ground water

Analytical Solution for Well Design with Respect to Discharge Ratio

by Ekkehard Holzbecher

Abstract

For a well in the vicinity of a surface water body, a formula is developed that relates the share of bank filtrate on total pumpage, that is, the discharge ratio, on one side, to basic well and aquifer characteristics on the other. The application of the formula is demonstrated for solving the inverse problem: for an aimed discharge ratio, well characteristics (pumping rate, distance to shore) can be determined. Other useful applications of the formula are outlined.

Introduction

In cases of groundwater withdrawal, there often is the practical task to optimize well operation with respect to a certain condition. There may be restrictions or constraints. A certain goal may have to be met, for example, a certain head or drawdown may be required, or a certain rate of aquifer discharge. Another criterion is optimization to obtain a required mix of waters, which are here characterized by the discharge ratio. In this article is explored, how to deal with that criterion especially.

An example is bank filtration (ISMAR5 2004). Wells are drilled in the vicinity of surface water bodies, mostly to obtain a mix of water originating from surface water and groundwater. Bank filtrate denotes the water originating from surface water body. Concerning its chemical characterization bank filtrate differs substantially from surface water. Bank filtration systems may be designed to deliver a certain value of bank filtration share, that is, the share of bank filtrate of the total pumped water. The remaining share on pumpage is pristine groundwater from the aquifer.

Such a requirement originates from the water quality of bank filtrate on one side and groundwater on the other. Both types of water may contain small amounts of unwanted in some respect harmful substances in concentrations near to the limit required by legislation. If a substance exceeds the required value in bank filtrate, the bank filtration share should be low. On the other hand, if a substance exceeds the required limit in groundwater the bank filtration share should be high. From this opposite requirement at certain locations a window of possible values of bank filtration share results, to fulfill the regulation. A numerical example of such a situation is given preceding the mathematical analysis.

From this background results the task to find appropriate values for the pumping rates or the distance of wells from the bank, in order to obtain mixed water with a required bank filtration share. For an existing well the pumping rate, or a window of pumping rates, should be determined delivering a wanted bank filtration share. For a new well gallery or a single well to be installed, one may ask for the required distance from the bank, if the total pumping rate is given.

In this work, a procedure is derived to perform the mentioned task easily. Subject of the study is the situation with a single well near a straight bank boundary. We use analytical solutions and show that a unique solution exists for each required value of bank filtration share. The solution can be obtained by using a graphical method, a table for most relevant values of bank filtration share, or a numerical inversion method.

For more general situations, that is, with several wells, inhomogeneities, 3D effects or the existence of a clogging layer, one may use analytical methods, if it is allowed, or a numerical model, to examine the situation

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in detail. An outline of the general situation is not aim of this work, in which we focus on a procedure for a first quick check.

Delineation of capture zones is a classical topic in groundwater publications and the mathematical tools to approach such problems are also classical. The fundamentals of the method are outlined by Strack (1989), and applied in several publications, as, for example, by Newsom and Wilson (1988), Bakker and Strack (1996), Shan (1999), Christ and Goltz (2002), and Fienen et al. (2005). As method is based on complex analysis as it is typically used for this type of problems, and as it was described already by Polubarinova-Kochina (1962) and Strack (1989). Most studies focus on the geometry and extension of the capture zone, in dependence of well locations, pumping rates, and ambient uniform flow.

For optimization tasks, the use of analytical solutions is very convenient. Analytical solutions can be computed quickly and such computations can be called easily within an optimization algorithm. Fitts (1994) describes well optimization in order to achieve to a certain drawdown, head or discharge at a certain location. Christ et al. (1999) use similar methods to design in situ remediation installations. Using capture zone type curves, Javandel and Tsang (1986) determine the optimum number of pumping wells for aquifer cleanup. Pump and treat technology also assigns optimization tasks, for which analytical methods can be applied (Matott et al. 2006). In this article, the "goal function" is the discharge ratio, and it turns out that no sophisticated or elaborated inversion tools are needed to solve the optimization problem, for a single well. Moreover, as we show in this note, analytical solutions may exist even for optimization problems that can be used as a benchmark for more complicated numerical codes of the optimization problems.

Bank Filtration Examples

In this article, the situation of a single well installed in the vicinity of a surface water body, that is, a river, lake, or reservoir, is studied. In most such cases, the ambient groundwater flow is in the direction toward the bank line. Thus water pumped in a nearby well will partly be groundwater, partly originate from the surface water, the so-called bank filtrate. For a given well, there is a critical pumping rate, which has to be exceeded to obtain bank filtrate (Holzbecher 2003; Asadi-Aghbolaghi et al. 2011). The single stagnation point that appears for situations with pure groundwater pumping, splits into two stagnations points at the bank in case of bank filtration (Lu et al. 2009). The analysis rests on the assumptions, that the aquifer is homogeneous, that groundwater and surface water are well connected and that ambient groundwater is 1D uniform reaching the bank at right angle.

We describe two example problems that can be handled easily by the procedure developed in the following. The first examines bank filtration at a site in Berlin, Germany. The entire public water supply in that city is obtained by pumping from well galleries in the vicinity of surface water bodies. The second example is hypothetical, nevertheless oriented on typical values for the relevant parameters.

In the Berlin case, trichloroacetate in surface water is measured at a concentration of 0.8 mg/l, while it is not found in groundwater. According to WHO guidelines (WHO 2008), a value of 0.2 mg/l should not be exceeded in public water supply systems. Thus the share of bank filtration on pumped groundwater should not exceed 25% to meet the guideline. On the other hand, the concentration of 1,2-dibromoethane is 5 µg/l in groundwater, while it is not found in surface water. The share of bank filtration should be 20% at least to obtain a limit concentration of 4 µg/l in mixed water, the limit according to WHO guidelines. In order to fulfill both criteria the window for bank filtration share lies between 20% and 25% of pumped water. Other examples can be found in the report on management alternatives in San Bernadino area (United States) (Danskin et al. 2006). The general situation is outlined by Rustler et al. (2010).

The hypothetical example illustrates the application for a situation that stems from shallow geothermics. In connection with an air conditioning installation water is pumped from a well near a river. For an effective cooling work, the temperature of the pumped water should not increase above 12.5 °C. The groundwater has a constant temperature of 10 °C, while on hot summer days the temperature of the bank filtrate may reach 20 °C. The problem is to find the minimum distance between the well and river shore to guarantee the effective operation of the air conditioning system.

Figure 1 shows the relation among pumping rate, well positioning and bank filtration share visually. A well is placed adjacent to the river or lake shore. The latter is located along the left side of the simulated region. Ambient groundwater flow is directed from the right to the left. The figures show head contours, streamlines, and bank filtrate flowpaths in an aerial view. Figure 1A shows a reference situation, in which bank filtrate is approximately 20% of the pumped water. This can be obtained from the figure by counting streamtubes: the streamlines (white) partition all water flow toward the well into 20 equal parts. A bit more than 4 of 20 originate at the river shore, providing roughly 20% of the well flow.

Figure 1B shows the flow pattern if the pumping rate is doubled: the region influenced by bank filtration increases, and the share of bank filtrate increases to more than 40%. In Figure 1C, one can also recognize an increase of bank filtration share to approximately 40%, here due to the location of the well, which is placed in half the distance from the shoreline in relation to the reference situation. All figures were obtained using the Bank Filtration Simulator (BFS), described by Rustler et al. (2009).

Mathematical Analysis

As in the previous examples, the aim is to achieve a given fixed ratio of bank filtrate in pumped water. If Q
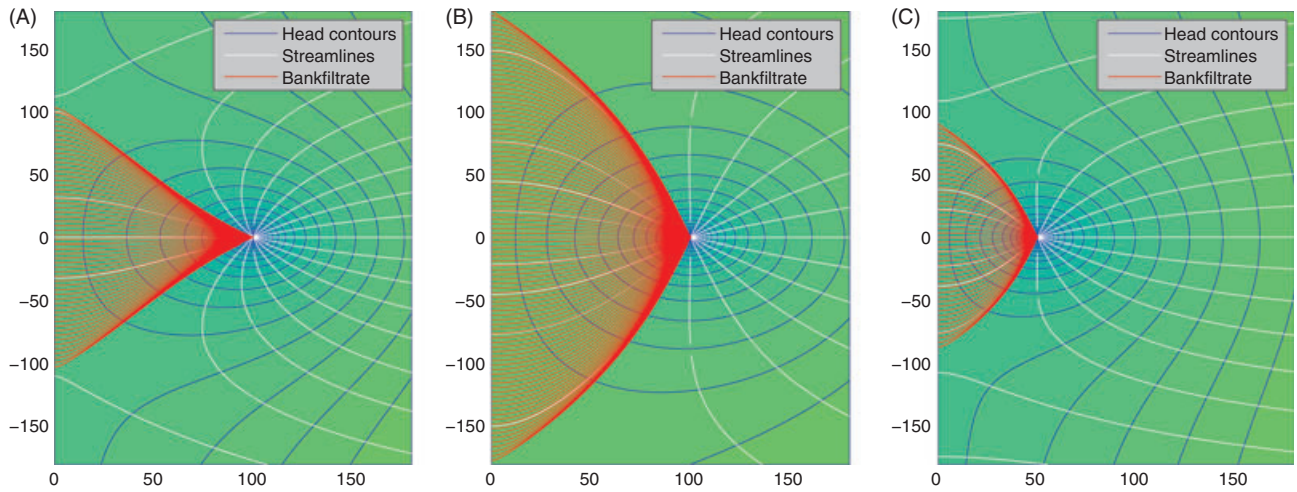
Figure 1. Simulations of groundwater flow toward a well in the vicinity of a surface water body. (A) Base case with discharge ratio of approximately 20%. (B) Pumping rate doubled, with a discharge ratio of approximately 40%. (C) Well moved half the distance to the shoreline, with a discharge ratio of approximately 40%.

denotes the pumped water per time unit and Q_{bf} denotes the amount of bank filtrate (both in units L^3/T), it is aimed to obtain a certain ratio:

$$\alpha = Q_{bf}/Q, \quad (1)$$

with given dimensionless ratio $\alpha \in (0, 1)$.

This study is about the situation with a single well and a straight bank line, the latter characterized by a fixed constant potential. The solution in form of the complex discharge potential $\Phi(z)$ in terms of the complex space variable $z = x + iy$ for the problem is given by the superposition of uniform flow, a solution for the real well and another solution for an image well:

$$\Phi(z) = -Q_{x0}z - \frac{Q}{2\pi}[\log(z-d) - \log(z+d)], \quad (2)$$

where d (unit L) denotes the distance of the well from the bank line and Q_{x0} (unit L^2/T) the absolute value of uniform flow in the x direction (Strack 1989; Holzbecher 2007). The stream function $\Psi(z)$ is the complex part of $\Phi(z)$, both with physical unit (L^3/T). Equation (2) is the solution in case of uniform flow directed toward the shore. Some authors prefer to use the uniform flow velocity U and the aquifer thickness B as parameters (Javandel and Tsang 1986; Shan 1999)—with these holds: $Q_{x0} = BU$. Note that the imaginary y axis is the isopotential line in this formulation, and that the well is located at position d on the x axis. The complex number z represents a position in the model region, here the half plane with $Re(z) \geq 0$.

The discharge vector (Strack 1989) **q** (unit L^2/T) can be calculated as the gradient of the potential and is given by:

$$\mathbf{q}(z) = -Q_{x0} - \frac{Q}{2\pi}\left(\frac{1}{z-d} - \frac{1}{z+d}\right). \quad (3)$$

For a single well that is pumping strong enough to induce bank filtration there are two stagnation points at the bank, marking the limits of the interval in which water from the surface water body enters the aquifer. At these stagnation points, denoted by $z_{stag} = \pm iy_{stag}$, the discharge vector vanishes, that is, there is the condition:

$$\mathbf{q}(\pm iy_{stag})$$
$$= -Q_{x0} - \frac{Q}{2\pi}\left(\frac{1}{\pm iy_{stag} - d} - \frac{1}{\pm iy_{stag} + d}\right) = 0. \quad (4)$$

or

$$Q_{x0} - \frac{Q}{\pi}\frac{d}{(y_{stag}^2 + d^2)} = 0 \quad (5)$$

Equation (5) can be resolved for y_{stag}^2:

$$y_{stag}^2 = -d^2 + \frac{d}{\pi}\frac{Q}{Q_{x0}}. \quad (6)$$

The condition for the existence of stagnations points is that the right-hand side of Equation (6) is nonnegative, that is, $Q/\pi d Q_{x0} \geq 1$. The limit case with a single stagnation point is given by the condition $Q/\pi d Q_{x0} = 1$, as already noted by Strack (1989). In order to determine the flux of bank filtrate one has to evaluate the stream function Ψ as imaginary part of the complex potential Φ at the stagnation points:

$$\Psi(\pm iy_{stag}) = \pm Q_{x0}y_{stag} - \frac{Q}{2\pi}\arg\left(\frac{\pm iy_{stag} - d}{\pm iy_{stag} + d}\right). \quad (7)$$

In this article, the principal logarithm with arguments arg in the interval $(-\pi, \pi)$ is used (Howie 2004), which

has a jump of size $2\pi i$ at the negative x axis. In the model domain with $x \geq 0$ a jump of size $-Q$ appears at the x axis with $x \leq d$ due to the first log term in Equation (2). The total amount of bank filtrate is then given by the difference between the stream function values at the two stagnation points

$$Q_{bf} = \Psi(iy_{\text{stag}}) - \Psi(-iy_{\text{stag}}) + Q \qquad (8)$$

for the usual definition of the stream function, in which the complex plane is sliced at the negative x axis. The last term in Equation (8) appears due to the facts that the stream function of the complex potential Φ given by Equation (2) has a jump of size $-Q$ at the x axis, and that the connecting line between both stagnation points crosses that axis. Together with Equation (1) results the condition

$$\Psi(-iy_{\text{stag}}) - \Psi(iy_{\text{stag}}) = Q(1-\alpha) \qquad (9)$$

and, by utilization of Equation (7):

$$2Q_{x0}y_{\text{stag}} - \frac{Q}{2\pi} \arg\left(\frac{-iy_{\text{stag}} - d}{-iy_{\text{stag}} + d}\right)$$
$$+ \frac{Q}{2\pi} \arg\left(\frac{iy_{\text{stag}} - d}{iy_{\text{stag}} + d}\right) = Q(1-\alpha) \qquad (10)$$

or, as the two arg terms deliver the same result:

$$\arg\left(\frac{iy_{\text{stag}} - d}{iy_{\text{stag}} + d}\right) = \pi\left(1 - \alpha - 2\frac{Q_{x0}}{Q}y_{\text{stag}}\right). \qquad (11)$$

The left-hand side can be rewritten as:

$$\arg\left(\frac{iy_{\text{stag}} - d}{iy_{\text{stag}} + d}\right) = 2\arg(iy_{\text{stag}} - d) - \pi, \qquad (12)$$

which leads to the condition:

$$\arg(iy_{\text{stag}} - d) = \pi\left(1 - \frac{\alpha}{2} - \frac{Q_{x0}}{Q}y_{\text{stag}}\right). \qquad (13)$$

As the argument has values in the interval $(\pi/2, \pi)$ one may also write:

$$-\arctan\left(\frac{y_{\text{stag}}}{d}\right) = \pi\left(1 - \frac{\alpha}{2} - \frac{Q_{x0}}{Q}y_{\text{stag}}\right). \qquad (14)$$

Utilizing Equation (6) yields:

$$-\arctan\left(\sqrt{\frac{1}{\pi d}\frac{Q}{Q_{x0}} - 1}\right)$$
$$= \pi\left(1 - \frac{\alpha}{2} - \frac{Q_{x0}d}{Q}\sqrt{\frac{1}{\pi d}\frac{Q}{Q_{x0}} - 1}\right). \qquad (15)$$

Note that the radicand on both sides is always positive, as the criterion for the onset of bank filtration

is given by $Q \geq \pi Q_0 d$ (Holzbecher 2003), which is identical to the condition for the existence of stagnation points, mentioned above. One can gather all physical parameters (in this formulation only three: d, Q, and Q_{x0}) in a dimensionless number

$$\beta = \frac{1}{\pi d}\frac{Q}{Q_{x0}} - 1 \qquad (16)$$

Introducing this number in the preceding formula (15) leads to:

$$-\arctan(\sqrt{\beta}) = \pi - \pi\frac{\alpha}{2} - \frac{\sqrt{\beta}}{\beta + 1}. \qquad (17)$$

For each bank filtration share α a corresponding value of β can be determined Equation (17) describes a unique correspondence between the bank filtration share α on one side and β which summarizes the physical setting on the other side. Below this correspondence will be used in some examples.

Before for a more detailed look at the correspondence (17), the formula can be rewritten as a zero condition for the function f, defined as:

$$f(\beta) := -\arctan(\sqrt{\beta}) + \frac{\sqrt{\beta}}{\beta + 1} + \pi\frac{\alpha}{2} = 0 \qquad (18)$$

As the arctan is not unique, but delivers values differing by a multiple of π, the first term on the right-hand side of Equation (17) can be omitted. The corresponding function $\alpha(\beta)$ looks as follows (Figure 2).

There is a unique correspondence between α and β. Some specific values of β are given in Table 1, obtained by numerical inversion of the function f.

In order to check the result we simulated bank filtration patterns using the BFS (Rustler et al. 2009) and compared the results of the software with the result of the analytical procedure described in this article. The results, gathered in Table 2, confirm the validity of the here derived formula.
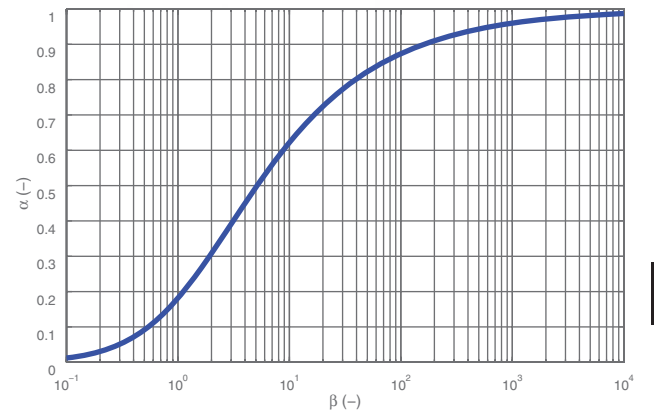

Figure 2. The function $\alpha(\beta)$; for each value of bank filtration share α the β value can be obtained, from which the solutions for values of d or Q can be calculated.

Table 1
β Values for Some Selected Values of Bank Filtration Share α (see Appendix for a Refined Table)

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
β	0.5417	1.1185	1.9207	3.1335	5.1277	8.7845	16.6716	39.1918	160.779

Table 2
Comparison Bank Filtration Shares, Computed Using BFS Software and Analytical Procedure Presented in this Article

Case	d	Q	Q_{x0}	β	α (%)	α from BFS (%)
A	100	0.004	6.10^{-6}	1.121	20.05	20.02
B	100	0.008	6.10^{-6}	3.2441	40.7	40.69
C	50	0.004	6.10^{-6}	3.2441	40.7	40.67

Note: Cases A, B and C refer to situations, shown in Figure 1.

Some important results can be derived directly from the described procedure:

- The discharge ratio depends on three parameters only: the distance between well d, and shoreline, the pumping rate Q, and uniform flow rate Q_{x0}.
- The change of pumping by a certain factor γ has the same effect as changing distance d or Q_{x0} using the γ^{-1} as a factor, concerning flow field and discharge ratio

Application Procedure

In order to apply the result of the previous analysis, a certain discharge ratio, α, is needed. In practice there are different ways to obtain the β-value for a required bank filtration share. One way is to use Figure 2. For given α the value of β can be taken from the graph. Another way is to use a value from a table (Table 1; a refined table for increments $\Delta\alpha = 0.01$ is given in Appendix). A third way is to use a numerical method for the computation of the zero of the function f in Equation (18).

After that first step one of the three remaining parameters, appearing in Equation (16) can be calculated from the others. One may use the reformulation:

$$\frac{1}{d}\frac{Q}{Q_{x0}} = \pi(\beta + 1), \qquad (19)$$

where the right-hand side becomes a constant for that problem setting. For example, one can obtain the optimal pumping rate for a given well and known uniform flow:

$$Q = \pi(\beta + 1)Q_{x0}d. \qquad (20)$$

Lets calculate the minimum and maximum pumping rates for the bank filtrate problem situation as described above for Berlin. The required window of bank filtrate shares between 20% and 25% which corresponds with β values of 1.1185 and 1.4823. If the distance from the shoreline is 100 m and the uniform flow amounts to 4.10^{-6} m^2/s, one thus obtains a window of pumping rates between 0.0027 and 0.0031 m^3/s.

Before well drilling, in the design phase of well construction one can use the above result to obtain the optimal distance of a well to the bank, if the pumping rate and uniform flow are known:

$$d = \frac{1}{\pi(\beta + 1)}\frac{Q}{Q_{x0}}. \qquad (21)$$

As third possible application is the following: if in the operating phase a well at a distance d is pumping at a rate Q and if the ratio α (with a corresponding β) can be derived from some chemical analysis, it is possible to obtain an estimate for uniform flow by the formula:

$$Q_{x0} = \frac{1}{\pi(\beta + 1)}\frac{Q}{d}. \qquad (22)$$

Let give a first example calculation for the input values $\frac{Q}{Q_{x0}} = 10^3$ and $\alpha = 0.33$. From a numerical computation of the zero of f (Equation 18), or the table in Appendix gives $\beta = 2.2297$ corresponding to α. From this value results, the optimal distance of the well from the bank $d = 98.6$ m, using Equation (21). Stagnation points in this example are at positions $y_{stag} = \pm 147.75$ and the values of the stream function at the stagnation points are $\Psi(\pm iy_{stag}) = \pm 0.3371 \cdot 10^{-3}$.

Similarly one can obtain the solution for the hypothetical geothermics application stated above. In the limit situation of a hot summer period with temperatures of 20 °C for the bank filtrate water the maximum share of bank filtrate should be 25% to keep the temperature of the pumped water below the required 12.5 °C. For that share a value of $\beta = 1.4823$ is obtained. If the uniform flow is $Q_{x0} = 10^{-6}$ m^2/s and the pumping rate is $Q = 10^{-4}$ m^3/s, one obtains from Equation (21) that the well should be at least 12.3 m away from the shoreline.

The alternate hypothetical application is to determine the uniform flow. This may be important in situations for which few measurements are available for the field site. For appropriate chemical substances, that is, for which the concentration is well known and not fluctuating, the mixing rate and thus the bank filtration share can be obtained from on-site measurements during constant pumping. As well location and pumping rate are also known in that situation, the only remaining unknown variable is the uniform flow. If bank filtrate and groundwater show typical different temperatures, the mixing rate could also be obtained from temperature measurements.

Summary and Outlook

A procedure is presented that relates the share of bank filtrate to total pumping rate, here referred to as discharge ratio, to basic well and aquifer characteristics. While the task to calculate the discharge ratio from the physical characteristics can easily be performed using well-known analytical solutions, the developed scheme works for the inverse task: for an aimed ratio to be obtained, the optimal well characteristics can be determined. It is shown that the method can be solved graphically or via a look-up table, using Figure 2 or Table A1. For the desired ratio, an intermediate dimensionless parameter combination β is used, which is the dimensionless combination of the basic parameters. Equation (16) relates β with the well location parameter d, the pumping rate Q, and the aquifer uniform flow Q_{x0}. Alternative formulations are given in Equations (19 through 22).

The developed one to one correspondence between α and β can be used for several problem settings of groundwater flow. This work outlines the utilization concerning well location and pumping rate for pumping in the vicinity of surface water bodies, so-called bank filtration. For the same situation one could also use the proposed procedure to calculate uniform flow, which often is an unknown aquifer characteristic that is not easy to determine.

A generalization of the entire method for more than two wells is difficult, as more well design parameters come into play: wells can be located at different distances from the shoreline and pump at different rates. However, for some situations with several wells a procedure similar to the one outlined in this article could possibly be derived.

The derived procedure can probably be generalized to situations, in which the ambient groundwater uniform flow direction and the shoreline do not meet at a right angle, as it is required in this article. However, the right-angle condition is often fulfilled in the direct vicinity of a surface water body. Note that the analytical solution, the generalization of Equation (2), in that case requires a head gradient on the y axis, that is, a slope in the river. Thus the situation considered in this article is likely more important than the generalized situation.

The outlined method can also be applied to dipole installations with a pumping and an injecting well. The formulae given above can be taken directly, as the bank filtration solution (2) actually includes two wells (one real and the other virtual). In that respect, the procedure derived here could be used for all situations in which a doublet is installed, that is, for groundwater lowering, groundwater sanitation, in geothermics and other application fields. The author hopes that the proposed procedure finds some useful application in some of these fields as well.

Appendix

Table A1

β **Values for Some Selected Values of Bank Filtration Share** $\alpha = \alpha_{row1} + \alpha_{column1}$ **(Refined)**

α	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	—	0.0879	0.1455	0.1976	0.2474	0.2962	0.3446	0.3931	0.4419	0.4914
0.1	0.5417	0.5929	0.6452	0.6988	0.7537	0.8102	0.8681	0.9279	0.9894	1.0529
0.2	1.1185	1.1863	1.2564	1.3290	1.4043	1.4823	1.5633	1.6475	1.7349	1.8259
0.3	1.9207	2.0194	2.1223	2.2297	2.3419	2.4591	2.5817	2.7100	2.8445	2.9855
0.4	3.1335	3.2889	3.4523	3.6243	3.8059	3.9964	4.1980	4.4109	4.6362	4.8748
0.5	5.1277	5.3961	5.6814	5.9850	6.3086	6.6537	7.0227	7.4176	7.8409	8.2955
0.6	8.7845	9.3115	9.881	10.4964	11.1641	11.8897	12.6803	13.5437	14.4892	15.5277
0.7	16.6716	17.9358	19.3378	20.8983	22.6423	24.5996	26.8066	29.3075	32.1572	35.4235
0.8	39.1918	43.5705	48.6991	54.7590	61.9903	70.7154	81.3762	94.5906	111.245	132.644
0.9	160.779	198.807	251.969	329.511	448.983	647.122	1011.878	1799.932	4051.51	16210.05

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