

Unlevering & Relevering Betas with Divergent Tax Rates

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Abstract

Whenever an unquoted company is to be valued the determination of cost of capital is performed by using observable betas of comparable peer group companies. However, these betas have to be adjusted for differing degrees of leverage. This so-called unlevering and relevering belongs to the standard repertoire of modern finance theory, but so far, the effect of divergent tax rates between the company being valued and the peer group has never been addressed in context of unlevering und relevering beta. We abstain from the implicit assumption of unique tax rates by deriving an adjustment formula for beta that takes into account not only divergent leverage ratios but also divergent tax rates. Moreover, in an numerical analysis we quantify the mispricings that result from a disregard of divergent tax rates and show that these distortions can be of significant extent.

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JEL Classification: G32, H25

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1 Problem

When determining costs of capital, e.g. in context of corporate valuations, one usually applies the (Tax) Capital Asset Pricing Model which states that costs of capital are composed of the risk-free rate and a risk premium which itself equals the market premium times beta. In case of a company that is publicly listed on a stock exchange beta can easily be measuring by using historical stock prices. If, however, the company is not quoted the determination of beta is based on the identification of an appropriate peer group consisting of comparable companies whose betas can be derived from the capital market. In order to apply these betas for the company which is to be valued two requirements are mentioned in literature that have to be fulfilled:¹

1. Identical operating/industry risk
2. Identical leverage risk

In order to meet the first requirement, one has to choose a peer group that resembles the (non-listed) company with respect to its operating figures like size and industrial sector.² The second condition has to be considered if peer group companies show differing debt ratios and are therefore subject to a differing degree of financial risk. In this case the peer group beta has to be adjusted properly for leverage effects before applying it to the company being valued. Arithmetically speaking this so-called „unlevering“ and „relevering“ is conducted with the help of adjustment formulas which provide a mathematical relation between beta of an unlevered firm and beta of a levered company. Such adjustment formulas belong to the standard repertoire of modern finance theory and have been discussed extensively in literature.³

However, very scant attention has been payed to the underlying tax regime when identifying the peer group and adjusting its beta for leverage effects. To the best of our knowledge the question of how to deal with divergent tax systems imposed on the peer group and the company that shall be valued has not been addressed in literature. That is surprising, especially in light of the growing internationalization of capital markets and in

¹See e.g. *Damodaran* (1999), p. 25 and *Hillier/Ross/Westerfield/Jaffe/Jordan* (2010), pp. 324–327.

²See *Alford* (1992), p. 106.

³See e.g. *Hamada* (1972), *Harris and Pringle* (1985), *Ruback* (2002), *Modigliani and Miller* (1963), *Damodaran* (1994), p. 31 and 277. The adjustment formulas are based on different assumptions concerning the consideration of default risk and the underlying financing policy. An overview is provided by *Fernandez* (2008).

context of the valuation of companies operating in niche industries for which appropriate peer group companies can only be found abroad. In this article we address this question by considering divergent corporate tax rates. Whenever the peer group and the enterprise being valued are not only subject to different debt ratios but also to different tax rates one has to adjust for leverage *and* tax effects.⁴ This article analyzes the influence of these tax effects on the adjusted beta and thus on the costs of capital and firm value. We show that a disregarding of divergent tax rates can lead to mispricings of non-negligible extent.

This article is structured as follows: Section 2 presents the derivation of an adjustment formula that accounts for divergent debt ratios and divergent tax rates. Section 3 focuses on a numerical analysis in order to quantify misevaluations which arise whenever untaxing and retaxing are not taken into account. The article closes with a summary.

2 Theoretical Analysis

2.1 Assumptions

For simplicity reasons we consider a peer group consisting of only one company which is, just like the company being valued, assumed to be free of default risk.⁵ From this it follows that the risk-free rate r_f can be applied for costs of debt, and debt beta has a zero value, $\beta^D = 0$.

Under these assumptions and for a financing strategy based on market values⁶ with nonvarying leverage ratios L the Miles-Ezzell-adjustment formula

$$\beta^l = \beta^u \left(1 + \frac{1 + r_f(1 - \tau)}{1 + r_f} L \right) \quad (1)$$

is widely used.⁷ It provides a connection between beta of a levered company β^l and beta of an unlevered firm β^u , both facing an identical corporate tax rate τ . This formula will be the starting point for our considerations. However, in contrast to equation (1) we assume that the peer group company is subject to a tax rate amounting to τ_p , whereas (e.g. due

⁴In analogy to the terms „unlevering“ and „relevering“ we speak of „untaxing“ and „retaxing“.

⁵Our results remain valid if even risky debt and several peer group companies are considered. However, these extensions will cause problems (like the assignment of correct weights for each member within the peer group) that we want to rule out.

⁶This financing policy implies deterministic future debt ratios measured in market values.

⁷See *Miles and Ezzell* (1980). For application conditions, the derivation and advantages over the Modigliani-Miller adjustment formula and its variants, see *Kruschwitz/Löffler* (2006), pp. 71–75.

to international tax differentials) a different tax rate τ_c is relevant for the company being valued.

2.2 Adjustment formula with divergent tax rates

In order to determine the unobservable beta of the (levered, non-listed) company the peer group's beta $\beta^{l,p}$ first has to be adjusted for its leverage L_p by using equation (1). The relevant tax rate is now that rate which is imposed on the peer group company, τ_p . By rearranging the Miles-Ezzell-formula beta of an unlevered firm is obtained.

$$\beta^u = \frac{\beta^{l,p}}{\left(1 + \frac{1+r_f(1-\tau_p)}{1+r_f} L_p\right)}. \quad (2)$$

In a second step β^u has to be relevered according to the leverage level of the company being valued, L_b , taking into account its relevant tax rate τ_c .

$$\beta^{l,c} = \beta^u \left(1 + \frac{1+r_f(1-\tau_c)}{1+r_f} L_c\right) \quad (3)$$

Inserting (2) in (3) und rearranging finally yields in

$$\beta^{l,c} = \beta^{l,p} \frac{1+r_f + L_c(1+r_f(1-\tau_c))}{1+r_f + L_p(1+r_f(1-\tau_p))}. \quad (4)$$

With the aid of this equation one can control not only for differing financial risks but also for divergent tax rates in context of adjusting beta. Whenever tax rates as well as debt ratios coincide equation (4) makes obvious that $\beta^{l,c}$ equals $\beta^{l,p}$ and no further adjustments are necessary. However, since this will rarely occur in practice we want to analyze the effects of a disregard of divergent tax rates in the following numerical analysis.

3 Numerical Analysis

In this section we perform a numerical analysis to measure the errors that occur when tax effects are neglected and a unique tax rate is applied instead. In the following, the parameter values taking into consideration re- and untaxing will be denoted as *true* values. They abstain from the simplifying assumption of unique tax rates. The following figures show the differences between these true values and those values assuming an equal tax rate amounting to τ_c , even though τ_p might be different. In this manner we show the extent of a possible distortion caused by disregarding tax differences.

The initial values of the numerical analysis are set as follows.

- The leverage ratio of the peer group (L_p) equals the leverage ratio of the company being valued (L_c), both amounting to $L_p = L_c = 9$. The equality of these values is necessary for separating the tax effects from financing effects.⁸
- In order to determine the firm value we assume a constant free cash flow $E(\widetilde{FCF}) = 1$ in form of a perpetual annuity. This assumption is appropriate because we take into account relative variations only. Furthermore, in a second step, the effects of positive growth rates g will be analyzed.
- Beta of the peer group is set to $\beta^{l,p} = 3$. The risk-free interest rate amounts to $r_f = 10\%$ and the market risk premium to $MRP = 8\%$.
- The tax rate of the company being valued (τ_c) and the tax rate of the peer group (τ_p) are of particular interest for the following analyzes and will be varied.

The distortion caused by the simplifying assumption of unique tax rates impacts relevant parameters of the valuation. However, it may affect the relevant parameters in a different extent and in a different direction. Therefore, the following figures show the distortions regarding the company's beta $\beta^{l,c}$, its costs of equity $k^{l,c}$ as well as the weighted average costs of capital $WACC^c$ and the firm value $V^{l,c}$ separately.

While the formula for beta was derived in the previous section (see equation (4)) the other parameters can be derived as follows. According to the Capital Asset Pricing Model costs of equity capital can be written as

$$k^{l,c} = r_f + MRP \cdot \beta^{l,c} \quad (5)$$

while the weighted average costs of capital results from

$$WACC^c = k^{l,c} \cdot \frac{1}{1 + L_c} + r_f(1 - \tau_c) \cdot \frac{L_c}{1 + L_c}. \quad (6)$$

For the company's market value on the basis of a perpetuity

$$V^{l,c} = \frac{E(\widetilde{FCF})}{WACC^c - g} \quad (7)$$

is applied. These parameters are of particular interest for every corporate valuation and will thus be considered in the following numerical analysis.

⁸We take a given debt policy for granted and we will not discuss the question of which leverage ratio maximizes the value of the levered company.

First, we focus on the impact of tax rate differentials. We assume a tax rate of the company being valued amounting to $\tau_c=50\%$ and vary the peer group's tax rate on an interval between 0% to 100% on the x-coordinate. This assumptions allows us to show a tax rate difference in a wide rage between -50%, to +50%.

Figure 1: Variation of Tax Rate Differences ($\tau_c = 50\%$)

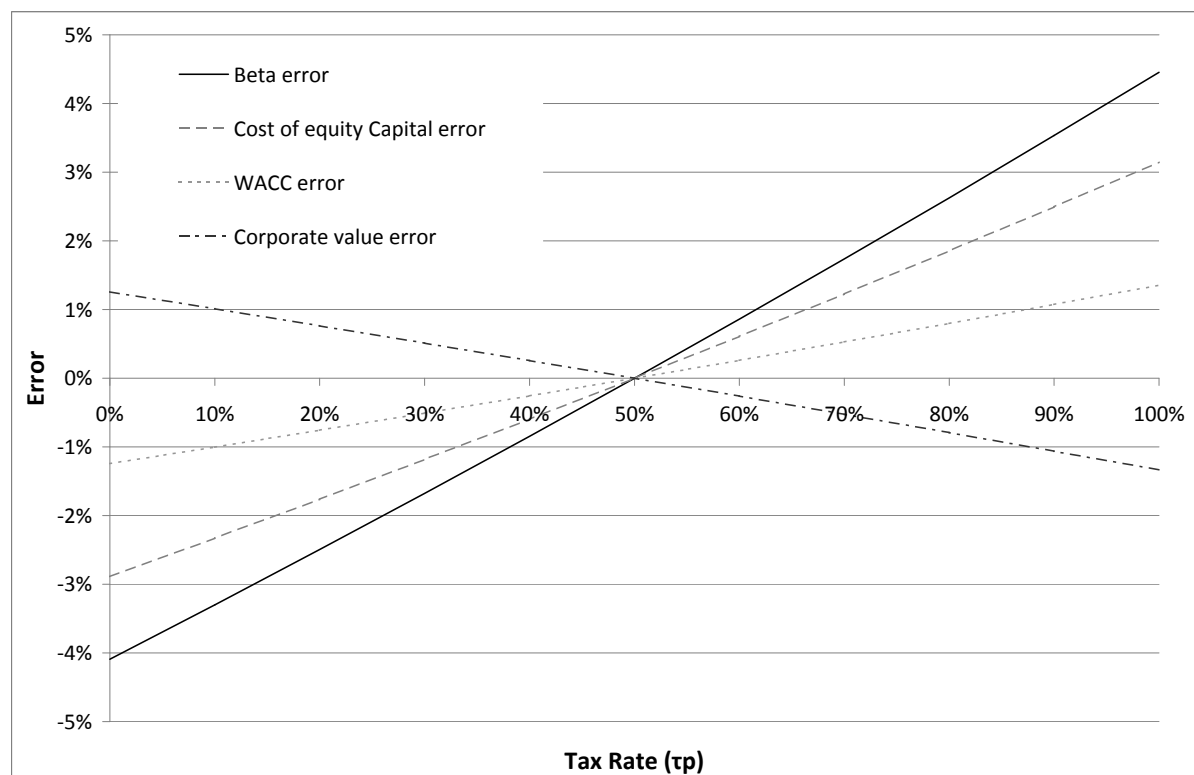


Figure 1 shows relevant distortions caused by a disregard of un- and retaxing depending on tax rate differences. A positive value means that the true value is higher than the simplified value. With increasing tax rate differences and $\tau_p > \tau_c$ beta, equity costs of capital as well as *WACC* are underestimated. For $\tau_p < \tau_c$ we find a contrary effect. Based on these distortions an over-/underestimation of the firm value results.

In a second step we show the impact of leverage on the valuation errors caused by the assumption of unique tax rates. In this case, we assume a constant tax rate difference of 50% with $\tau_p < \tau_c$. Furthermore, we vary the leverage ratio of the peer group company (L_p) on an interval between 0 to 9. The leverage ratio of the company being valued is constant

and set to $L_c = 4.5$. In this manner we get results for positive and negative deviations.

Figure 2: Variation of Differences in Leverage Ratio ($L_c = 4.5$)

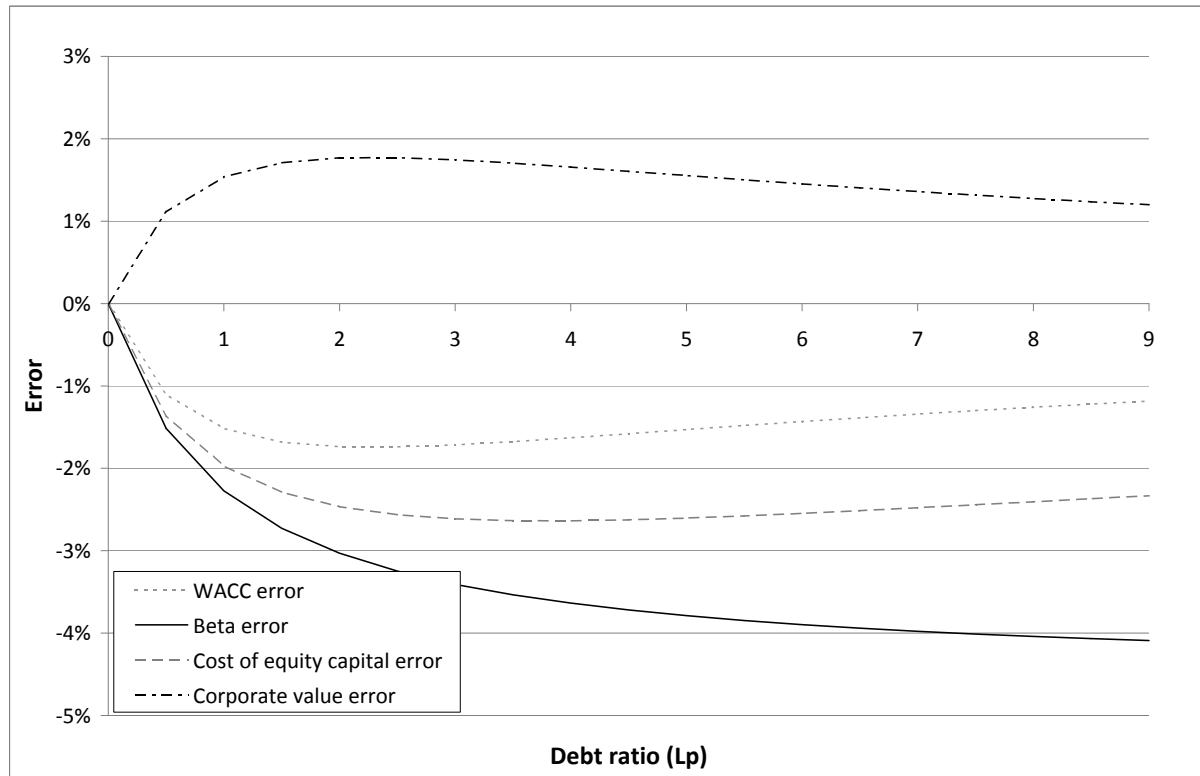
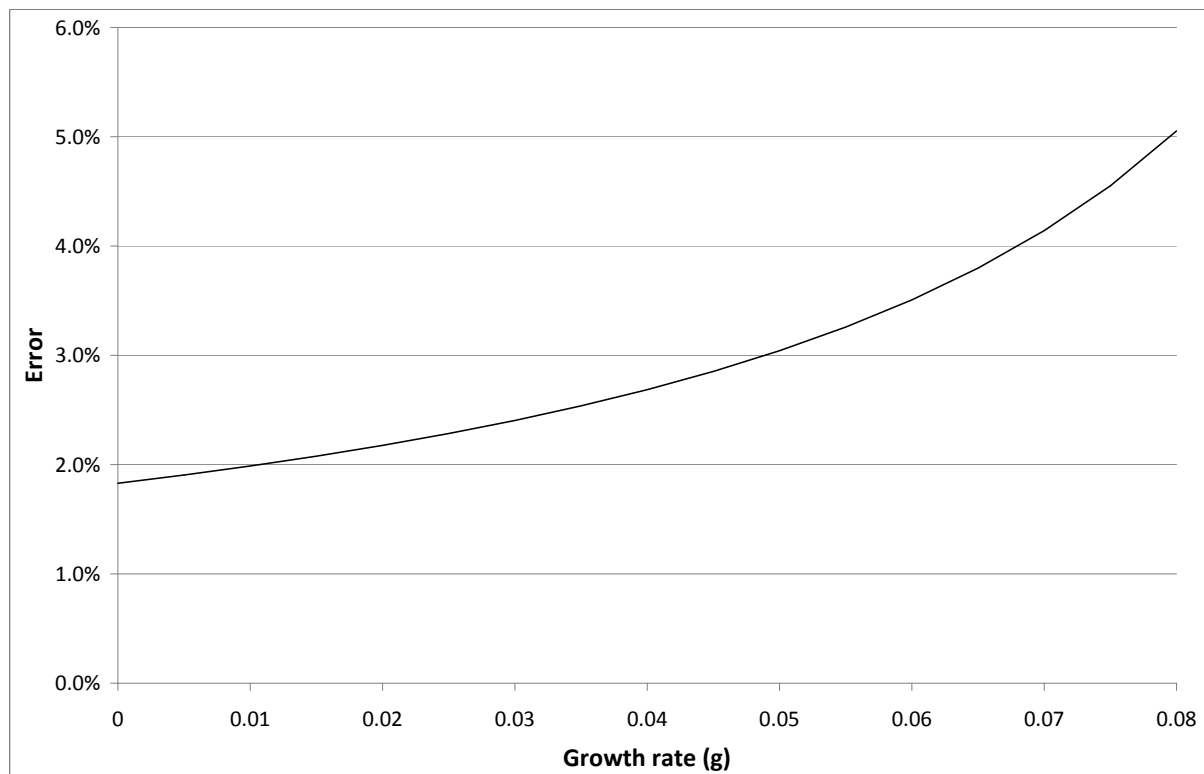


Figure 2 shows overestimation on the whole interval for beta, *WACC* and the costs of equity. In contrast, firm value is underestimated by neglecting re- and untaxing effects. Furthermore, the figure makes obvious that *WACC* as well as the equity costs of capital are not a monotonous decreasing function in leverage difference. For a peer group leverage ratio of 2.27 (3.78) we find a minimum for *WACC* (equity costs). As expected, the firm value function has a maximum analogous to the *WACC* minimum. The explanation for these extreme value is based on the opposing effects between un- and relevering on the one hand and un- and retaxing on the other hand. In the zero origin there is no difference between the parameters because the peer group did not raise any debt and leverage ratio is zero. This leads to equity costs of capital and beta of an unlevered firm which makes unlevering and relevering redundant. Raising leverage L_p increases the unlevering effect which is additionally strengthened by the untaxing effect. When continuing the increase of leverage ratio, the relative tax rate effect decreases because of the peer group's leverage

ratio effect. See equation (2) for this contrary effects. These opposite effects are causal for the minimum (maximum) values of the functions. Figure 2 shows that distortions for the firm value amount to approximately 2% when an exemplary tax rate difference of 50% is applied.

The assumption of constant free cash flows is not very realistic. For that reason we take into account positive growth rates in a last step and show how they affect the extent of our results concerning firm value. For the numerical analysis we again assume a tax rate difference of 50% with $\tau_p < \tau_c$. The peer group leverage ratio is set to its above calculated maximum value of $L_p = 2.27$, while L_c amounts to 9. We vary the growth rate in figure 3 on an interval between 0% and 8%.

Figure 3: Variation of Growth Rate



In this case, the true firm value is higher than the simplified value. This distortion increases with growth in cash flows. For example, a growth rate of 8% leads to a higher true firm value of 5.05%⁹ This simple numerical analysis shows that the disregard of

⁹For a tax rate difference of 50% and the assumption that $\tau_p > \tau_c$, we find an overestimation in absence

untaxing and retaxing leads to relevant mispricings in corporate valuation. Particularly, the extent of the distortions depends on differences in the actual tax rate and leverage ratios as well as on the growth of cash flows.

4 Conclusion

The problem of untaxing and retaxing arises whenever a peer group company is chosen that underlies a tax regime that differs from the regime being relevant for the company that is to be valued. Due to high tax differentials between but also within countries that problem is likely to occur and makes an adjustment of the peer group beta for divergent financial risk and for differing tax regimes inevitable. Insofar it is surprising that divergent corporate tax rates in the standard procedure of unlevering and relevering beta have not been addressed in literature before, implying the unrealistic assumption of a unique tax rate being relevant.

This article provides evidence that significant mispricings can occur when companies are valued based on peer group betas that are not adjusted for divergent tax rates. Consequently, such a disregard of tax effects can distort investors' decision making. In particular, we show that for relatively high tax rates of the peer group company ($\tau_c < \tau_p$) beta, the cost of equity as well as the weighted costs of capital are underestimated, whereas comparatively low tax rates ($\tau_c > \tau_p$) lead to an overestimation of these parameters. This in turn results in incorrect firm values of substantial extent depending on the underlying parameter values. Moreover, the numerical analysis demonstrates amplifying effects: The difference between the company's and peer group's debt ratios as well as the growth rate of future cash flows reinforces the extent of the distortions caused by a disregard of untaxing and retaxing. Particularly with regard to strongly growing environments with high growth rates the valuation error is substantial and reaches up to 5 % based on the numbers of our numerical example.

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