Rapid Communications

Probing the energy reactance with adiabatically driven quantum dots

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The tunneling Hamiltonian describes a particle transfer from one region to another. Although there is no particle storage in the tunneling region itself, it has an associated amount of energy. The corresponding energy flux was named reactance since, such as an electrical reactance, it manifests itself in time-dependent transport only. We show here that the existence of the *energy reactance* leads to the universal response of a mesoscopic thermometer, a floating contact coupled to an adiabatically driven quantum dot.

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Introduction. A very exciting experimental activity is lately taking place in search of controlling on-demand quantum coherent charge transport in the time domain. The recent burst of activity started with the experimental realization of quantum capacitors in quantum dots under ac driving [1], single-particle emitters [2], and was followed by the generation of quantum charged solitons over the Fermi sea (levitons) [3]. A controlled manipulation of flying single electrons [4–6] and their time-resolved detection [7] has already been reported [8]. These marvelous developments, along with the identically impressive progress in the field of fast thermometry [9–11], are opening an avenue towards the study and control of the concomitant time-dependent energy flow in the quantum realm.

The relevant systems are characterized by small (nanoscale) components confining a small number of particles in contact with macroscopic reservoirs. This puts the description of the energy transport and heat generation beyond the scope of usual thermodynamical approaches, motivating a number of formal theoretical developments in statistical mechanics [12] and condensed-matter physics [13]. At the heart of this problem, there is the proper definition of the quantum heat current in the time domain. The concept of heat looks very intuitive, and anyone can provide a definition for it. Formally, it is a clear and well-established concept in macroscopic systems close to equilibrium. However, its accurate definition on the nanoscale and in situations away from equilibrium is a deep and subtle issue, in particular, due to the coupling between a nanosystem and macroscopic reservoirs; see, e.g., Refs. [14–23]. In fact, although charge and energy are concepts obeying strict fundamental conservation laws, the definition of heat implies the proper identification of a portion of the total energy.

An appealing scenario to address this problem from the theoretical point of view is a periodically driven single level in contact with an electron reservoir. This is the most basic and meaningful setup to analyze the interplay of charge and energy dynamics. At the same time, this is the simplest model for a quantum resistor-capacitor (RC) circuit [24], which has been experimentally realized [1]. A sketch is presented in Fig. 1 where we stress that the driven level represents a quantum dot. The nonequilibrium ingredient is provided by the time-dependent gate voltage $V(t) = V\cos(\Omega t)$ locally applied to the single level. The reservoir is an electron gas with temperature T and chemical potential μ, and the strength of the coupling between the two subsystems is arbitrary. The setup also includes a floating contact, which we will discuss in detail later on.

The effect of the periodic driving is twofold. On one hand, it induces a charge current that periodically flows between the dot and the reservoir. On the other hand, it performs

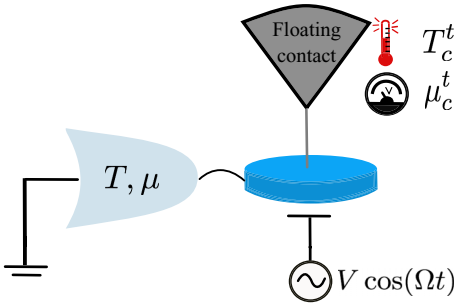

FIG. 1. Schematic of our proposal. The quantum RC circuit consists of a quantum dot (the dark blue disk in the middle) coupled to a fermionic reservoir (the light blue region in the left) with well-defined temperature T and chemical potential μ. Electrons can be transferred between the dot and the reservoir (the black curve). The dot is capacitively coupled to a gate terminal where an ac potential of amplitude V and frequency Ω is applied. A floating contact is also attached to the dot. For every time snapshot, the temperature T_c^t and chemical potential μ_c^t of the floating contact adjust themselves to cancel both the charge and the heat currents flowing through it. This instantaneous reaction allows for an experimental test of the energy reactance, namely, the variation of the stored energy at the tunneling region between the floating contact and the dot (the gray line).

work on the system, thus injecting energy that is ultimately dissipated as heat deep inside the reservoir. Importantly, due to charge conservation, the electronic current is defined as the change in time of the electron number either at the reservoir or on the dot. No contribution of the tunneling region on the charge current exists. In contrast, the energy delivered by the external ac source is temporarily stored in three different parts of the setup: the dot, the reservoir, and in the dot-reservoir tunneling region. The role of the tunneling region is typically disregarded in classical thermodynamics because it is a surface term that is negligible when both the system and the reservoir are macroscopic [25]. Yet, in the nanoscale setup studied here the amount of energy stored in the dot is comparable to that of the tunneling region and the latter can no longer be neglected.

In a recent work [15] we have coined the name of *energy reactance* to characterize the energy temporarily stored at the tunneling region. This is a thermal analog of an electrical reactance (due to electrical capacitance and inductance), which manifests itself in a time-dependent setup only. We have argued that it is physically meaningful to take the energy reactance into account as a contribution to the time-dependent heat current flowing into the reservoir. We have shown that this is in full agreement with the laws of thermodynamics [13,23]. Although some recent works raised some concerns [18,22], other works arrived at conclusions similar to our analysis [14,17,21]. The aim of the present Rapid Communication is to take one step further by proposing a measurement scheme that is able to test the effect of the energy reactance onto a time-dependent heat flux.

Proposed experiment and predictions. The setup is sketched in Fig. 1 where we introduce a floating contact attached (e.g., via tunneling) to the quantum dot [26]. When a periodic gate voltage $V(t)$ is applied, charge and heat currents enter not only the reservoir, but also the floating contact. We will focus on slow "adiabatic" driving, which corresponds to a driving period much larger than any characteristic time scale for the system. The floating contact is a small conductor which is assumed to have a charge and energy relaxation rate much faster than any other characteristic time so that it can adjust time by time its chemical potential μ_c^t and if necessary its temperature T_c^t to prevent charge and heat accumulation on it. Thus, we assume the floating contact to be in thermal equilibrium at every instant of time with both chemical potential and temperature satisfying the simultaneous condition of vanishing charge and vanishing heat currents. Here the index t stresses that the chemical potential and temperature of the floating contact do depend on time but in a frozen picture in the sense that the local equilibrium condition is satisfied for every time snapshot. This is justified in the adiabatic regime (very low driving frequency Ω), mostly accessible in experiments [1]. In contrast, the reservoir is a massive electrode that keeps its temperature and chemical potential constant independent of the ac potential. In practice, this can be achieved grounding the reservoir as indicated in Fig. 1. Its temperature variations would be suppressed if the reservoir has in addition a large heat capacity.

The evolution of the chemical potential and temperature of the floating contact as the dot is adiabatically driven can be sensed by means of a voltage probe and a thermometer [27–35] as indicated in the figure. We predict different behaviors for μ_c^t and T_c^t depending on whether the energy reactance is considered or not in the heat flux into the floating contact. In this way, the proposed experiment would help to discern the proper definition of the heat current and test the existence of the energy reactance.

The results are the following: (i) By defining the heat flux into the floating contact, taking into account the energy reactance as in Ref. [15], we find that the temperature of the floating contact is not changing in time. The outcome is

$$T_c^t = T, \tag{1}$$

where T is the background temperature. The chemical potential of the contact μ_c^t does vary with time in a periodic fashion with a period dictated by the electrical current flowing through the dot. (ii) We demonstrate that any other definition of the heat current, that does not properly account for the energy reactance, necessarily leads to a change in both quantities, T_c^t and μ_c^t as functions of time.

Heat current into the floating contact and quantum energy reactance. Let the rates of change for the charge and the internal energy stored in the floating contact due to exchanges with the rest of the device be $\dot{N}_c(t)$ and $\dot{U}_c(t)$, respectively. Similarly, the rate of change for the energy stored at the tunneling region between the dot and the floating contact is denoted by $\dot{U}_{\mathcal{T}_c}(t)$. The meaningful definition for the instantaneous heat current entering the floating contact is [15]

$$\dot{Q}_c(t) = \dot{U}_c(t) + \frac{\dot{U}_{\mathcal{T}_c}(t)}{2} - \mu_c^t \dot{N}_c(t). \tag{2}$$

The energy reactance $\dot{U}_{\mathcal{T}_c}(t)/2$ contributes to the heat flux only instantaneously and as such vanishes when averaged over one driving period.

From the theoretical point of view, the energy reactance is necessary to derive an instantaneous Joule law for the heat current into a (single-channel) floating contact at low temperatures $\dot{Q}_c(t) = R_q[\dot{N}_c(t)]^2$ with the universal charge relaxation resistance, the Büttiker resistance $R_q = h/2e^2$ [1,24]. This universal Joule law would be then observable in the same regime as the Büttiker resistance. The energy reactance is also necessary to both reconcile the relation between the Green's function and the scattering matrix formalisms [36] for the instantaneous heat current [15] and to obtain correct frequency parity properties of the response functions [17].

Temperature and chemical potential of the floating contact. Our goal is to explicitly show that the definition of Eq. (2) can be verified by measuring the temperature and chemical potential of the floating contact. As discussed above, the latter is at local equilibrium for every time snapshot with a chemical potential μ_c^t and a temperature T_c^t which simultaneously fulfill the condition of zero charge and heat currents, i.e., $\dot{Q}_c(t) = \dot{N}_c(t) = 0$. Deviation of the floating contact temperature and chemical potential from their equilibrium values are denoted by $\delta T_c^t = T_c^t - T$ and $\delta\mu_c^t = \mu_c^t - \mu$, respectively. In the adiabatic regime, these quantities are small $\delta T_c^t, \delta\mu_c^t \propto \hbar\Omega$. As a consequence, we can evaluate both charge and heat fluxes in linear response in these quantities (whereas the amplitude of the ac driving potential is arbitrary).

Following Refs. [23,37] we expand the fluxes $\mathbf{J}(t) \equiv (\dot{N}_c, \dot{Q}_c)$ in the affinities $\mathbf{X}^t = (\delta\mu_c^t, \delta T_c^t, \hbar\Omega)$ with

coefficients $\Lambda_{ij}(t)$ as

$$J_i(t) = \sum_{j=1}^{3} \Lambda_{ij}(t) X_j^t, \tag{3}$$

where $i = 1,2$ ($j = 1$–3) label the different components of the vectors $\mathbf{J}$ and $\mathbf{X}^t$, respectively. The coefficients of the above expansion are response functions evaluated with the frozen Hamiltonian at time t and have the following physical interpretation: Λ_{11} and Λ_{22} are the usual electric and thermal conductances. On the other hand, Λ_{12} (related to the Seebeck effect) and Λ_{21} (related to the Peltier effect) capture the thermoelectric transport, and they satisfy the reciprocity relation $\Lambda_{21} = T\Lambda_{12}$ [38–41]. Finally, Λ_{13} and Λ_{23}, respectively, describe the generation of charge and heat currents by the ac driving. They also obey Onsager relations with the coefficients entering the work flux (not considered here) [37]. Explicit expressions of these coefficients will be supplied below for the specific model.

Here, we notice that the conditions of vanishing fluxes to the floating contact amounts to finding the solution of the 2×2 linear set of equations $\sum_{j=1}^{2} \Lambda_{ij} X_j^t = -\Lambda_{i3}\hbar\Omega$, $i = 1,2$. The solutions are

$$\begin{aligned} \delta\mu_c^t &= \frac{\Lambda_{12}\Lambda_{23} - \Lambda_{13}\Lambda_{22}}{\det \Lambda'}\hbar\Omega, \\ \delta T_c^t &= \frac{\Lambda_{13}\Lambda_{21} - \Lambda_{11}\Lambda_{23}}{\det \Lambda'}\hbar\Omega, \end{aligned} \tag{4}$$

where $\det \Lambda'$ corresponds to the determinant of the 2×2 matrix determined by the condition $j \neq 3$.

The coefficients Λ can be calculated for the system considered in Fig. 1 (see the details in the Supplemental Material [42]),

$$\Lambda_{ij}(t) = \begin{cases} \int \frac{(\varepsilon-\mu)^{i+j-2}}{hT^{(j-1)}} \mathcal{T}(t,\varepsilon)\partial_\varepsilon f \, d\varepsilon, & \text{if } j \neq 3, \\ -\frac{\Gamma_c \dot{V}}{(\Gamma+\Gamma_c)\hbar\Omega} \int (\varepsilon-\mu)^{i-1} \rho_f(t,\varepsilon)\partial_\varepsilon f \, d\varepsilon, & \text{if } j = 3, \end{cases} \tag{5}$$

The distinction between $j \neq 3$ and $j = 3$ is important. In the former case, the response depends on the instantaneous transmission probability $\mathcal{T}(t,\varepsilon)$ for electrons traversing the quantum dot between the reservoir and the floating contact. Physically, this corresponds to dc transport. In the latter case, the response is a function of the time derivative of the potential applied to the gate $\dot{V} = -\Omega V \sin(\Omega t)$ and the instantaneous local density of states of the dot $\rho_f(t,\varepsilon)$. Physically, this is pumping and, as such, of the ac nature. Both coefficients are time dependent because the system adiabatically reacts to the instantaneous ac driving potential [46]. Finally, in Eq. (5) f is the Fermi-Dirac distribution of the reservoir, whereas $\Gamma_c = |w_c|^2\rho_c$ and $\Gamma = |w|^2\rho$ are the hybridization functions with w_c as the dot-floating contact couplings and w as the dot-reservoir couplings. The density of states of the floating contact is ρ_c, and that of the reservoir is ρ.

Interestingly, we readily find that the coefficients of Eq. (5) satisfy the relations: (i) $\Lambda_{13}\Lambda_{21} - \Lambda_{11}\Lambda_{23} = 0$ and (ii) $\Lambda_{j3} = -\Lambda_{j1}\frac{\dot{V}}{\Gamma\Omega}$ with $j = 1,2$, leading to the solution,

$$\delta T_c^t = 0, \qquad \delta\mu_c^t = \frac{\hbar}{\Gamma} e\dot{V}. \tag{6}$$

These equations tell us that changing only the chemical potential is sufficient to satisfy the balance of change and energy for the floating contact. Notice that this conclusion is independent of coupling to the floating contact Γ_c and the base temperature T, provided that the adiabaticity condition $\Gamma, \Gamma_c \gg \hbar\Omega$ is satisfied [47,48]. This is true even for temperatures close to zero in which case the second-order contributions in the affinities should be added to Eq. (3) (see the Supplemental Material [42]). Note that, whereas the universality of the Büttiker resistance can be lifted with increasing temperature, the universality of our result holds for finite T.

To summarize, the floating contact fulfills the conditions of vanishing heat and charge fluxes by changing $\delta\mu_c^t$ in time according to Eq. (6) while keeping its temperature constant and equal to the background temperature as indicated in Eq. (1).

Examine the energy reactance. We would like to stress now that Eq. (6), in particular, the prediction of a constant temperature of the floating contact expressed in Eq. (1), constitutes a proof for the existence of the energy reactance $\dot{U}_{\mathcal{T}_c}(t)/2$ and the definition of the heat current as in Eq. (2). This can be easily understood by noting that we would arrive at completely different conclusions on the behavior of the temperature of the floating contact if we consider a definition of the heat flux that does not take into account the energy reactance.

As a proof, let us analyze the consequence of adopting a commonly used definition, that does not take into account the energy reactance. This corresponds to the following expression for the heat flux into the floating contact:

$$\dot{\tilde{Q}}_c(t) = \dot{U}_c(t) - \mu_c^t \dot{N}_c(t). \tag{7}$$

We need to recalculate the coefficients $\Lambda_{2j}(t)$ by using the above equation. We denote the so-defined coefficients by $\tilde{\Lambda}_{2,j}(t)$. From Eq. (4) where we replace $\Lambda_{2,j}(t) \to \tilde{\Lambda}_{2,j}(t)$, $j = 1$–3, we find the floating contact temperature $\delta\tilde{T}_c^t$ and chemical potential $\delta\tilde{\mu}_c^t$. In contrast to Eq. (6), now we find that both the temperature $\delta\tilde{T}_c^t$ and the chemical potential $\delta\tilde{\mu}_c^t$ of the floating contact change in time. In the case of the chemical potential, $\delta\tilde{\mu}_c^t$ evolves in time in a different pattern from that described by Eq. (6) (the corresponding behavior is shown in the Supplemental Material [42]). We turn to focus on the behavior of the temperature $\delta\tilde{T}_c^t$, which is shown in Fig. 2. It is worth noting that the amplitude of the $\delta\tilde{T}_c^t$ oscillations decreases as T increases, which shows that the two definitions of the heat current agree in the high-temperature limit. These results show that the role of the energy reactance is particularly relevant in the quantum regime. In the classical high-temperature limit the temperature of the floating contact is independent of time either with the heat current defined as in Eq, (2) or with the definition of Eq. (7).

Conclusion. We have shown that the behavior of the time-resolved chemical potential and temperature of a floating contact coupled to an adiabatically driven quantum dot is strongly sensitive on the definition of the instantaneous heat flux. For this reason, sensing these quantities would provide an experimental test for the relevance of the energy reactance introduced in Ref. [15] as a component of the time-dependent heat flux.

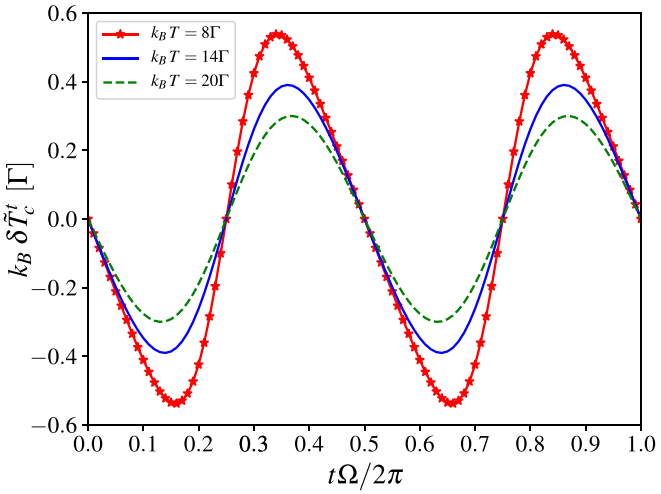

FIG. 2. Deviation of the temperature of the floating contact $\delta\tilde{T}_c^t$ as a function of time for different background temperatures T. The ac potential is $20\Gamma\cos(\Omega t)$ with $\hbar\Omega = 0.07\Gamma$. The hybridization between the floating contact and the quantum level is $\Gamma_c = 0.6\Gamma$. All energies are expressed in units of the hybridization Γ with the reservoir. The temperature of the floating contact displays oscillations that depend on the background temperature. As T increases, the oscillations become less pronounced, and the maxima positions deviate from the moment when the level is aligned with the chemical potential of the reservoir, which in this case corresponds to $t\Omega/2\pi = 0.25$ and $t\Omega/2\pi = 0.75$.

Specifically, for an adiabatically driven quantum dot with a single active level coupled to a single reservoir, we have shown that: (i) If the energy reactance is taken into account, then the temperature of the floating contact is constant and equal to that of the reservoir, whereas its chemical potential follows the time derivative of the driving potential $\dot{V}$ as expressed in Eq. (6). Instead, (ii) if the energy reactance is not taken into account, these two quantities follow a nonuniversal time-dependent pattern.

The experiment we propose is close to the scope of present-day experimental techniques. In fact, typical level spacing for quantum dots is around 100 μeV [49]. Thus, by keeping driving amplitudes below this energy, we would basically have a single active level. On the other hand, typical parameters for single-particle emitters have $\Gamma \simeq 1$ μeV ($\simeq 1$ GHz) and are operated at frequencies of $\Omega \simeq 0.1$ GHz [1], which satisfy the adiabatic condition $\hbar\Omega < \Gamma$. As a consequence, a fast thermometer [9] is able to follow temperature changes in the floating contact on the nanosecond scale. Experiments are typically performed at temperatures close to $T \sim 100$ mK. For this temperature, the oscillations in the temperature shown in Fig. 2 have an amplitude of $\delta\tilde{T}_c^t \simeq 10$ mK.

Since the universal behavior of R_q at $T = 0$ remains valid when the system behaves as a Fermi liquid (both in the linear [50,51] and in the nonlinear regimes [52,53]), we expect that our prediction will also remain valid under the same conditions.

We emphasize that the question about the role of the energy reactance in the definition of a time-dependent heat flux is a fundamental one. It is not restricted to slowly driven systems of noninteracting electrons but is also relevant for interacting models for fast drivings and for weakly and strongly coupled systems. So far this question has been addressed only theoretically. The present proposal shows that a thermometer probe response will experimentally demonstrate the existence of the energy reactance.

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