	2.389	459.456	0.132
1SU(453)	4325.2	3.583	831.067	0.085	2740.1	0.975	841.897	0.059
4SU(447)	4410.1	0.906	836.454	0.062	2734.3	0.000	779.174	0.052
E5(8)	4716.6	1.200	292.243	0.181	3050.0	0.720	317.893	0.191
8HBP(21)	4182.1	2.830	700.572	0.113	2709.3	0.000	443.190	0.047
1VSF(225)	3886.9	2.934	653.575	0.107	2370.7	1.225	476.317	0.062
E1(410)	5020.2	1.055	176.818	0.153	3240.1	0.672	164.918	0.174
E1(412)	4970.7	1.198	278.439	0.178	3221.6	0.665	176.181	0.131

Table F.4: Refitted best fit parameter for P and S-wave velocities for Jones (1995) data set. Porosity ϕ is dimensionless, density ρ is given in [kg/m^3], parameter A in [km/s], K in [km/s/MPa], B in [km/s], and D in [1/MPa].

Sample	ϕ	ρ	AP	KP	BP	DP	AS	KS	BS	DS
1SU(453)	0.117	2656	4.420	0.002	0.912	0.072	2.618	0.003	0.731	0.072
4SU(447)	0.091	2632	4.471	0.000	0.892	0.057	2.673	0.001	0.721	0.057
E5(8)	0.135	2640	4.714	0.001	0.291	0.186	3.052	0.001	0.319	0.186
8HBP(21)	0.159	2668	4.322	0.000	0.808	0.080	2.549	0.002	0.295	0.080
1VSF(225)	0.161	2661	3.974	0.001	0.721	0.085	2.274	0.003	0.391	0.085
E1(410)	0.105	2635	5.016	0.001	0.175	0.164	3.243	0.001	0.166	0.164
E1(412)	0.107	2638	4.983	0.001	0.284	0.154	3.211	0.001	0.169	0.154
8VBP(20)	0.136	2680	4.409	0.001	0.673	0.077	2.670	0.001	0.482	0.077
1DYK(101)	0.133	2662	4.218	0.002	0.860	0.120	2.492	0.003	0.399	0.120
2MYK(77)	0.159	2722	3.837	0.005	0.520	0.149	2.328	0.002	0.856	0.149
B79	0.238	2648	3.628	0.003	0.859	0.148	2.209	0.002	0.874	0.148
5SU(452)	0.104	2652	4.363	0.003	0.504	0.103	2.705	0.002	0.591	0.103
FD(444)	0.099	2688	4.596	0.002	0.550	0.142	2.532	0.003	0.438	0.142
HW(448)	0.033	2680	5.167	0.000	0.329	0.061	3.029	0.000	0.332	0.061
RS(443)	0.158	2736	4.321	0.002	0.766	0.114	2.512	0.002	0.427	0.114
TS(451)	0.090	2684	4.410	0.003	0.340	0.154	2.640	0.003	0.443	0.154

Table F.5: Best fit parameters for data given by Freund (1992) obtained from second fit. Parameter A is given in [km/s], K in [km/s/MPa], B in [km/s], and D in [1/MPa]. χ^2 is dimensionless. P and S indicate best fit parameters for P-wave and S-wave fit.

Sample	AP	KP	BP	DP	χ^2	AP	KP	BP	DP	χ^2
8	5.017	0.000	0.608	0.023	0.004	3.286	0.000	0.267	0.023	0.000
10	4.644	0.000	0.598	0.023	0.007	3.087	0.000	0.263	0.023	0.000
17	4.796	0.000	0.715	0.018	0.002	2.671	0.000	0.232	0.018	0.000
21	4.567	0.000	0.590	0.016	0.003	3.006	0.000	0.263	0.016	0.002
22	5.193	0.000	1.142	0.035	0.018	3.205	0.000	0.348	0.035	0.000
24	5.348	0.000	1.096	0.016	0.007	3.390	0.000	0.409	0.016	0.000
25	5.068	0.000	0.771	0.026	0.003	3.390	0.000	0.390	0.026	0.001
27	4.839	0.000	0.560	0.014	0.006	3.083	0.000	0.172	0.014	0.000
34	4.362	0.000	0.828	0.016	0.012	2.604	0.000	0.236	0.016	0.001
38	4.987	0.000	0.674	0.018	0.001	3.305	0.000	0.415	0.018	0.001
41	4.685	0.000	0.505	0.007	0.001	2.970	0.000	0.174	0.007	0.000
43	5.006	0.000	0.580	0.021	0.001	3.038	0.000	0.253	0.021	0.000
44	4.757	0.000	0.417	0.026	0.008	3.138	0.000	0.214	0.026	0.001
48	5.011	0.001	0.597	0.016	0.007	3.468	0.000	0.399	0.016	0.002
51	4.615	0.000	0.443	0.017	0.001	3.110	0.000	0.268	0.017	0.000
52	4.631	0.000	0.621	0.019	0.001	3.108	0.000	0.377	0.019	0.000
53	4.705	0.000	0.273	0.010	0.001	3.039	0.000	0.144	0.010	0.000
55	4.631	0.000	0.472	0.017	0.009	3.082	0.000	0.242	0.017	0.002
56	5.300	0.001	1.559	0.026	0.023	3.390	0.000	0.666	0.026	0.004
58	4.790	0.000	0.690	0.022	0.012	3.036	0.000	0.227	0.022	0.001
60	4.426	0.000	0.730	0.028	0.010	2.740	0.000	0.351	0.028	0.003
62	4.835	0.000	0.456	0.009	0.002	3.197	0.000	0.380	0.009	0.001
65	4.722	0.000	0.665	0.028	0.021	3.120	0.000	0.310	0.028	0.006
73	4.037	0.000	0.683	0.022	0.023	2.570	0.000	0.241	0.022	0.001
74	4.221	0.000	0.384	0.019	0.001	2.778	0.000	0.128	0.019	0.001
78	4.575	0.000	0.605	0.028	0.007	3.091	0.000	0.467	0.028	0.002
81	4.555	0.000	0.709	0.028	0.014	2.918	0.000	0.256	0.028	0.002
90	4.064	0.000	0.647	0.025	0.007	2.522	0.000	0.159	0.025	0.000
91	5.122	0.000	1.748	0.023	0.032	3.283	0.000	0.831	0.023	0.004
95	5.403	0.000	0.944	0.018	0.006	3.104	0.000	0.313	0.018	0.007
102	5.333	0.000	1.670	0.022	0.020	3.239	0.000	0.615	0.022	0.002
107	5.159	0.001	1.905	0.032	0.000	3.358	0.000	0.887	0.032	0.008
108	5.174	0.001	2.317	0.023	0.017	3.548	0.000	1.298	0.023	0.003
116	5.212	0.000	1.789	0.025	0.021	3.190	0.000	0.840	0.025	0.002
120	5.265	0.001	1.738	0.021	0.003	3.534	0.000	0.924	0.021	0.001
123	5.365	0.000	0.724	0.012	0.004	3.453	0.000	0.379	0.012	0.001
126	4.142	0.000	0.520	0.016	0.012	2.472	0.000	0.224	0.016	0.000
131	4.935	0.001	2.345	0.036	0.002	3.334	0.000	1.321	0.036	0.006
134	4.730	0.001	1.535	0.036	0.009	3.022	0.001	0.654	0.036	0.001
135	4.382	0.000	1.567	0.022	0.006	3.043	0.000	0.981	0.022	0.003

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Sample	AP	KP	BP	DP	χ^2	AP	KP	BP	DP	χ^2
139	5.195	0.000	2.977	0.029	0.024	3.244	0.000	1.433	0.029	0.005
140	4.402	0.000	0.927	0.021	0.003	2.907	0.000	0.426	0.021	0.001
146	4.671	0.000	1.007	0.017	0.008	2.840	0.000	0.484	0.017	0.000
159	4.865	0.000	0.648	0.018	0.005	3.136	0.000	0.287	0.018	0.000
165	4.778	0.001	0.430	0.040	0.004	3.067	0.000	0.231	0.040	0.001
172	4.035	0.000	0.720	0.023	0.015	2.452	0.000	0.281	0.023	0.000
191	4.966	0.001	1.474	0.046	0.018	3.262	0.000	0.751	0.046	0.011
196	3.949	0.000	0.592	0.009	0.009	2.417	0.000	0.138	0.009	0.001
202	5.091	0.001	2.086	0.032	0.017	3.210	0.000	0.610	0.032	0.003
205	4.572	0.000	0.458	0.029	0.002	3.013	0.000	0.195	0.029	0.000
206	3.839	0.000	0.532	0.017	0.001	2.585	0.000	0.260	0.017	0.001
213	4.673	0.000	2.328	0.052	0.008	3.106	0.000	1.148	0.052	0.003
216	4.667	0.001	1.970	0.030	0.013	3.164	0.001	1.145	0.030	0.004
218	4.955	0.000	0.913	0.036	0.014	3.345	0.000	0.458	0.036	0.003
219	4.745	0.000	2.134	0.041	0.013	3.233	0.000	1.211	0.041	0.006
221	4.910	0.000	1.625	0.037	0.008	3.309	0.000	0.877	0.037	0.005
222	5.082	0.000	3.307	0.064	0.027	2.871	0.000	0.916	0.064	0.002
223	4.304	0.001	0.920	0.042	0.020	2.586	0.001	0.284	0.042	0.000
230	5.084	0.001	1.816	0.029	0.002	3.518	0.000	1.046	0.029	0.004
235	5.671	0.000	2.120	0.030	0.055	3.240	0.001	0.632	0.030	0.003
240	4.793	0.000	1.545	0.035	0.033	2.822	0.000	0.373	0.035	0.001
246	4.933	0.000	0.612	0.024	0.011	2.895	0.000	0.132	0.024	0.000
253	5.422	0.000	2.403	0.035	0.024	3.172	0.000	0.770	0.035	0.003
254	5.436	0.000	2.093	0.034	0.050	3.321	0.000	0.680	0.034	0.001
256	4.686	0.000	2.083	0.040	0.012	3.121	0.000	1.034	0.040	0.007
258	4.442	0.000	2.078	0.052	0.005	2.924	0.000	1.051	0.052	0.001
261	5.283	0.000	2.231	0.028	0.060	3.236	0.001	0.912	0.028	0.003
265	4.879	0.001	1.537	0.067	0.014	2.952	0.000	0.381	0.067	0.004
266	5.047	0.000	2.110	0.032	0.008	3.040	0.000	0.794	0.032	0.001
267	4.598	0.001	1.225	0.041	0.021	3.023	0.000	0.355	0.041	0.001
272	5.271	0.001	3.176	0.061	0.037	3.243	0.001	1.268	0.061	0.003
281	5.375	0.000	2.602	0.036	0.083	3.324	0.001	1.271	0.036	0.024
287	4.478	0.000	1.230	0.033	0.019	2.720	0.000	0.363	0.033	0.003
288	4.293	0.001	1.834	0.027	0.002	2.988	0.000	1.099	0.027	0.004
294	4.338	0.001	0.945	0.033	0.002	2.893	0.000	0.434	0.033	0.002
296	4.678	0.000	1.333	0.019	0.025	2.492	0.001	0.122	0.019	0.000
299	5.035	0.000	2.729	0.053	0.072	2.933	0.001	1.006	0.053	0.005
302	4.606	0.001	1.690	0.031	0.011	3.180	0.000	0.867	0.031	0.010
303	4.981	0.001	2.195	0.048	0.003	3.153	0.000	0.982	0.048	0.001
304	5.118	0.000	2.248	0.038	0.008	3.383	0.000	0.932	0.038	0.005
305	4.382	0.001	1.470	0.038	0.022	2.959	0.000	1.045	0.038	0.003
306	4.690	0.000	1.622	0.026	0.014	3.018	0.000	0.683	0.026	0.001
307	4.203	0.001	1.565	0.031	0.008	2.670	0.001	0.505	0.031	0.005
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Sample	AP	KP	BP	DP	χ^2	AP	KP	BP	DP	χ^2
308	4.847	0.000	1.212	0.021	0.004	3.281	0.000	0.838	0.021	0.002
309	4.854	0.000	2.265	0.039	0.040	3.350	0.000	1.312	0.039	0.014
314	5.649	0.000	2.475	0.027	0.014	3.367	0.000	1.016	0.027	0.001
320	5.519	0.000	2.262	0.037	0.029	3.478	0.000	0.995	0.037	0.001

Table F.6: Stress sensitivity parameters inverted from velocity best fit parameters. Velocity data from Freund (1992).

Sample	Porosity [—]	Density [kg/m^3]	K_{dryS} [GPa]	μ_{dryS} [GPa]	θ_c [—]	$\theta_{c\mu}$ [—]	ϕ_{c0} [—]
8	0.037	2620	28.232	28.291	657.054	306.059	0.001
10	0.051	2590	22.937	24.688	518.429	230.881	0.001
25	0.018	2660	27.550	30.575	727.456	403.516	0.001
38	0.026	2620	27.010	28.612	486.822	411.076	0.001
44	0.017	2720	25.836	26.781	661.880	394.445	0.000
48	0.022	2650	24.055	31.868	377.012	342.729	0.001
51	0.037	2630	22.101	25.431	381.265	295.863	0.001
52	0.050	2600	22.269	25.120	423.730	335.502	0.001
56	0.050	2640	33.717	30.332	887.745	423.865	0.001
58	0.022	2670	28.461	24.607	620.814	207.790	0.001
60	0.050	2640	25.282	19.824	709.376	446.156	0.001
65	0.044	2620	24.422	25.497	690.441	345.846	0.001
74	0.036	2640	19.868	20.369	381.467	115.189	0.001
78	0.026	2670	21.877	25.504	621.770	913.125	0.000
81	0.033	2660	24.977	22.653	702.173	259.875	0.001
90	0.066	2560	20.561	16.286	507.097	122.627	0.001
91	0.069	2560	30.378	27.590	710.966	401.630	0.001
102	0.037	2610	37.704	27.387	822.748	360.926	0.001
108	0.026	2640	26.346	33.237	594.059	371.033	0.002
116	0.037	2640	35.892	26.864	884.960	551.052	0.001
120	0.036	2660	29.458	33.214	612.250	369.637	0.001
134	0.090	2440	24.887	22.285	895.376	426.717	0.001
165	0.043	2620	26.946	24.643	1065.802	744.676	0.000
202	0.026	2630	32.018	27.107	1021.572	294.819	0.001
205	0.026	2670	23.484	24.239	685.357	296.968	0.000
216	0.067	2510	21.179	25.125	635.334	444.239	0.002
218	0.067	2510	24.166	28.088	875.181	465.280	0.001
219	0.078	2480	21.275	25.927	863.338	565.824	0.001
221	0.073	2530	24.056	27.697	879.744	539.742	0.001
240	0.069	2500	30.871	19.913	1074.494	292.082	0.001
253	0.026	2590	41.408	26.053	1453.165	577.676	0.001
254	0.030	2610	38.741	28.789	1306.338	474.410	0.001
256	0.089	2470	22.164	24.052	893.994	486.949	0.001
266	0.064	2500	32.874	23.101	1059.993	490.119	0.001
267	0.106	2350	21.056	21.470	862.103	215.656	0.001
281	0.048	2560	36.255	28.282	1311.063	849.622	0.001
287	0.123	2350	23.928	17.392	793.062	257.087	0.001
288	0.112	2370	15.467	21.161	419.381	288.318	0.003
294	0.149	2280	17.470	19.077	581.111	275.813	0.001
302	0.097	2410	18.629	24.371	579.337	297.475	0.002

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Sample	Porosity	Density	K_{dryS}	μ_{dryS}	θ_c	$\theta_{c\mu}$	ϕ_{c0}
304	0.091	2410	26.342	27.582	1011.514	417.009	0.001
305	0.130	2350	17.694	20.577	664.280	760.748	0.001
307	0.127	2330	19.014	16.604	586.644	189.746	0.002
309	0.086	2450	21.076	27.494	823.188	540.679	0.001
320	0.041	2570	36.849	31.084	1369.537	714.220	0.001
131	0.041	2570	24.495	28.569	879.669	582.687	0.001
107	0.026	2660	30.810	29.998	997.361	521.117	0.001
123	0.022	2680	34.539	31.960	420.790	278.417	0.001
126	0.011	2780	25.040	16.990	398.955	229.628	0.001
135	0.116	2370	16.240	21.946	349.670	267.524	0.002
139	0.047	2610	33.822	27.462	983.477	607.393	0.001
140	0.143	2340	18.966	19.776	399.742	195.279	0.002
146	0.086	2460	27.221	19.846	472.006	309.500	0.001
159	0.044	2610	27.539	25.667	489.816	242.354	0.001
191	0.047	2560	26.831	27.232	1235.545	735.931	0.001
196	0.032	2720	21.229	15.886	200.447	47.348	0.002
206	0.043	2680	15.606	17.915	262.068	133.994	0.002
213	0.078	2480	22.253	23.924	1151.787	624.081	0.001
21	0.036	2660	23.436	24.034	374.084	175.226	0.001
222	0.063	2470	36.653	20.359	2331.910	829.996	0.001
22	0.033	2630	34.906	27.011	1233.971	399.610	0.001
230	0.036	2690	25.126	33.296	717.818	460.236	0.001
235	0.022	2680	48.680	28.137	1472.862	561.772	0.001
246	0.036	2650	34.862	22.217	847.807	201.767	0.000
24	0.033	2650	35.182	30.461	557.101	222.427	0.001
258	0.107	2440	20.329	20.867	1051.065	613.222	0.001
261	0.038	2560	35.712	26.809	1011.464	506.695	0.001
265	0.073	2460	29.977	21.431	2022.637	529.567	0.000
272	0.035	2540	34.953	26.710	2119.854	1013.172	0.001
172	0.010	2770	22.906	16.651	525.434	250.720	0.001
223	0.112	2420	23.251	16.188	983.906	348.865	0.001
27	0.018	2680	28.784	25.475	390.508	116.704	0.001
296	0.110	2350	31.974	14.594	604.917	69.301	0.001
299	0.075	2480	34.423	21.332	1836.867	892.491	0.001
303	0.079	2450	28.310	24.356	1372.822	726.543	0.001
306	0.092	2430	23.943	22.128	627.752	288.243	0.002
308	0.114	2360	21.577	25.403	457.892	483.842	0.001
314	0.014	2690	45.181	30.504	1210.071	651.233	0.001
34	0.043	2650	26.448	17.973	419.076	135.950	0.001
41	0.036	2670	27.210	23.555	191.747	68.545	0.002
53	0.043	2630	25.823	24.290	267.420	178.467	0.001
55	0.044	2630	23.092	24.989	388.553	225.602	0.001
62	0.048	2600	25.356	26.572	238.421	471.070	0.001
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Sample	Porosity	Density	K_{dryS}	μ_{dryS}	θ_c	$\theta_{c\mu}$	ϕ_{c0}
73	0.036	2660	19.936	17.569	437.368	159.168	0.001
95	0.011	2680	43.790	25.824	800.530	346.088	0.001
43	0.018	2680	34.196	24.730	704.296	397.625	0.000

Appendix G

Common conversion factors in rock physics

The subject of pressure and stress and their spatio-temporal evolution is important for geologists, geopysicists, and engineers. As a result of the wide spread 'pressure community', many different units used for quantification of physical units can be found in the literature. The listed conversion factors should help to deal with this diversivity. Conversion factors taken from Mavko *et al.* (1998) are marked with A and those taken from Messinger & Langenscheidt-Redaktion (2001) with B. If a mark is missing for a certain factor, the factor belongs to the last mark given above within the reference column.

Table G.1: Conversion factors.

Weights			
1 g	(gramm)	0.001 kg	A
1 kg	(kilogramm)	2.204623 lb	
1 lb	(pound)	0.4535924 kg	
		16 oz (avior.)	
1 ton (USA)		2000 lb	
		907.2 kg	
1 ton (imperial)		2240 lb	
		1016 kg	
1 ton (metric)		1000 kg	
		2.204.662 lb	
1 oz (avior.)	(ounce)	28.3495 g	
1 oz (troy)	(ounce)	31.10348 g	
Length units.			
1 m	(meter)	100 cm	A
		0.001 km	
		39.37 in	
		3.2808399 ft	
1 cm	(centimeter)	0.01 m	
		10^{-5} km	
		0.3937 in	
		0.032808399 ft	
1 km	(kilometer)	1000 m	
		0.62137 mi	
1 in	(inch)	2.540005 cm	
1 ft	(foot)	30.48006 m	
		12 in	
1 mi	(mile)	1.60935 km	
1 mi (nautic)		1.852 km	
		1.15077 miles	
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Square measures.			
1 cm^2	(square centimeter)	0.0001 m^2	B
		0.154991 square inch	
		0.001076 square foot	
1 m^2	(square meter)	10^4 cm^2	
1 km^2	(square kilometer)	10^6 m^2	
1 square inch		6.452 cm^2	
1 square foot		144 square inches	
		929.029 cm^2	
1 square mile		2.59 km^2	
Cubic measures			
1 cm^3	(cubic centimeter)	$10^{-6}m^3$	
		0.0610238 in^3	
1 m^3	(cubic meter)	10^4 cm^2	
1 km^3	(cubic kilometer)	10^9 m^3	
1 l	(liter)	0.001 m^3	
		0.264172 gls	
		0.035315 ft^3	
1 in^3	(cubic inch)	16.387 cm^3	
1 ft^3	(cubic foot)	1728 in^3	
		0.02832 m^3	
1 gl (brit.)	(gallon)	4.5459 l	
1 gl (amer.)			
dry measure		4.405 l	
liquid measure		3.7853 l	
1 bl (brit.)	(barrel)	36 gls	
		163.6561 l	
1 bl (amer.)			
liquid measure		31.5 gls	
		119.228 l	
petroleum		42 gls	
		158.971 l	
Density measures			
1 g/cm^3		1000 kg/m^3	A
		0.036127 lb/in^3	
		62.42797 lb/ft^3	
1 lb/in^3		27.6799 g/cm^3	
1 lb/ft^3		0.016018 g/cm^3	
Forces			
1 N	Newton	1 kgm/s^2	A
1 dyne		10^{-5} N	
Pressure and stress			
1 Pa	(pascal)	1 N/m^2	A

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1 MPa	(mega pascal)	10^6 Pa	
		145.0378 psi	
		10 bars	
1 psi	(pounds per square inch)	0.006895 MPa	
		6894.6497 Pa	
1 bar		10^5 Pa	
1 kbar		100 MPa	
1 atm	(76 cm Hg)	1.01325 bars	
		$\approx$ 0.1 MPa	
Permeability			
1 Darcy		$9.86923x10^{-13}m^2$	A
		$1.06x10^{-11}ft^2$	
1 m^2		$1.01325x10^{12}$ Darcy	