Is ‘Dark Matter” made entirely of the Gravitational Field

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We argue that part of “dark matter” is not made of matter, but of the singular world lines and world-surfaces in the solutions of *Einstein's vacuum field equation* $G_{\mu\nu} = 0$. Their Einstein-Hilbert action governs, in a slightly modified form, also their quantum fluctuations in a partition function fromed from a sum over all line and surface configurations. For world sufaces, the Einstein-Hilbert action coincides with that of closed bosonic “strings” in four spacetime dimensions, which appear here in a new physical context.

PACS numbers: 95.35+d.04.60,04.20C,04.90

The surpisingly large orbital velocities of galaxies in clusters induced F. Zwicky in 1933 to postulated the existence of dark matter. Confirmation came from plots of the orbital velocities of stars versus distance inside individual galaxies, whose explanation asked for large amounts of invisible matter in each galaxy. The Friedmann model of the evolution of the universe indicates that dark matter constitutes a large percentage, roughly 23%, of the mass energy of the universe. If dark matter is added to the so-called ‘dark energy”, which accounts for roughly 70% of the energy, the visible matter is practically negligible, which is the reason for ignoring it complетly in the most extensive computer simulation of the evolution of cosmic structures so far [1], the so-called “Millenium Simulation”.

There are many speculations as to its composition and we want to propose, in this note, the simplest possible explanation of at least a part of it.

Let us remember that all static electric fields in nature may be considered as originating from the nontrivial solutions of the Poisson equation for the electric potential $\phi(x)$ as a function of $x = (t, \mathbf{x})$:

$$\Delta\phi(x) \equiv \boldsymbol{\nabla} \cdot \boldsymbol{\nabla}\, \phi(x) = 0. \qquad (1)$$

The simplest of them has the form e/r, where $r = |\mathbf{x}|$, and is attributed to a pointlike electric charges, whose size e can be extracted from the pole strength of the singularity of the electric field $\mathbf{E}$ which points radially outward and has a strenght e/r^2. This becomes visible by performing an area integral over the $\mathbf{E}$ field around the singularity which, by the famous Gauss integral theorem,

$$\int_V d^3x\, \boldsymbol{\nabla} \cdot \mathbf{E} = \int_A d^2\mathbf{a} \cdot \mathbf{E}, \qquad (2)$$

is equal to the volume integral over $\boldsymbol{\nabla} \cdot \mathbf{E} = -\Delta\phi(x)$. Thus a field that solves the homogeneous Poisson equation can have can have a nonzero integral $\int_V d^3x\, \Delta\phi(x) = -4\pi e$. This fact is more properly be expressed with the help of a Dirac-delta function $\delta^{(3)}(\mathbf{x})$ as

$$\Delta\phi(x) = -4\pi e \delta^{(3)}(\mathbf{x}). \qquad (3)$$

For the sequel, it is useful to re-express the Gauss theorem (2) in a one-dimensional form as

$$\int^R dr \partial_r E(r) = E(R). \qquad (4)$$

This appears in the radial part of the Gaussian relation

$$\int d^3x\, \boldsymbol{\nabla}\cdot\mathbf{E} = -4\pi\int^R dr\, r^2 \boldsymbol{\nabla}\cdot\boldsymbol{\nabla} e/r = 4\pi\int^R dr\, \partial_r e, \qquad (5)$$

so that we find the electric charge e from the one-dimensional equation

$$\int^R dr\, \partial_r e = e, \qquad (6)$$

showing once more that $\boldsymbol{\nabla} \cdot \mathbf{E} = 4\pi e \delta^{(3)}(\mathbf{x})$.

For celestial objects, the situation is quite similar. The Einstein equation in the vacuum, $G_{\mu\nu} = 0$, possesses simple nontrivial solutions in the form of the Schwarzschild metric defined by

$$ds^2 = B(r)c^2dt^2 - A(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \qquad (7)$$

with $B(r) = 1 - r_S/r$, $A(r) = 1/B(r)$, where $r_S \equiv 2G_{\rm N}M/c^2$ is the Schwarzschild radius and $G_{\rm N}$ Newton's gravitational constant. Its Einstein tensor has the component

$$G_t{}^t = A'/A^2r - (1 - A)/Ar^2. \qquad (8)$$

This vanishes. However, if we calculate the volume integral $\int_V d^3x\, \sqrt{-g} G_t{}^t$, we find $\int_V d^3x \sqrt{B/A}[A'/Ar - (1 - A)/r^2]$, which is equal to $\int^R dr\partial_r(r - r/A) = (2G_{\rm N}/c)\int^R dr\partial_r M = (2G_{\rm N}/c)M$. Using the one-dimensional form (4) of Gauss' integral theorem we find, also here, a nonzero integral

$$\int_V d^3x\sqrt{-g}\, G_0{}^0 = \kappa c M, \qquad (9)$$

where κ is defined in terms of the Planck length $l_{\rm P}$, as

$$\kappa \equiv 8\pi l_{\rm P}^2/\hbar = 8\pi G_{\rm N}/c^3. \qquad (10)$$

From (9) we identify the mass of the object as being M.

If the mass point moves through spacetime along a trajectory parametrized by $x^\mu(\tau)$, it has an energy-momentum tensor

$$T^{\mu\nu}(y) = M \int_{-\infty}^{\infty} d\tau \dot{x}^\mu(\tau)\dot{x}^\nu(\tau)\delta^{(4)}(y - x(\tau)), \quad (11)$$

where a dot denotes the τ-derivative. We may integrate the associated solution of the homogeneous Einstein equation $G_{\mu\nu} = 0$ over spacetime, and find, using $\dot{x}^2 = 1$, that that its Einstein-Hilbert action

$$\mathcal{A}_{\rm EH} = -\frac{1}{2\kappa} \int d^4x\sqrt{-g}R, \quad (12)$$

is proportional to the classical action of a point-like particle:

$$\mathcal{A}_{\rm EH}^{\rm worldline} \propto -Mc \int ds. \quad (13)$$

A slight modification of (13), that is the same classically, but different for fluctuating orbits, describes also the quantum physics of a spin-0 particle [2] in a path integral over all orbits. Thus Einstein's action for a singular world line in spacetime can be used to define also the quantum physics a spin-0 point particle.

In addition to pointlike singularities, the homogeneous Einstein equation will also possess singularities on surfaces in spacetime. These may be parametrized by $x^\mu(\sigma, \tau)$, and their energy-momentum tensor has the form

$$T^{\mu\nu}(y) \propto \int_{-\infty}^{\infty} d\sigma d\tau (\dot{x}^\mu \dot{x}^\nu - x'^\mu x'^\nu)\delta^{(4)}(y - x(\sigma, \tau)), (14)$$

where a prime denotes a σ-derivative. In the associated vanishing Einstein tensor, the δ-function on the surface manifests itself in the nonzero spacetime integral [5]:

$$\int d^4x\sqrt{-g}\, G_\mu{}^\mu \propto \int d^2a \equiv \int_A d\sigma d\tau \sqrt{(\dot{x}x')^2 - \dot{x}'^2 x'^2}. \quad (15)$$

By analogy with the line-like case we obtain for such a singular field an Einstein-Hilbert action (12)

$$\mathcal{A}_{\rm EH}^{\rm worldsurface} \propto -\frac{1}{2\kappa} \int_A d^2a = -\frac{\hbar}{16\pi l_{\rm P}^2} \int_A d^2a. \quad (16)$$

Apart from a numerical proportionality factor of order one, this is precisely the Nambu-Goto action of a bosonic closed string in four spacetime dimensions:

$$\mathcal{A}_{\rm NG} = -\frac{\hbar}{2\pi l_{\rm s}^2} \int_A d^2a, \quad (17)$$

where $l_{\rm s}$ is the so-called string length $l_{\rm s}$, related to the slope parameter $\alpha' = dl/dm^2$ in the string tension $T \equiv 1/2\pi\alpha'\hbar c$ by $l_{\rm s} = \hbar c\sqrt{\alpha'}$. Note that in contrast to the world lines, there is no extra mass parameter M.

The original string model was proposed to describe color-electric flux tubes and their Regge trajectories whose slopes α' lie around 1 GeV^{-2}. However, since the tubes are really fat objects, as fat as pions, only very long flux tubes are approximately line-like. Short tubes degenerate into spherical "MIT-bags" [6]. The flux-tube role of strings was therefore abandoned, and the action (17) was re-interpreted in a completely different fashion, as describing the fundamental particles of nature, assuming $l_{\rm S}$ to be of the order of $l_{\rm P}$. Then the spin-2 particles of (17) would interact like gravitons and define Quantum Gravity. But also the ensuing "new string theory" [3] has been criticized by many authors [7]. One of its most embarrassing failures is that it has not produced any experimentally observable results. The particle spectra of its solutions have not matched the existing particle spectra. The proposal of this note cures this problem. If "strings" describe "dark matter", there would be no need to reproduce other observed particle spectra. Instead, their celebrated virtue, that their spin-2 quanta interact like gravitons, can be used to fix the proportionality factor between the Einstein action action (16), and the string action (17).

It must be kept in mind that just as $-Mc\int ds$ had to be modified for fluctuating paths [2], also the Nambu-Goto action (17) needs a modification, if the surfaces fluctuate. That was found by Polyakov when studying the consequences of the conformal symmetry the theory. He replaced the action (17) by a new action that is equal to (17) at the classical level, but contains in $D \neq 26$ dimensions another spin-0 field with a Liouville action.

Since the singularities of Einstein's fields possess only gravitational interactions, their identification with "dark matter" seems very natural. All visible matter consists of singular solutions of the Maxwell equations and the field equations of the standard model. A grand-canonical ensemble of these and the smooth wave solutions of the standard model explain an important part of the matter in the Friedmann model of cosmological evolution.

But the main contribution to the energy comes from the above singularities of Einstein's equation. Soon after the universe was created, the temperature was so high that the configurational entropy of the surfaces overwhelmed completely the impeding Boltzmann factors. Spacetime was filled with these surfaces in the same way as superfluid helium is filled with the world-surface of vortex lines. In hot helium, these lie so densely packed that the superfluid behaves like a normal fluid [8, 9]. The Einstein-Hilbert action of such a singularity-filled turbulent geometry behaves like the action of a grand-canonical ensemble of world surfaces of a bosonic closed-string model. Note that here these are two-dimensional objects living in four spacetime dimensions, and there is definite need to understand their spectrum by studying the associated Polyakov action, without circumventing the accompanying Liouville field by escaping into unphysical dimensions

It should be noted that in the immediate neighborhood of the singularities, the curvature will be so high, that Einstein's linear approximation $-(1/2\kappa)R$ to the Lagrangian must break down and will have to be corrected by some nonlinear function of R, that starts out like Einstein's, but continues differently. A possible modification has been suggested a decade ago [10], and many other options have been investigated since then [11].

After the big bang, the universe expanded and cooled down, so that large singular surfaces shrunk by emitting gravitational radiation. Their density decreased, and some phase transition made the cosmos homogeneous and isotropic on the large scale [12]. But it remained filled with gravitational radiation and small singular surfaces that had shrunk until their sizes reached the levels stabilized b quantum physics, i.e., when their fluctuating action decreased to order $\hbar$. The statistical mechanics of this cosmos is the analog of a spacetime filled with superfluid helium whose specific heat is governed by the zero-mass phonons and by rotons. Recall [13], that in this way Landau discovered the fundamental excitations called rotons, whose existence he deduced from the temperature behavior of the specific heat. In the universe, the role of rotons is played by the smallest surface-like singularities of the homogeneous Einstein equation, whose existence we deduce from the cosmological requirement of dark matter.

The situation can also be illustrated by a further analogy with a many-body system. The defects in a crystal whose "atoms" have a lattice spacing $l_{\rm P}$ simulate precisely the mathematics of a Riemann-Cartan spacetime, in which disclinations and dislocations define curvature and torsion [9, 14, 15]. Thus we may imagine a model of the universe as a "floppy world crystal" [16], a liquid-crystal-like phase [17] in which a first melting transition has led to correct gravitational $1/r$-interactions between disclinations. The initial hot universe was filled with defects—it was a "world-liquid". After cooling down to the present liquid-crystal state, there remained plenty of residual defects around, which form our "dark matter".

We know that the cosmos is filled with a cosmic microwave background (CMB) of photons of roughly 2.725 Kelvin, the remnants of the big bang. They contribute to the Friedmann equation of motion a constant $\Omega_{\rm rad}h^2 = (2.47 \pm 0.01) \times 10^{-5}$, where $h = 0.72 \pm 0.03$ is the Hubble parameter, defined in terms of the Hubble constant H by $h \equiv H/(100 \text{ km/Mpc sec})$. The symbol Ω denotes the energy density divided by the so-called critical density $\rho_{\rm c} \equiv 3H^2/8\pi G_{\rm N} = 1.88 \times 10^{-26} h^2\,\text{kg/m}^3$ [18]. The baryon density contributes $\Omega_{\rm rad}h^2 = 0.0227 \pm 0.0006$, or 720 times as much, whereas the dark matter contributes $\Omega_{\rm dark}h^2 = 0.104 \pm 0.006$, or 4210 as much. If we assume for a moment that all massive strings are frozen out, and that only the subsequently emitted gravitons form a thermal background [19] then, since the energy of massless states is proportional to T^4, the temperature of this background would be $T_{\rm DMB} \approx 4210^{1/4} \approx 8T_{\rm CMB} \approx 22$K. In general we expect the presence of also the other singular solutions of Einstein's equation to change this result.

There is an alternative way of deriving the above-described properties of the fluctuating singular surfaces of Einstein's theory. One may rewrite Einstein's theory as a gauge theory [9, 15], and put it on a spacetime lattice [20]. Then the singular surfaces are built explicitly from plaquettes, as in lattice gauge theories of asymptotically-free nonabelian gauge theories [21]. In the abelian case, the surfaces are composed as shown in Ref. [22], for the nonablian case, see [23]. An equivalent derivation could also be given in the framework of *loop gravity* [24]. But that would require a separate study beyond this letter.

Summarizing we have seen that the Einstein-Hilbert action governs not only the classical physics of gravitational fields but also, via the fluctuations of its line- and surface-like singularites, the quantum physics of dark matter. A string-like action, derived from it for the fluctuating surface-like singularites, contains interacting spin-2 quanta that define a finite Quantum Gravity.

Acknowledgment: I am grateful to R. Kerr, N. Hunter-Jones, F. Linder, F. Nogueira, A. Pelster, R. Ruffini, B. Schroer, and She-Sheng Xue for useful comments.

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